A novel projected shell model method for nuclear level density

Yang Sun

Toward complete set of nuclear states

- Nuclear level density (NLD), is defined as the number of nuclear levels per unit energy interval

$$\rho(E) = \Delta N / \Delta E$$

- To obtain the levels, one should solve the **many-body** eigenvalue equation

$$H |\Psi\rangle = E |\Psi\rangle$$

and count **all** energy levels in the Hilbert space.

- However, this is only possible in a strongly truncated space for lighter nuclei, or for nuclei in the vicinity of shell closures.

V. Zelevinsky, B. A. Brown, N. Frazier, and M. Horoi, Phys. Rep. 276, 85 (1996)
N. Shimizu et al., Phys. Lett. B 753, 13 (2016)
W. E. Ormand and B. A. Brown, Phys. Rev. C 102, 014315 (2020).

- For general applicability, one must seek more effective methods.

Shell-model based on deformed basis

- Most nuclei in the nuclear chart are deformed. For deformed nuclei, working with a spherical basis has no advantages.
- One can start from a deformed basis in which the rotational symmetry is broken spontaneously.
- In deformed bases, intrinsic states can be classified with **pair-breaking configurations.**
- One then applies angular-momentum-projection technique to recover the broken symmetry, which transforms intrinsic states to the laboratory frame.
- One finally diagonalizes the Hamiltonian in the projected basis. The diagonalization step is similar as in the spherical shell model.
- This is the concept of the **Projected Shell Model (PSM).**

K. Hara, Y. Sun, Int. J. Mod. Phys. E 4 (1995) 637
Y. Sun, Phys. Scr. 91 (2016) 043005

Pair-breaking configurations

Hypothesis:

A “complete” shell-model basis can be constructed by pair-breaking configurations

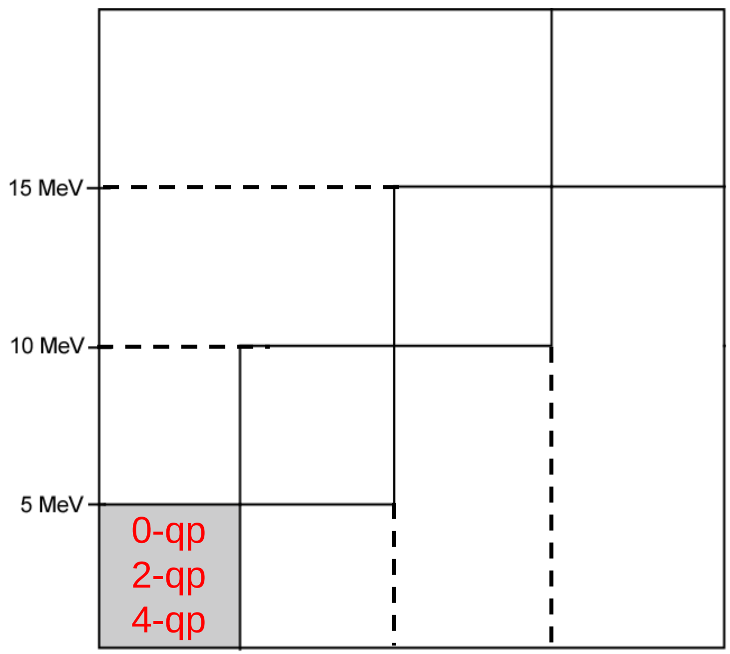
Configuration based on deformed qp-states

- Quasiparticle (qp) energy:

$$e = \sqrt{(\varepsilon - \lambda)^2 + \Delta^2}$$

$$\Delta \approx 1\,\mathrm{MeV}$$

- For even-even rare-earth nuclei, 5 MeV contains 0, 2-, 4-qp, and some 6-qp states.
- If we include up to 8-qp states, we can calculate a full set of excited states of ~ 10 MeV.



- This is the Tamm-Dancoff approximation. Wavefunction: $\psi_M^I = \sum_\kappa f_\kappa \hat{P}^I_{MK_\kappa} |\phi_\kappa\rangle$

