Microscopic aspects of γ-softness in atomic nuclei

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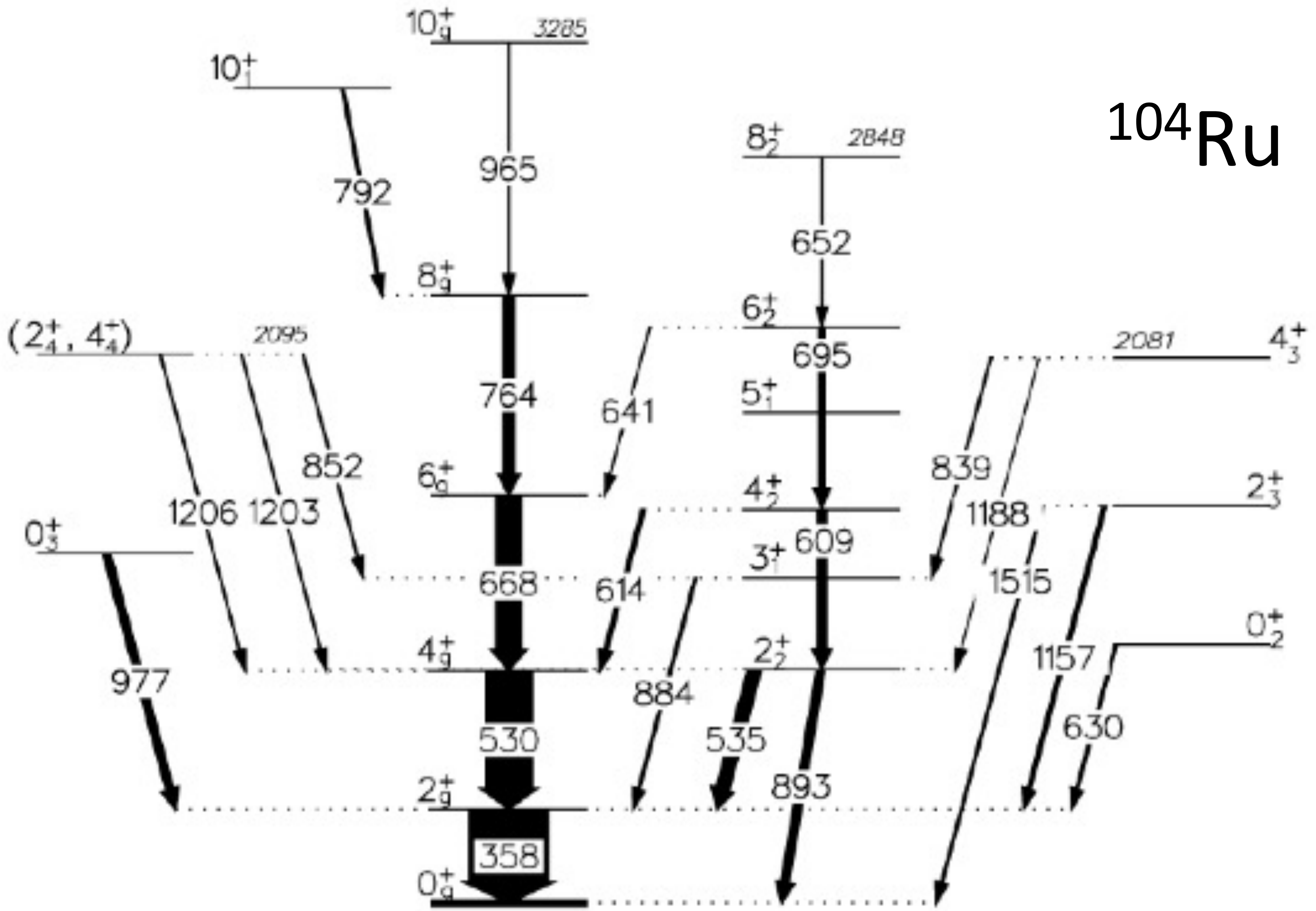
^{104}Ru

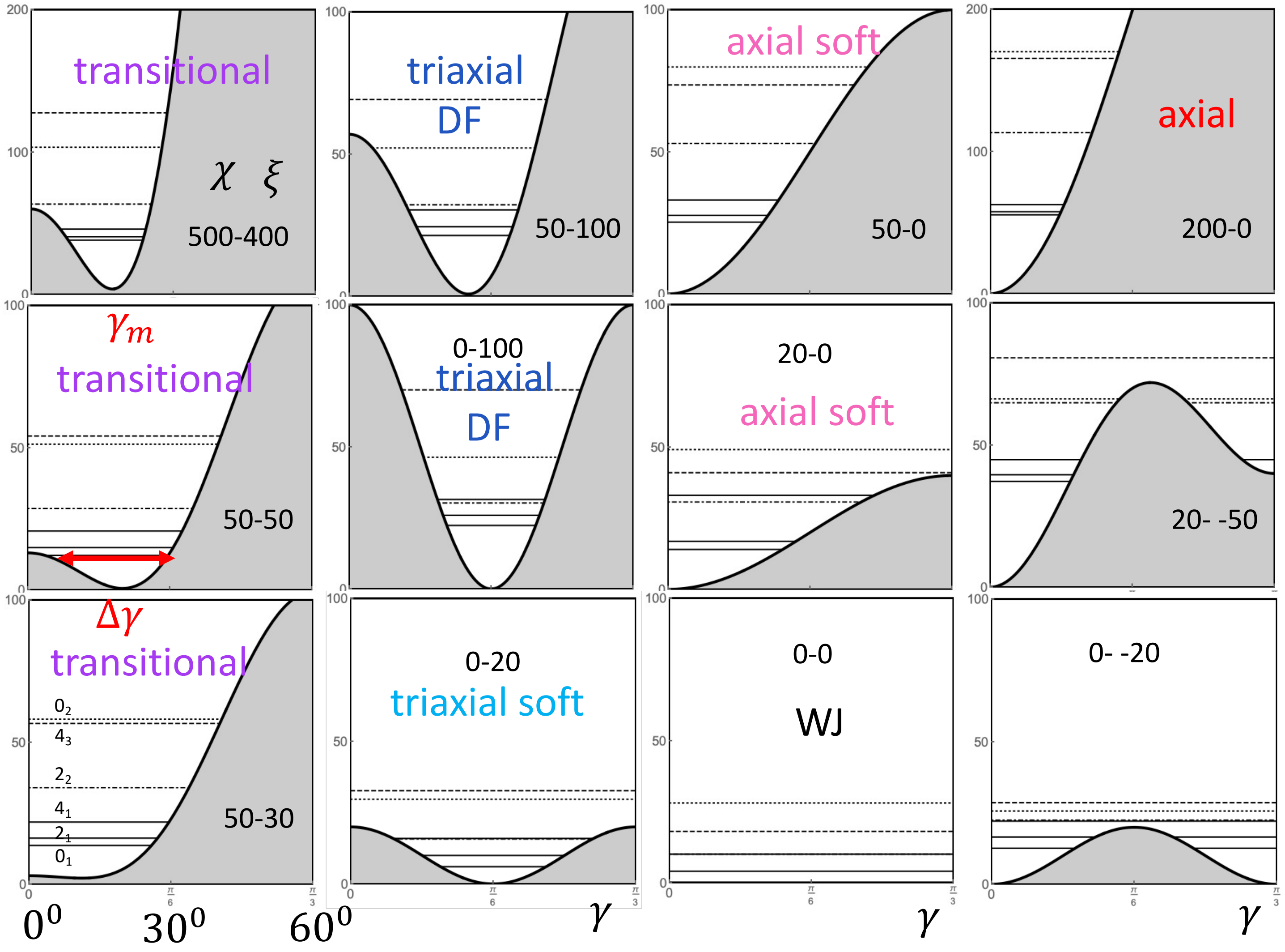
How to quantify triaxiality and γ softness?