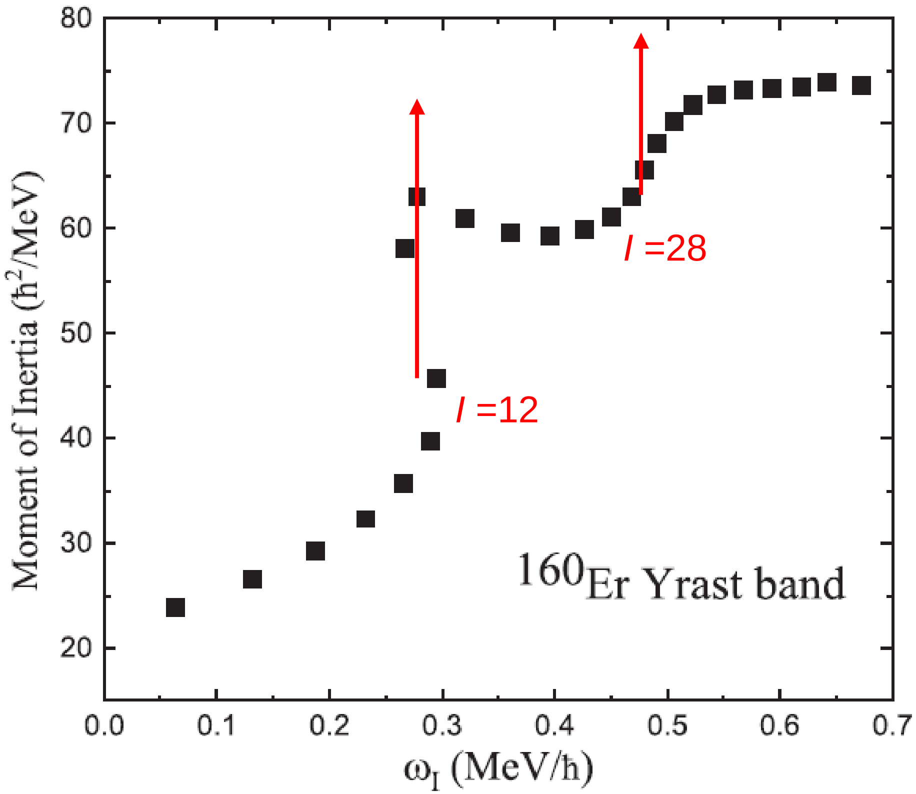
$$\left\{ \hat{P}^I_{MK} |0\rangle, \hat{P}^I_{MK} \alpha_\nu^+ \alpha_\nu^+ |0\rangle, \hat{P}^I_{MK} \alpha_\pi^+ \alpha_\pi^+ |0\rangle, \hat{P}^I_{MK} \alpha_\nu^+ \alpha_\nu^+ \alpha_\pi^+ \alpha_\pi^+ |0\rangle, \cdots \right\}$$

- Maybe regarded as generalization of seniority concept.

P. Van Isacker, AIP Conf. Proc. 1323, 141 (2010).

1. Pair-breaking along the Yrast line

$$\mathscr{J}(I) = \frac{2I-1}{E(I)-E(I-2)}$$



- Moment of inertia (MoI) changes violently with rotation.
- At I ~ 12, MoI suddenly increases. This is the effect of first nucleon-pair breaking.

  F. S. Stephens and R. S. Simon, Nucl. Phys. A 183, 257 (1972)

- At I ~ 28, an additional nucleon pair is broken simultaneously. MoI increases again.

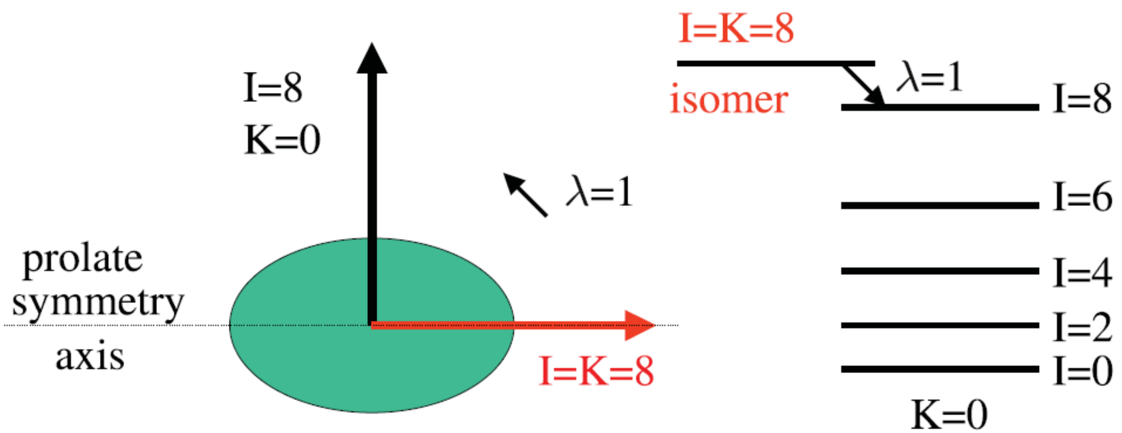
  I. Y. Lee et al. Phys. Rev. Lett. 38, 1454 (1977)

- **One includes 0-qp, 2-qp, 4-qp states in the model space.**

2. Formation of multi-qp *K*-isomers

- Experimental evidence: the structure of high-*K* isomers is built from broken nucleon pairs that generate high *K* along the nuclear symmetry axis.
- There are evidences for 6 and 8 quasi-particle high-*K* isomers.
- **Each of the isomers correspond to a 6-or 8- qp state.**

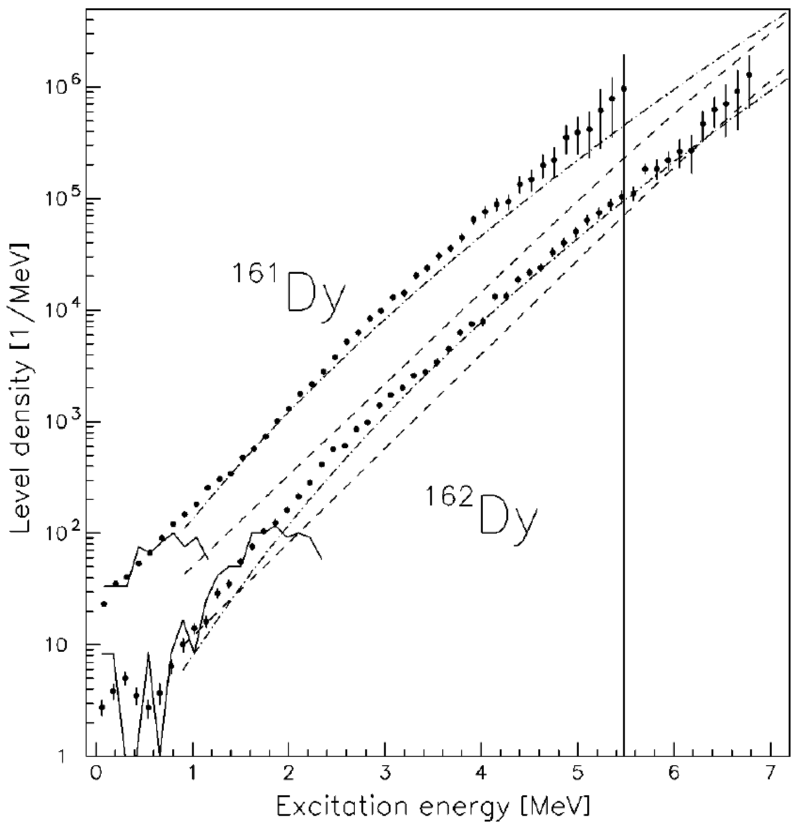
C. S. Purry, et al., Phys. Rev. Lett. 75, 406 (1995).
G. D. Dracoulis, et al., Phys. Rev. C 71, 044326 (2005).



Ph. Walker, Phys. Scr. 92 (2017) 054001
G. D. Dracoulis, P. M. Walker, F. G. Kondev, Rep. Prog. Phys. 79, 076301 (2016)

3. Increase of entropy in odd-mass systems

- Experimental evidence: For a given excitation, level density of odd-mass nuclei increases by a pairing gap Δ.
- Guttormsen et al. examined 280 nuclei and concluded it as the odd-even entropy differences.
- This can be understood as in odd-mass nuclei, the number of one last nucleon is blocked from the pair formation.
- **This effect is naturally included in a pair-breaking configuration space:**
  even-even (0-qp, 2-qp, 4-qp, …);
  odd-mass (1-qp, 3-qp, 5-qp …);
  odd-odd (2-qp, 4-qp, 6-qp, …).



M. Guttormsen, et al.,
Phys. Rev. C 61, 067302 (2000);
Phys. Rev. C 63, 044301 (2001).

Multi-qp configurations in even-even nuclei

- Many-body configuration space consists of such ordered qp states, for example: 0-, 2-, 4-, 6, 8, up to 10-qp states for even-even nuclei.
- Indices cover all possible single-particles subject to Pauli Principle.

$$
\begin{aligned}
\Big\{ & |\Phi\rangle,\ a^\dagger_{\nu_i}a^\dagger_{\nu_j}|\Phi\rangle,\ a^\dagger_{\pi_i}a^\dagger_{\pi_j}|\Phi\rangle,\ a^\dagger_{\nu_i}a^\dagger_{\nu_j}a^\dagger_{\pi_k}a^\dagger_{\pi_l}|\Phi\rangle,\\
& a^\dagger_{\nu_i}a^\dagger_{\nu_j}a^\dagger_{\nu_k}a^\dagger_{\nu_l}|\Phi\rangle,\ a^\dagger_{\pi_i}a^\dagger_{\pi_j}a^\dagger_{\pi_k}a^\dagger_{\pi_l}|\Phi\rangle,\\
& a^\dagger_{\nu_i}a^\dagger_{\nu_j}a^\dagger_{\nu_k}a^\dagger_{\nu_l}a^\dagger_{\nu_m}a^\dagger_{\nu_n}|\Phi\rangle,\ a^\dagger_{\pi_i}a^\dagger_{\pi_j}a^\dagger_{\pi_k}a^\dagger_{\pi_l}a^\dagger_{\pi_m}a^\dagger_{\pi_n}|\Phi\rangle,\\
& a^\dagger_{\pi_i}a^\dagger_{\pi_j}a^\dagger_{\nu_k}a^\dagger_{\nu_l}a^\dagger_{\nu_m}a^\dagger_{\nu_n}|\Phi\rangle,\ a^\dagger_{\nu_i}a^\dagger_{\nu_j}a^\dagger_{\pi_k}a^\dagger_{\pi_l}a^\dagger_{\pi_m}a^\dagger_{\pi_n}|\Phi\rangle,\\
& a^\dagger_{\pi_i}a^\dagger_{\pi_j}a^\dagger_{\nu_k}a^\dagger_{\nu_l}a^\dagger_{\nu_m}a^\dagger_{\nu_n}a^\dagger_{\nu_o}a^\dagger_{\nu_p}|\Phi\rangle, a^\dagger_{\pi_i}a^\dagger_{\pi_j}a^\dagger_{\pi_k}a^\dagger_{\pi_l}a^\dagger_{\nu_m}a^\dagger_{\nu_n}a^\dagger_{\nu_o}a^\dagger_{\nu_p}|\Phi\rangle,\\
& a^\dagger_{\pi_i}a^\dagger_{\pi_j}a^\dagger_{\pi_k}a^\dagger_{\pi_l}a^\dagger_{\nu_m}a^\dagger_{\nu_n}a^\dagger_{\nu_o}a^\dagger_{\nu_p}a^\dagger_{\nu_q}a^\dagger_{\nu_r}|\Phi\rangle,\\
& a^\dagger_{\nu_i}a^\dagger_{\nu_j}a^\dagger_{\nu_k}a^\dagger_{\nu_l}a^\dagger_{\pi_m}a^\dagger_{\pi_n}a^\dagger_{\pi_o}a^\dagger_{\pi_p}a^\dagger_{\pi_q}a^\dagger_{\pi_r}|\Phi\rangle \Big\}
\end{aligned}
\qquad (1)
$$