Example for a γ soft nucleus, many E2 matrix elements by COULEX

Phenomenology by Bohr Hamiltonian: $\Lambda^2 - \chi cos3\gamma + \xi cos^2 3\gamma$

$$\hat{\Lambda}^2 = -\left(\frac{1}{\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4}\sum_{i=1,2,3}\frac{\hat{L}_i'^2}{\sin^2(\gamma - \frac{2}{3}\pi i)}\right)$$ scaled kinetic energy



Observables characterizing triaxiality/softness

Staggering of the γ band energies

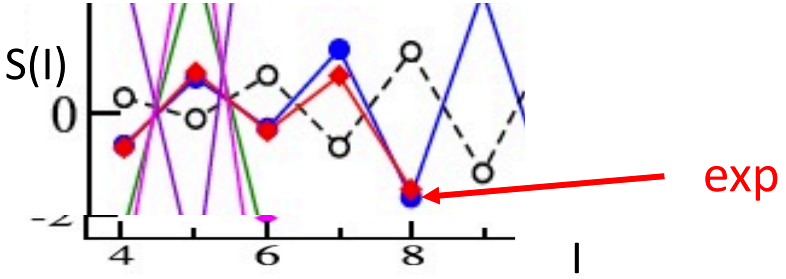
$$S(I) = \frac{[E(I)-E(I-1)]-[E(I-1)-E(I-2)]}{E(2_1^+)}, \quad \bar{S}(I) = \frac{S(I)-S(I+1)}{2}.$$

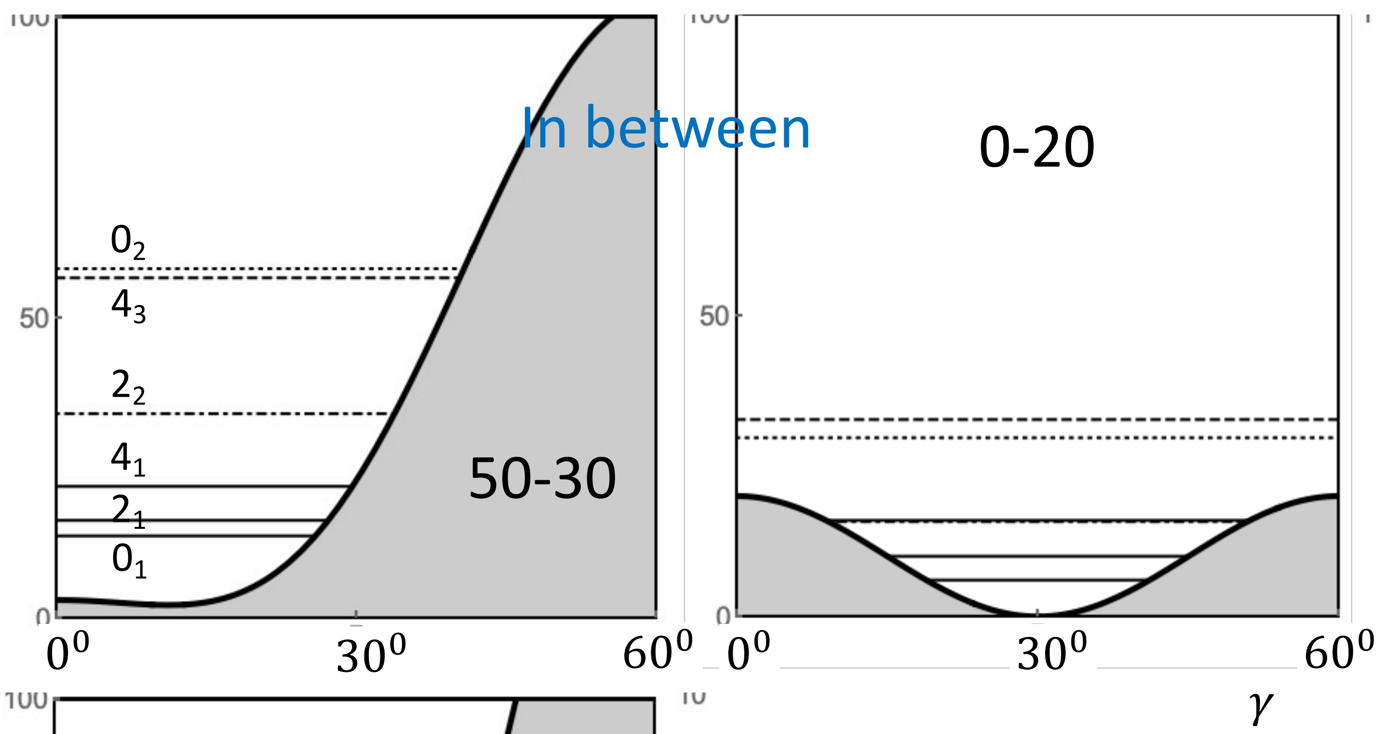
$$\left[\frac{E(2_2^+)}{E(2_1^+)}\right], \left[\frac{E(2_2^+)}{E(4_1^+)}\right] \quad Q(2_1^+) \; Q(2_2^+) \; B(E2, 2_2^+ \to 0_1^+)$$

$\chi-\kappa$	$E(2_1^+)_{GR}$	γ_m	$\Delta\gamma$	$\left[\frac{E(2_2^+)}{E(2_1^+)}\right]$	$\left[\frac{E(2_2^+)}{E(4_1^+)}\right]$	$\bar{S}(6)$	$Q(2_1^+)$	$Q(2_2^+)$	$B(E2,2_2^+\to 0_1^+)$
200-0	2.15	0	14	26.8	8.07	-0.03	-0.888	0.873	0.033
100-0	2.24	0	17	18.0	3.31	-0.14	-0.878	0.859	0.047
50-0	2.39	0	20	11.6	3.55	-0.51	-0.861	0.840	0.064
20-0	2.87	0	24	5.82	1.93	-1.75	-0.797	0.789	0.079
10-0	3.42	0	27	3.61	1.32	-2.49	-0.656	0.655	0.060
0-0	4.00	30	60	2.50	1.00	-2.75	0.000	0.000	0.000
0-200	3.05	30	16	2.11	0.81	3.87	0.000	0.000	0.000
0-100	3.57	30	19	2.21	0.86	3.12	0.000	0.000	0.000
0-50	3.80	30	21	2.34	0.92	1.61	0.000	0.000	0.000
0-20	3.95	30	22	2.45	0.98	0.50	0.000	0.000	0.000
50-100	3.10	25	19	3.50	1.20	1.85	-0.693	0.684	0.084
500-400	2.34	17	15	10.8	3.17	0.32	-0.861	0.851	0.066
50-50	2.73	20	26	6.01	1.92	0.30	-0.807	0.789	0.091
50-30	2.58	11	26	7.78	2.44	-0.35	-0.835	0.815	0.082
0- -20	3.39	30	35	2.42	0.96	-5.79	0.000	0.000	0.000
20- -50	2.35	34	17	11.74	3.62	-5.55	-0.868	0.874	0.032

^{104}Ru is a triaxial γ soft nucleus

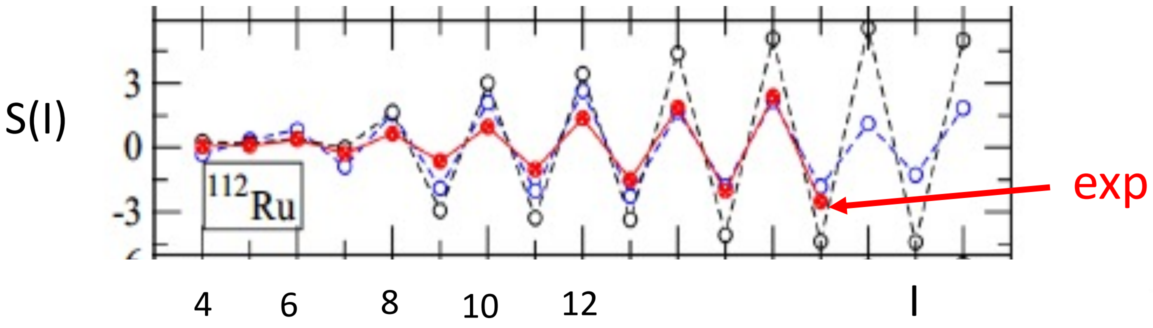
$\bar{S}(6)$=-0.15, $\left[\frac{E(2_2^+)}{E(2_1^+)}\right]$=2.5 $\left[\frac{E(2_2^+)}{E(4_1^+)}\right]$=1.0

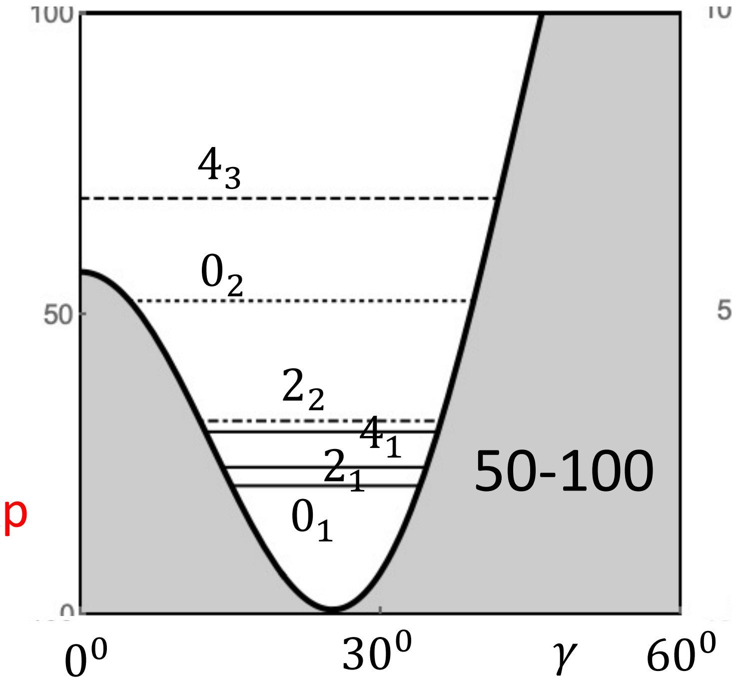




^{112}Ru is a triaxial γ "rigid" nucleus

$\bar{S}(6)$=0.31, $\left[\frac{E(2_2^+)}{E(2_1^+)}\right]$=2.2 $\left[\frac{E(2_2^+)}{E(4_1^+)}\right]$=0.8





Microscopic descriptions of γ soft nuclei

5DBH: Microscopic Bohr Hamiltonian based on ATDHF/ATDEDF

Generator Coordinate Method + Angular momentum projection

Spherical (Monte Carlo) Shell Model

Triaxial Projected Shell Model

Triaxial Projected Shell Model

For even-even systems, the TPSM basis space is composed of projected 0-qp state (or qp-vacuum $|\Phi>$), 2-proton, 2-neutron, and 4-qp configurations, ...

$$\hat{P}^I_{MK}\,|\Phi>;$$
$$\hat{P}^I_{MK}\,a^\dagger_{p1}a^\dagger_{p2}\,|\Phi>;$$
$$\hat{P}^I_{MK}\,a^\dagger_{n1}a^\dagger_{n2}\,|\Phi>;$$
$$\hat{P}^I_{MK}\,a^\dagger_{p1}a^\dagger_{p2}a^\dagger_{n1}a^\dagger_{n2}\,|\Phi>,\quad \ldots$$

where the three-dimensional angular-momentum operator is given by

$$\hat{P}^I_{MK} = \frac{2I+1}{8\pi^2}\int d\Omega\, D^I_{MK}(\Omega)\,\hat{R}(\Omega),$$

Odd-A

$$\hat{P}^I_{MK}a^\dagger_{\pi_1}|\Phi\rangle,$$
$$\hat{P}^I_{MK}a^\dagger_{\pi_1}a^\dagger_{\nu_1}a^\dagger_{\nu_2}|\Phi\rangle,$$
$$\hat{P}^I_{MK}a^\dagger_{\pi_1}a^\dagger_{\pi_2}a^\dagger_{\pi_3}|\Phi\rangle,$$
$$\hat{P}^I_{MK}a^\dagger_{\pi_1}a^\dagger_{\pi_2}a^\dagger_{\pi_3}a^\dagger_{\nu_1}a^\dagger_{\nu_2}|\Phi\rangle \;\ldots$$

The vacuum $|\Phi>$ is the BCS ground state with the deformations ε and γ as model parameters.
The Hamiltonian is of the paring + quadrupole-quadrupole type.