L.-J. Wang et al, Phys. Rev. C 90 (2014) 011303(R)
L.-J. Wang et al, Phys. Rev. C 93 (2016) 034322

Breakthrough in computation

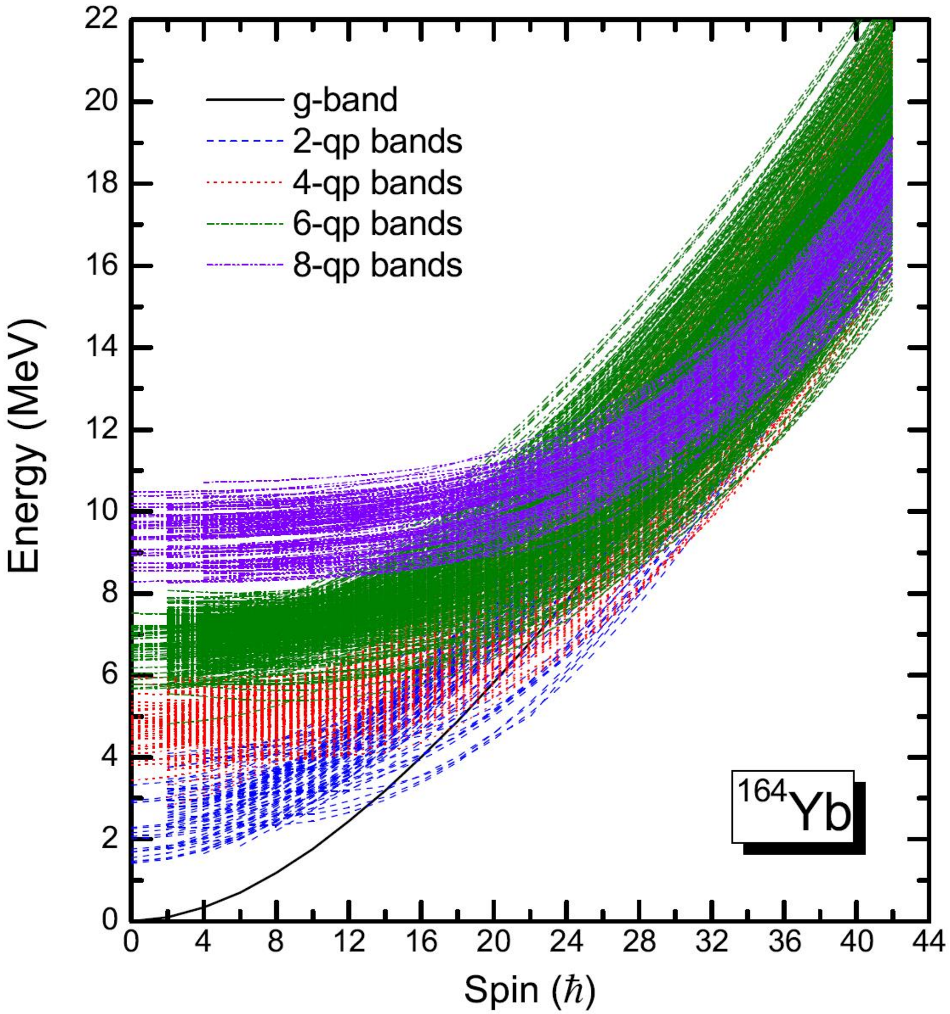
- Calculation of projected matrix elements usually applies the generalized Wick theorem.
- A matrix element having n (n') qp creation or annihilation operators respectively on the left- (right-) sides of the rotation operator contains $(n + n - 1)!!$ terms in the expression