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi\sum_\mu \hat{Q}^\dagger_\mu\hat{Q}_\mu - G_M\hat{P}^\dagger\hat{P} - G_Q\sum_\mu\hat{P}^\dagger_\mu\hat{P}_\mu,$$

The coupling constants are determined by the self-consistency conditions

$$\Delta = G_M < P >, \qquad G_Q = 0.16G_M, \qquad 0.66\hbar\omega_0\varepsilon = \chi < Q\_0 > .$$

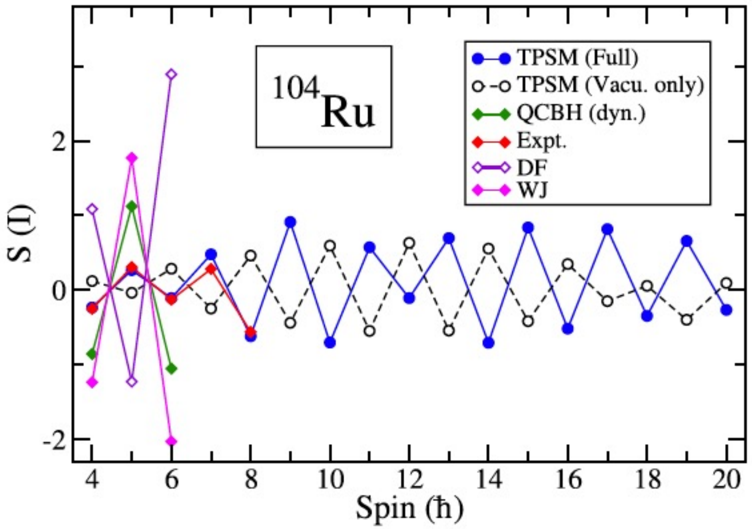
Δ is adjusted to the even-odd mass differences.
ε is taken from $B(E2, 2^+_1 \to 0^+_1)$systematics or mean field equilibrium deformations.
γ is adjusted to reproduce the γ band head energy $E(2^+_2)$.

Step 1:
The triaxial angular momentum projected basis incorporates the correlations that generate 3D rotational behavior.
Step 2:
Diagonalization takes care of the details, as small- vs. large- scale shape fluctuations.
Few parameters:
$B(E2, 2^+_1$->$0^+_1)$ fixes ε
$E(2^+_2)$ fixes γ.
Hamiltonian couplings constants from self-consistency.

Results for ^{104}Ru published in

PHYSICAL REVIEW C 107, L021303 (2023)

Energies very well reproduced



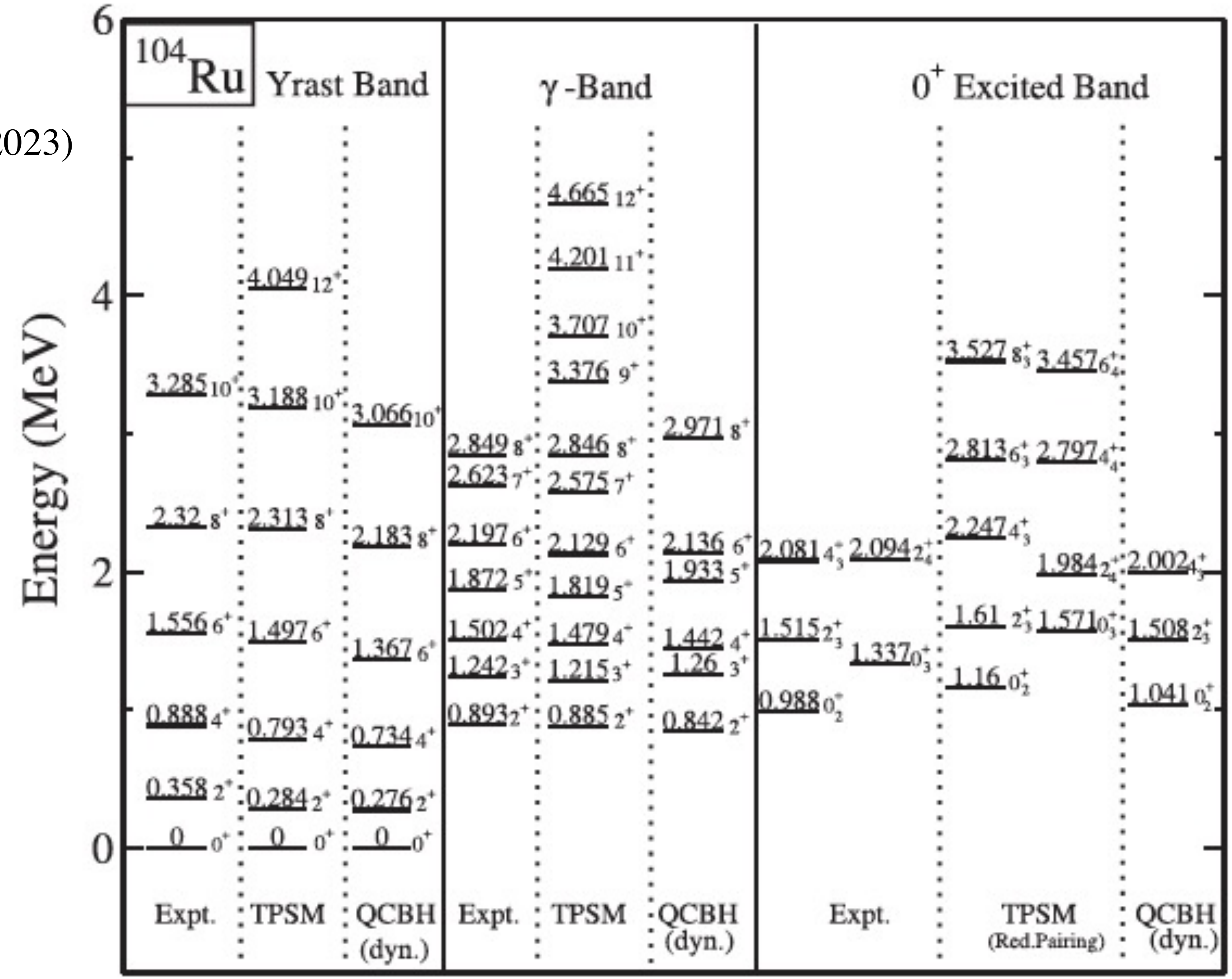
TPSM calculation:

$\varepsilon = 0.258$ adjusted to $B(E2, 2_1^+ \rightarrow 0_1^+)$

$\gamma = 30.2^0$ adjusted to $E(2_2^+)$

G_M reduced by factor of 0.5

for 0^+ excited bands



Quantum Bohr Hamiltonian calculation (QCBH) by Version of 5DBH based on micro-macro DFT.