A problem of combinatorial complexity

- The introduction of Pfaffian algorithm:
  - L.M. Robledo, Phys. Rev. C 79 (2009) 021302(R).
  - G. Bertsch, L.M. Robledo, Phys. Rev. Lett. 108 (2012) 042505.
  - T. Mizusaki, M. Oi, Phys. Lett. B 715 (2012) 219.
  - M. Oi, T. Mizusaki, Phys. Lett. B 707 (2012) 305.
  - T. Mizusaki, M. Oi, F.-Q. Chen, Y. Sun, Phys. Lett. B 725 (2013) 175
  - Q.-L. Hu, Z.-C. Gao, Y. S. Chen, Phys. Lett. B 734 (2014) 162.

Realization in computation

- The example includes ~2000 model states with good spin-parity.
- A cut-off energy of 2 MeV is applied to each order of multi qp-state.
- By using multi-qp states, we can include more than 10^7 configurations.
- We can calculate state-to-state transitions well beyond 10 MeV of excitations.
- We have developed a tool that can calculate excited nuclear states for arbitrarily heavy, deformed nuclei.

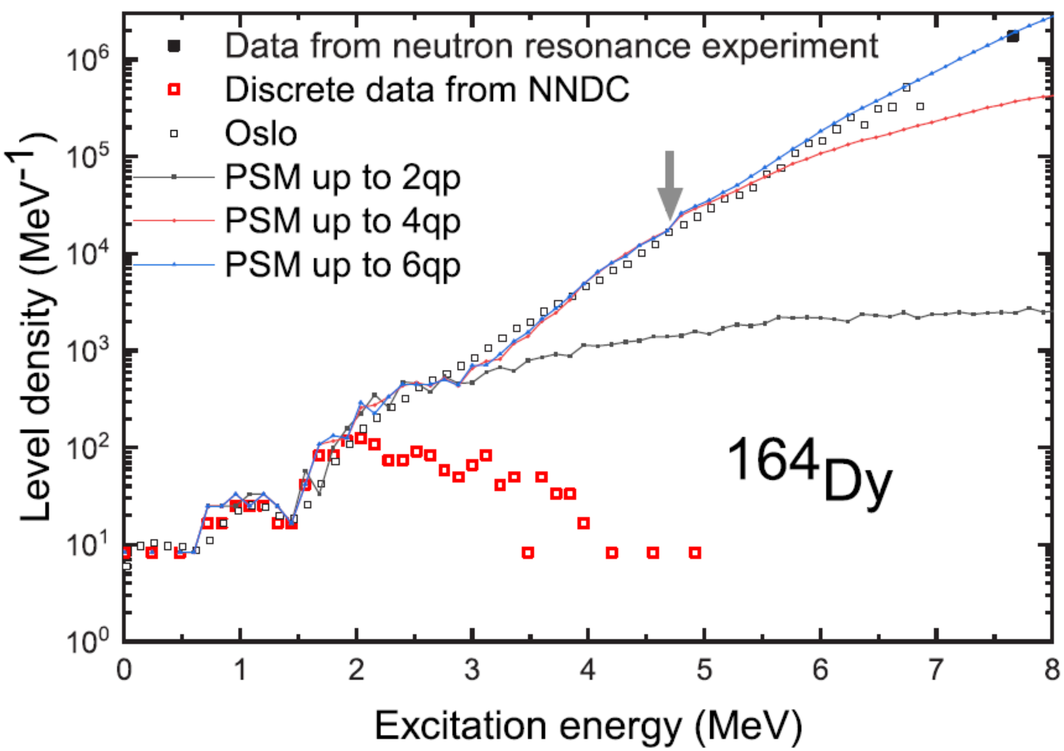


L.-J. Wang, et al., J. Phys. G 46 (2019) 105102

Level density: testing our algorithm

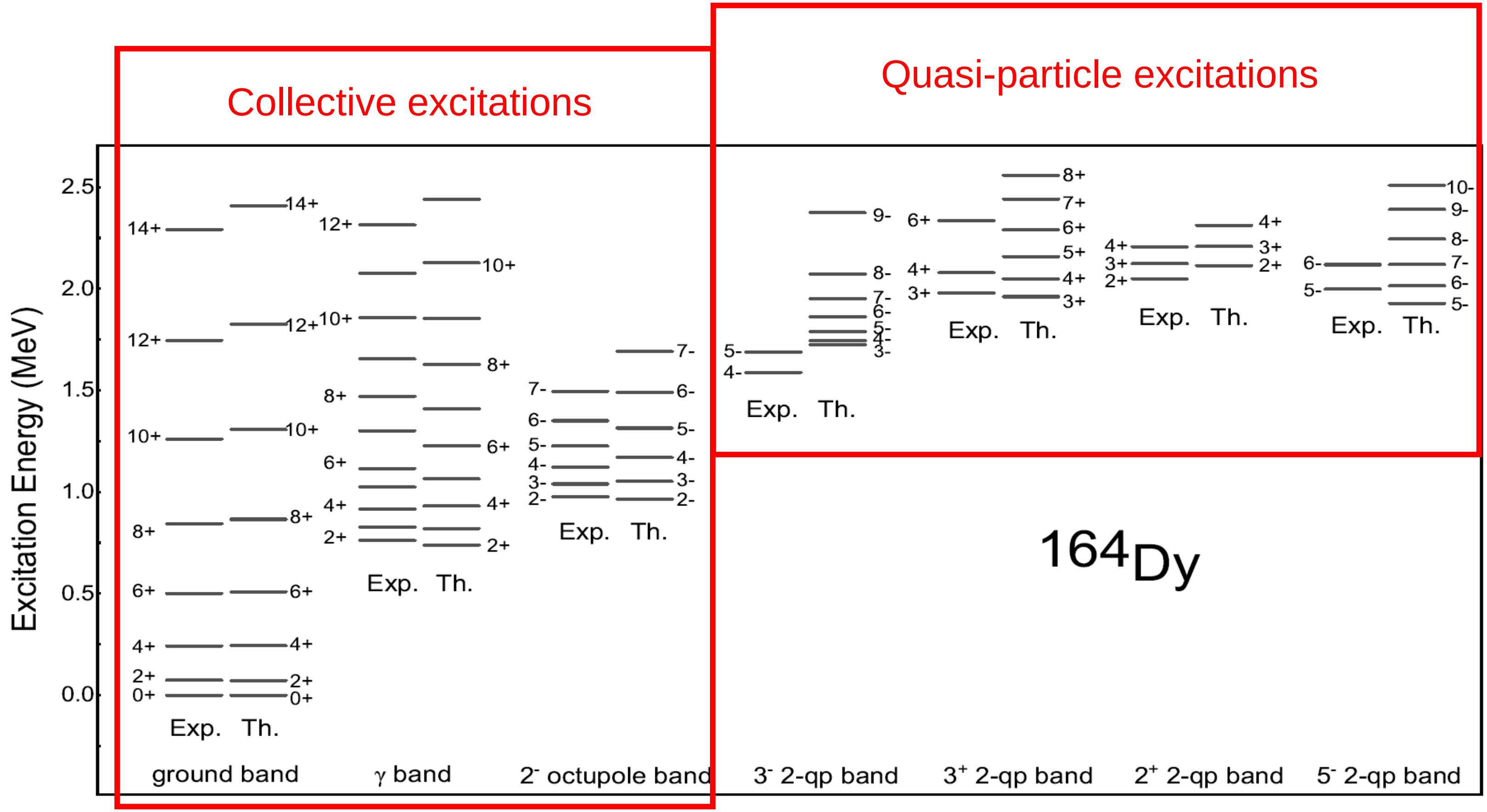
- Taking ^{164}Dy as example where we have the Oslo NLD curve to compare.
- Multi-qp configurations include up to 6-qp states.
- The pair-broken algorithm clearly demonstrates the contribution of different orders of qp-states for a NLD curve.
- It reproduces both low-energy spectra and high-energy data from neutron resonance spacing measurement.

J.-Q. Wang et al. to be published.