K. Zając, L. Próchniak, K. Pomorski, S. Rohoziński, and J. Srebrny, Nucl. Phys. A 653, 71 (1999).

TABLE I. Comparison of all known experimental reduced $E2$ diagonal, in-band, and interband matrix elements $\langle I_i \| E2 \| I_f \rangle (e\,\mathrm{b})$, (associated errors are in parentheses) and calculated ones for yrast and γ bands of ^{104}Ru.

$I_i \to I_f$	Expt.	TPSM (Full)	TPSM (Vacu.)	$I_i \to I_f$	Expt.	TPSM (Full)	TPSM (Vacu.)
$2_1 \to 2_1$	−0.71(*11*)	−0.817	−0.634	$4_2 \to 3_1$	±0.68 (*5*)	−0.787	−0.597
$4_1 \to 4_1$	−0.79(*15*)	−0.906	−0.437	$5_1 \to 3_1$	1.22(*4*)	1.184	0.697
$6_1 \to 6_1$	$-0.70(^{+30}_{-20})$	−0.868	−0.342	$6_2 \to 4_2$	1.52 (*12*)	1.521	0.682
$8_1 \to 8_1$	$-0.6(^{+3}_{-5})$	−0.855	−0.297	$8_2 \to 6_2$	2.02(*4*)	2.056	0.747
$2_2 \to 2_2$	0.62(*8*)	0.648	0.633	$2_2 \to 0_1$	−0.156 (*2*)	−0.141	−0.225
$4_2 \to 4_2$	−0.58(*18*)	−0.749	−0.534	$2_2 \to 2_1$	−0.75(*4*)	−0.722	−0.612
$6_2 \to 6_2$	±1.0(*3*)	−1.105	−0.763	$2_2 \to 4_1$	ϵ [−0.1, 0.1]	−0.090	−0.001
$2_1 \to 0_1$	0.917 (*25*)	0.973	0.901	$3_1 \to 2_1$	0.22(*10*)	0.254	0.302
$4_1 \to 2_1$	1.43 (*4*)	1.591	1.456	$3_1 \to 4_1$	−0.57	−0.517	−0.559
$6_1 \to 4_1$	2.04 (*8*)	2.081	1.830	$4_2 \to 2_1$	−0.107 (*8*)	−0.113	−0.054
$8_1 \to 6_1$	$2.59\ (^{+24}_{-9})$	2.486	1.902	$4_2 \to 4_1$	−0.83(*5*)	−0.840	−0.505
$10_1 \to 8_1$	2.7 (*6*)	2.668	1.623	$6_2 \to 4_1$	$-0.22(^{+6}_{-12})$	−0.230	−0.682
$3_1 \to 2_2$	−1.22 (*10*)	−1.241	−0.935	$6_2 \to 6_1$	>−0.84	−0.947	−0.411
$4_2 \to 2_2$	1.12(*5*)	1.095	0.510				

The TPSM very well reproduces the individual COULEX E2 matrix elements.

TABLE III. Comparison of all known experimental reduced $M1$ matrix elements $\langle I_i \| M1 \| I_f \rangle (\mu_N)$, in-band and interband values (associated errors in parentheses), and calculated ones for ^{104}Ru.

$I_i \to I_f$	Expt.	TPSM	$I_i \to I_f$	Expt.	TPSM
$2_1 \to 3_1$	$-0.054(^{-9}_{+9})$	−0.044	$2_1 \to 2_1$	0.82(*10*)	0.791
$2_1 \to 2_2$	<0.02	−0.038	$4_1 \to 4_2$	$-0.15(^{-3}_{+3})$	−0.136

Experiment: COULEX

K. Zając, L. Próchniak, K. Pomorski, S. Rohoziński, and J. Srebrny, Nucl. Phys. A **653**, 71 (1999).

TABLE II. Comparison of all known experimental reduced $E2$ matrix elements $\langle I_i \| E2 \| I_f \rangle (e.b)$, diagonal, in-band, and interband values (associated errors are in parentheses), and calculated ones for excited 0^+ bands of ^{104}Ru.

$I_i \to I_f$	Expt.	TPSM	$I_i \to I_f$	Expt.	TPSM
$2_3 \to 0_2$	0.71(*4*)	0.682	$2_3 \to 4_1$	−0.370(*4*)	−0.311
$4_3 \to 2_3$	0.75(*25*)	0.613	$2_3 \to 2_2$	$\pm 0.22(^{+25}_{-5})$	−0.237
$0_2 \to 2_1$	−0.266(*8*)	−0.221	$2_3 \to 4_2$	$0.31(^{+13}_{-6})$	0.221
$0_2 \to 2_2$	0.08 (*3*)	0.099	$2_3 \to 4_4$	$0.53(^{+32}_{-14})$	0.481
$2_3 \to 0_1$	−0.071(*3*)	−0.048	$0_3 \to 2_1$	>−0.1	−0.201
$2_3 \to 2_1$	±0.07(*3*)	−0.031	$2_3 \to 2_3$	$-0.08(^{11}_{25})$	−0.631

E2 matrix elements from life time measurements by Esmaylzadeh et al. PRC 106, 064323 (2022) (see his presentation at this conference) agree within experimental uncertainties with the tabulated COULEX values.

The quadrupole shape invariants

D. Cline, Annu. Rev. Nucl. Part. Sci. **36**, 683 (1986).
K. Kumar, Phys. Rev. Lett. **28**, 249 (1972).

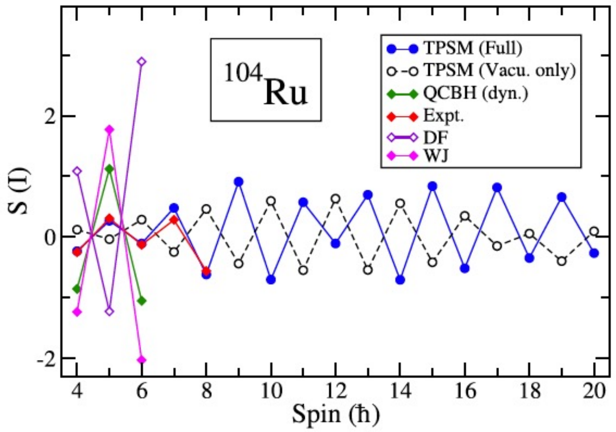
Method to extract intrinsic quadrupole moments from matrix elements in the lab. frame.
They provide a comprehensive characterization of the triaxiality.
The mean value $< cos3\delta >$ indicates the amount of triaxiality.
The dispersion σ $< cos3\delta >$ indicates the fluctuations around the mean value, i.e. how soft the shape is.

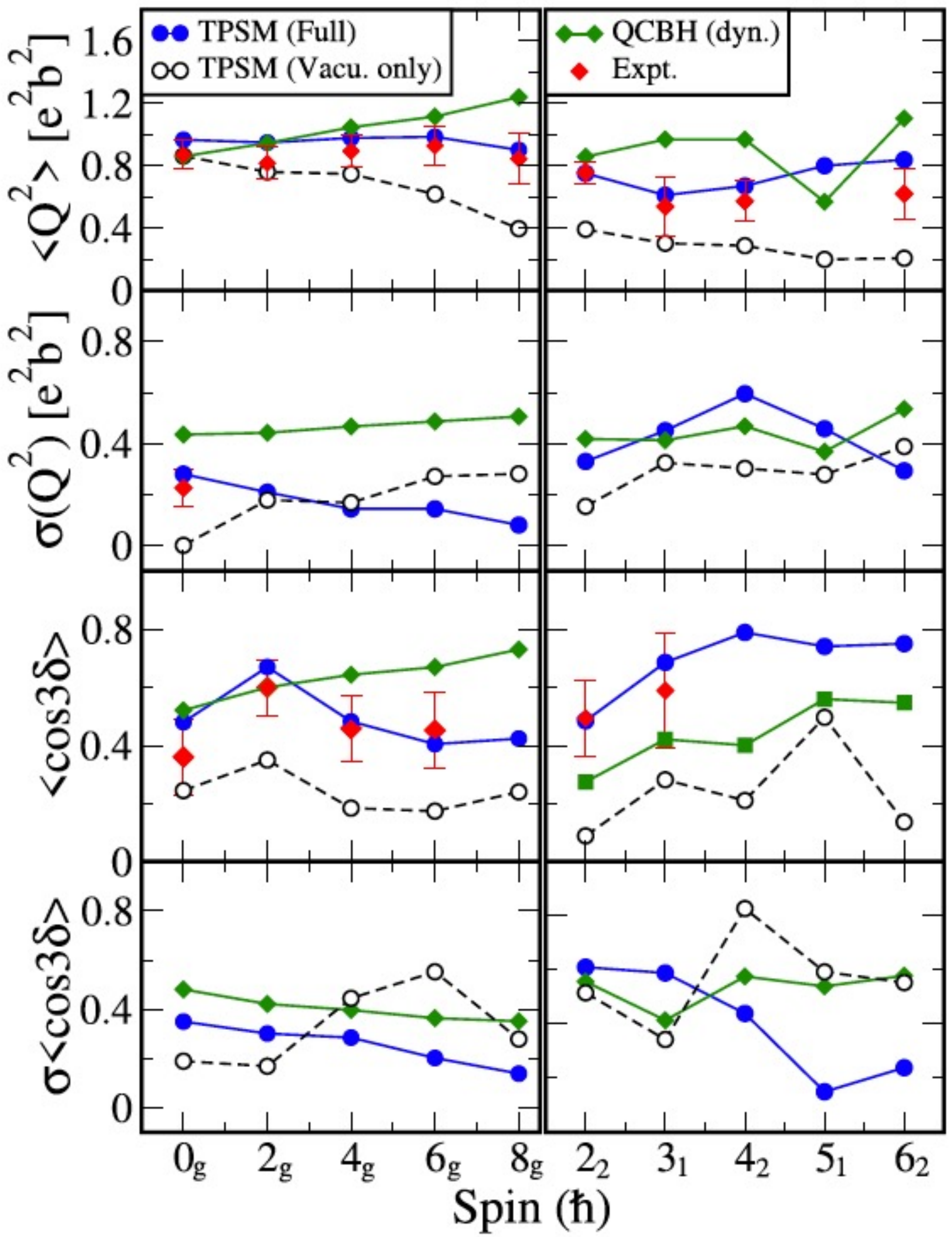
The γ soft nucleus ^{104}Ru

The experimental values are very well reproduced.

The value $< cos3\delta > \approx 0.6$ corresponds to $\delta \approx 18^0$.
The dispersion σ $< cos3\delta > \approx 0.4$ corresponds to fluctuations within the wide range $9^0 < \delta < 24^0$.
The microscopic Bohr Hamiltonian states (QCBH) give comparable results, which account less well for the experiment.
Results are consistent with the phenomenology.

TPSM without quasiparticle admixtures (-----) gives soft triaxiality $\delta \approx 27^0$, $\bar{S}(6)$=0.13.
Two-quasiparticle admixtures flip the staggering phase to γ soft.





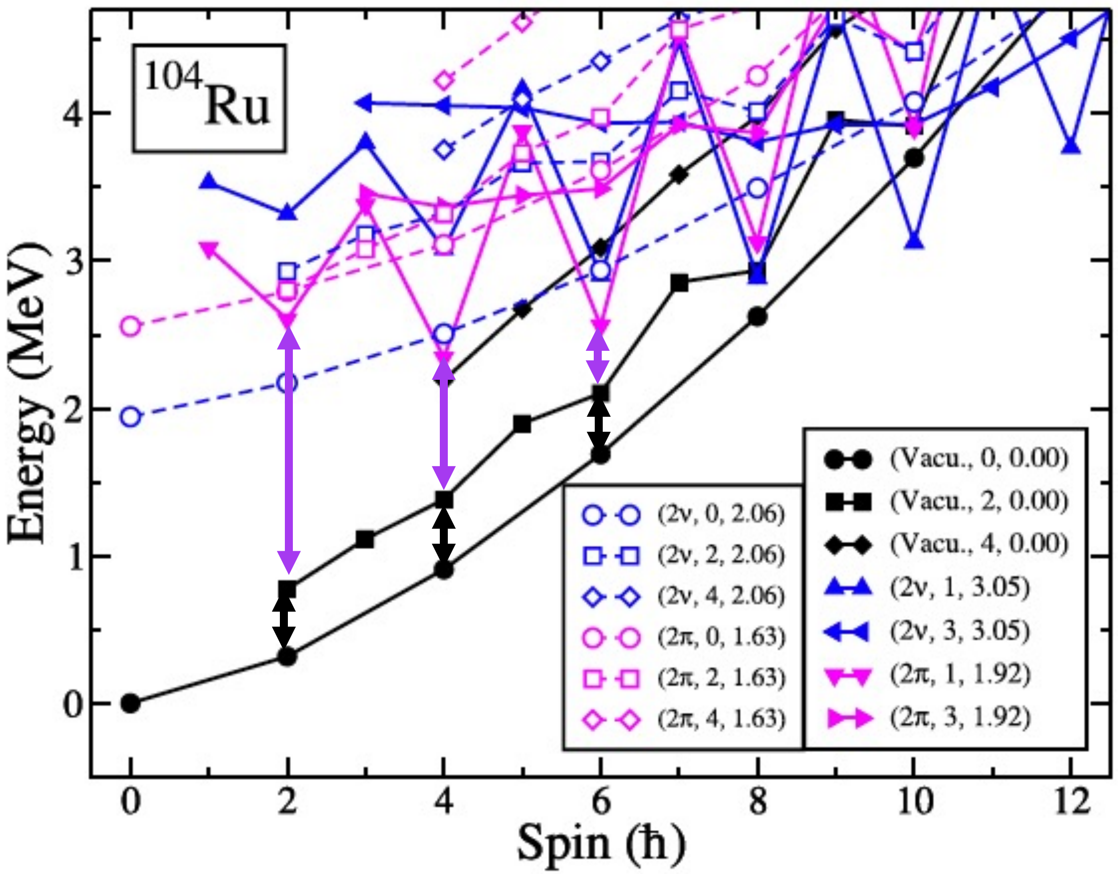
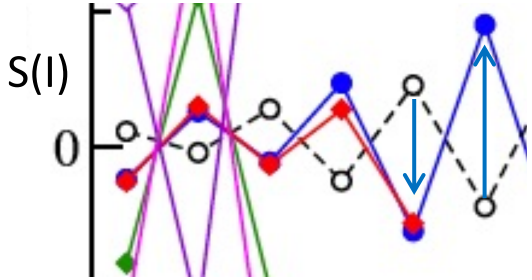