Nuclear level density: Discrete data

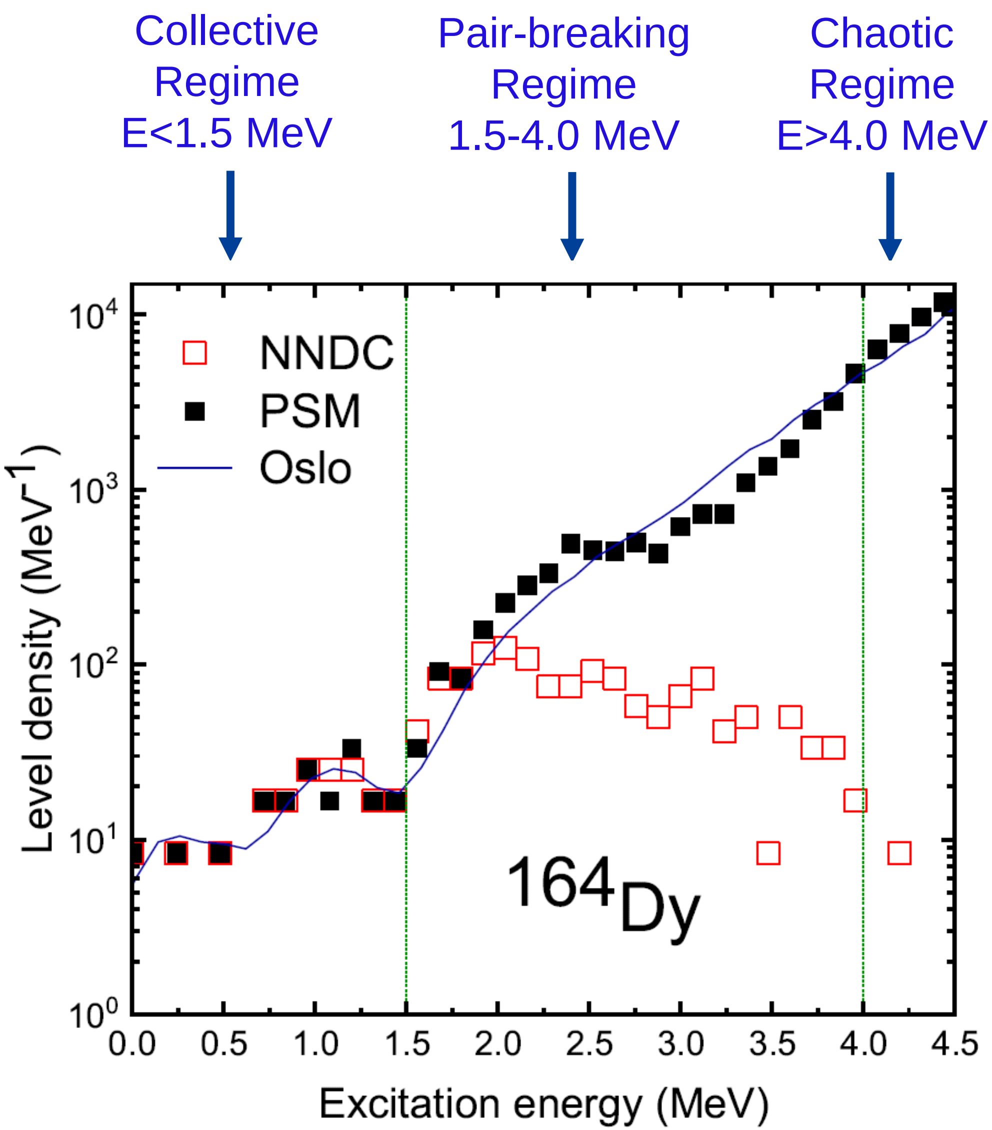
- For low-energy excitations, PSM can describe the known discrete data quantitatively.