FIG. 4. $\boxed{\text{TPSM projected energies before band mixing.}}$ The bands are labeled by three quantities: quasiparticle character, K quantum number, and energy of the two-quasiparticle state. For instance, $(2\nu, 1, 3.05)$ designates the $K = 1$ state projected from the $h_{11/2}$ two-quasineutron configuration with the energy of 3.05 MeV. The $K = 0$, 2, 4 states projected from the quasiparticle vacuum are labeled with Vacu. The four-quasiparticle states lie above 5 MeV.

Softness by band mixing

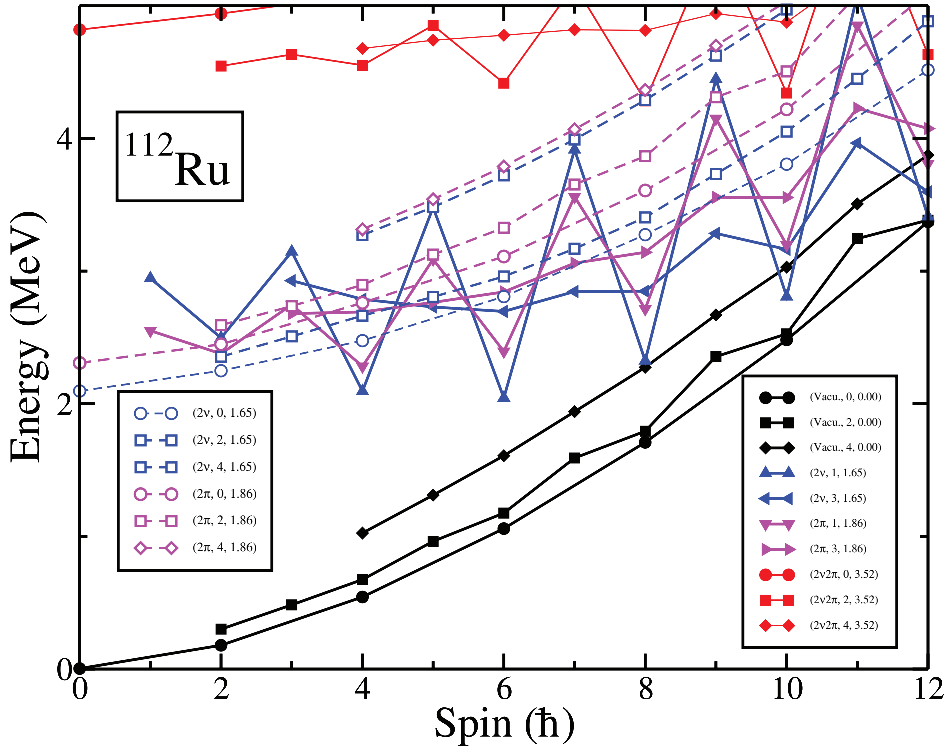
Mixing of the projected vacuum states pushes only the even I states of the K=2 (γ) band up (no repulsion for odd I)
→ Triaxial rotor pattern: even-I-up

Admixing of the lowest projected two quasiparticle states
pushes the even I states of the K=2 (γ) band down (less repulsion for odd I)
If prevails
→ Triaxial rotor pattern changes to γ soft pattern : even-I-down

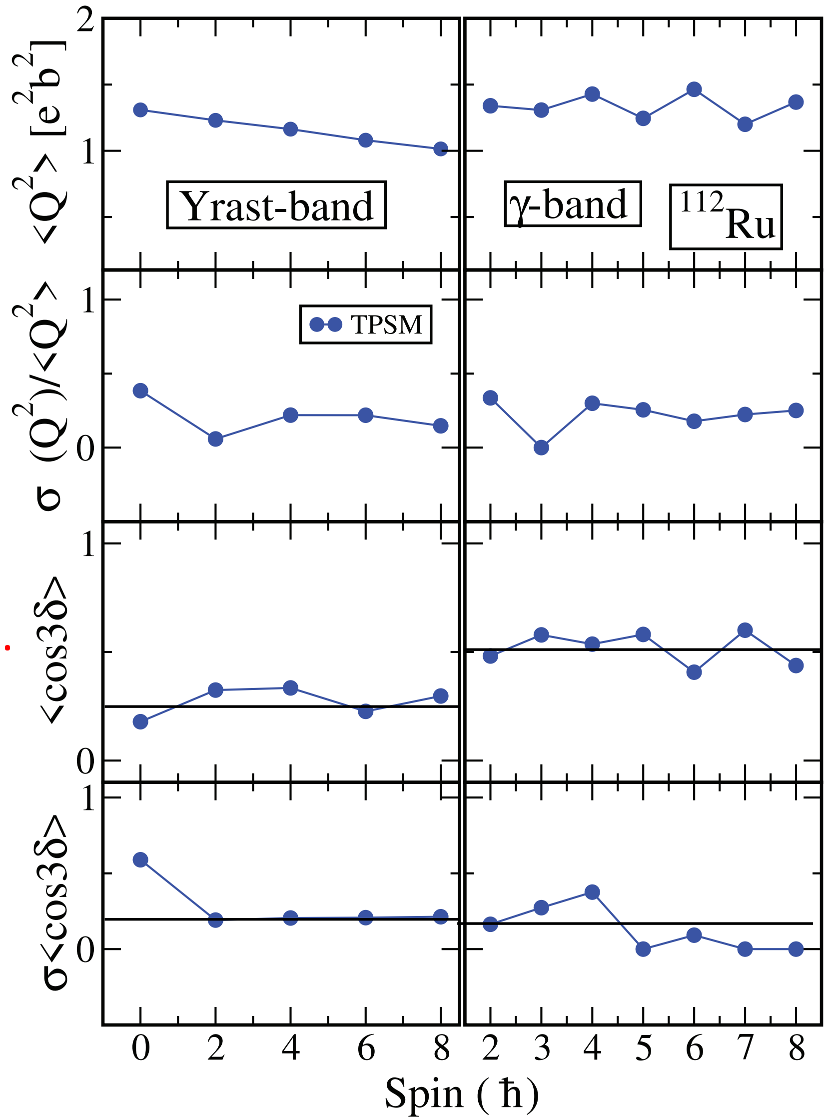


S(I) flips

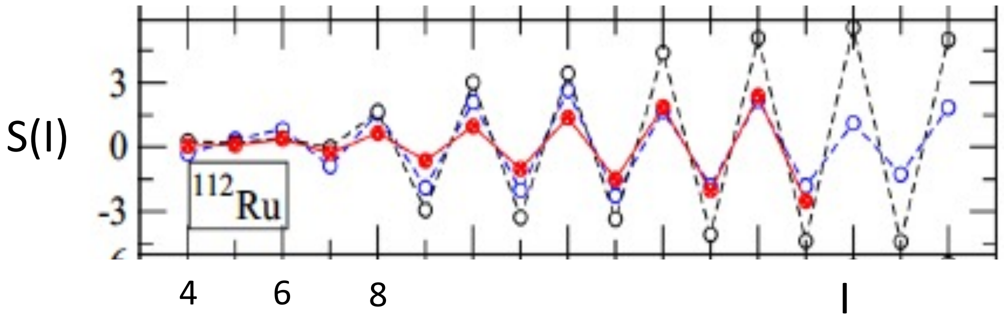
The γ “rigid” nucleus nucleus ^{112}Ru



$\delta_{yrast} = 26^0 \pm 4^0$
$\delta_{\gamma\ band} = 30^0 \pm 3^0$
Small fluctuations around the mean value.



Strong up push by ground band, $S_{vacuum}(6)=0.9$
prevails over down push by two quasiparticle bands $S(6)=0.8$



TPSM STUDIES OF THE COLLABORATION

Systematic study of 30 nuclei with good experimental data

Staggering of the γ bands: Eur. Phys. J. A (2021) 57, 308

E2 transition probabilities: Eur. Phys. J. A (2023), submitted, arxiv: 2370.06670

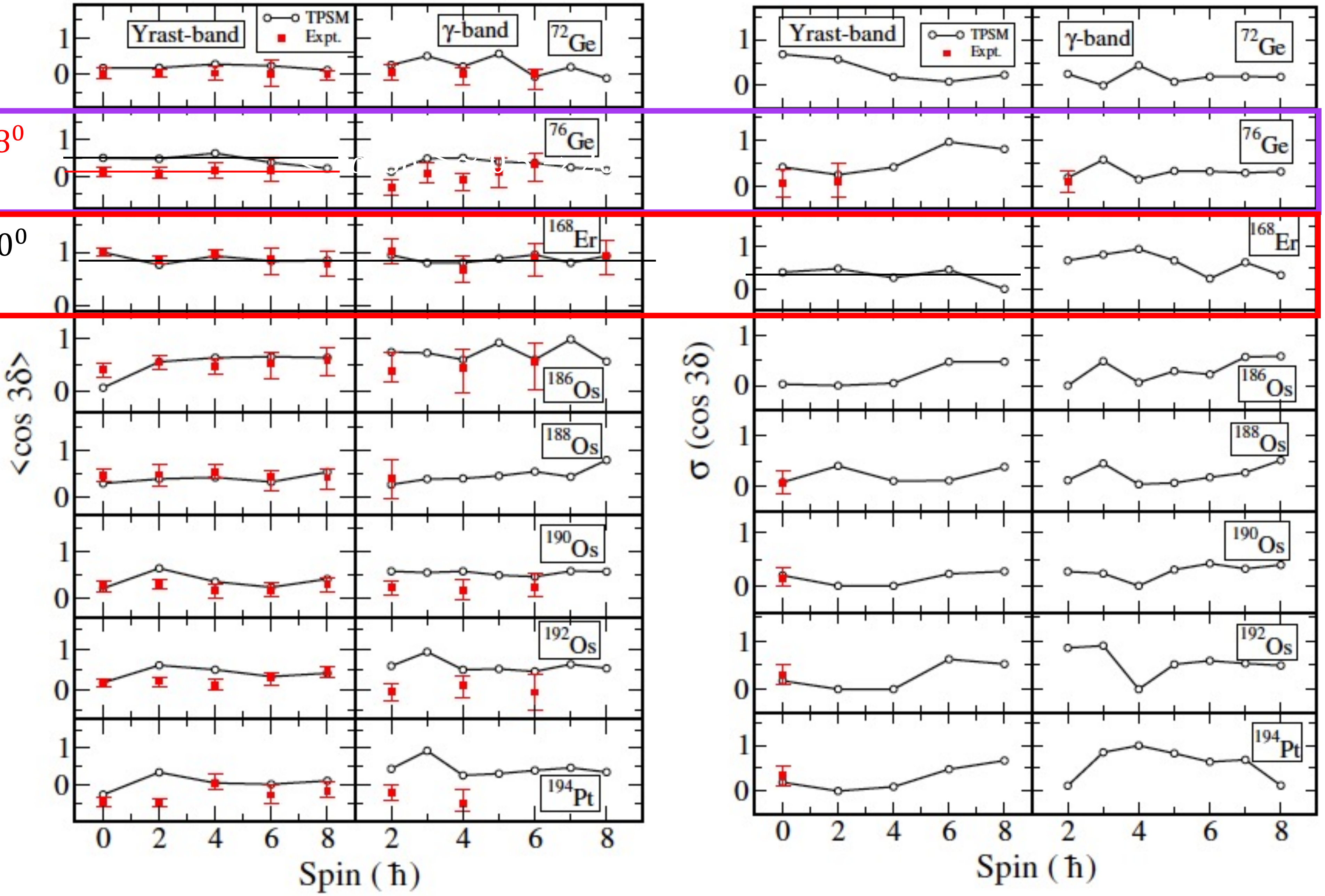
Ongoing: Shape invariants from TPSM compared to COULEX experiment

$< cos3\delta > \approx 0.1 \rightarrow \delta \approx 28^0$

$< cos3\delta > \approx 0.5 \rightarrow 20^0$

$< cos3\delta > \approx 0.85 \rightarrow \delta \approx 10^0$

substantial fluctuations



Quadrupole Shape invariants

To be published Phys. Rev. C

Summary by

Editor’s suggestion   Letter

Phys. Rev. C **107**, L021303

-TPSM: efficient many body method, which includes major correlations in triaxial mean field,
and treat details by quasiparticle excitations.

Improving/extending the TPSM:
-band diagrams for rotating quasiparticles
-SCS maps for TPSM
-fixing the P+QQ Hamiltonian parameters from EDFs
-individual minimalization of the deformation parameters of the quasiparticle configurations.