NNDC database at https://www.nndc.bnl.gov/nudat3/

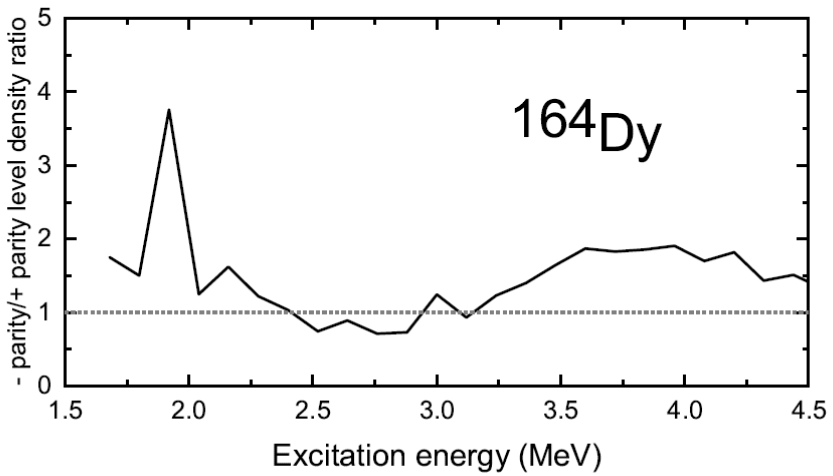
Nuclear level density: Structure effect

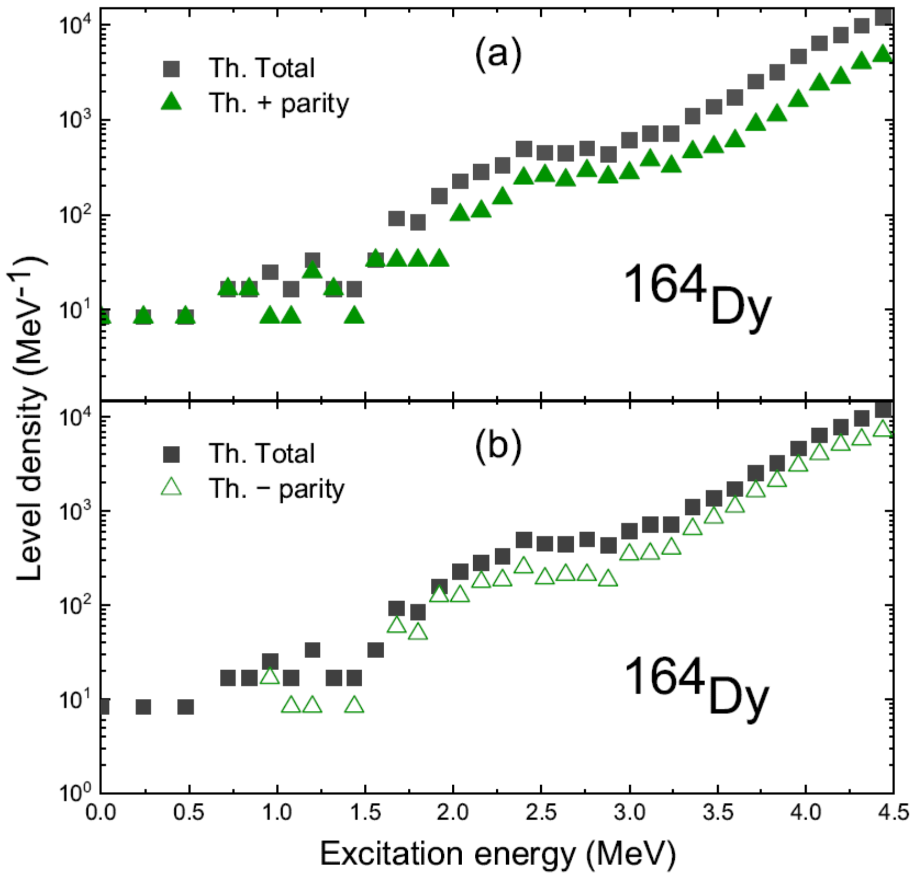
- Taking the ^{164}Dy example to compare with the Oslo curve.
- Three NLD regimes:
- **Collective regime:** a few known collective bands.
- **Pair-breaking regime:** two rises – one pair breaking (2-qp states) and simultaneous breaking of a neutron- and a proton-pair (4-qp states).
- **Chaotic regime:** all pairs are broken. Regular band structures die away completely.



Nuclear level density: Parity dependence

- Statistical method neglecting the shell effect would suggest that NLD is divided into two equal groups of even and odd parity.
- Our calculation for ^{164}Dy shows sensitive parity-dependence on structure.



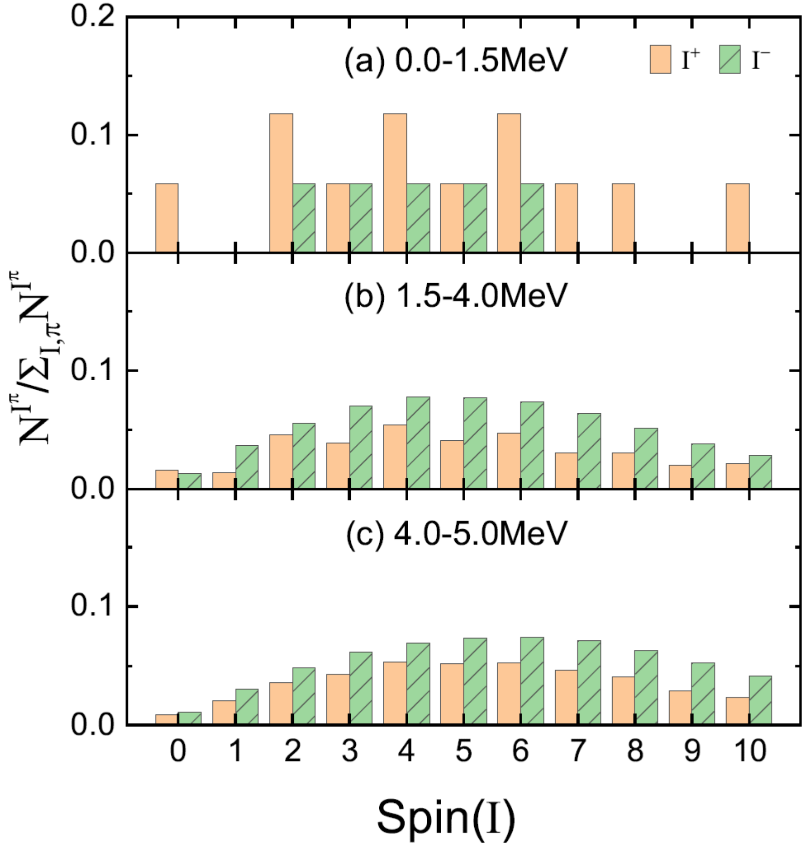


Nuclear level density: Spin dependence

- For ^{164}Dy, spin/ parity distribution patterns in three regimes.

  Spin cut-off at I=10.

- **0.0-1.5 MeV:** Collective regime, only 17 known levels, irregular pattern.
- **1.5-4.0 MeV:** Pair-breaking regime. For even-parity states, irregular distribution with odd-even effect in spin. For odd parity, the distribution tends to become bell-shaped.
- **4.0-5.0 MeV:** Chaotic regime. Total level density approaches 10^4, nearly perfect Gaussians for both parities.



Nuclear level density: Spin dependence

- In chaotic regime (4.5~5.5 MeV), our calculation indicates a Gaussian distribution as suggested by Ericson.
- The Ericson formula contains a parameter σ^2, which can now be obtained by fitting our shell-model results to the Ericson formula.

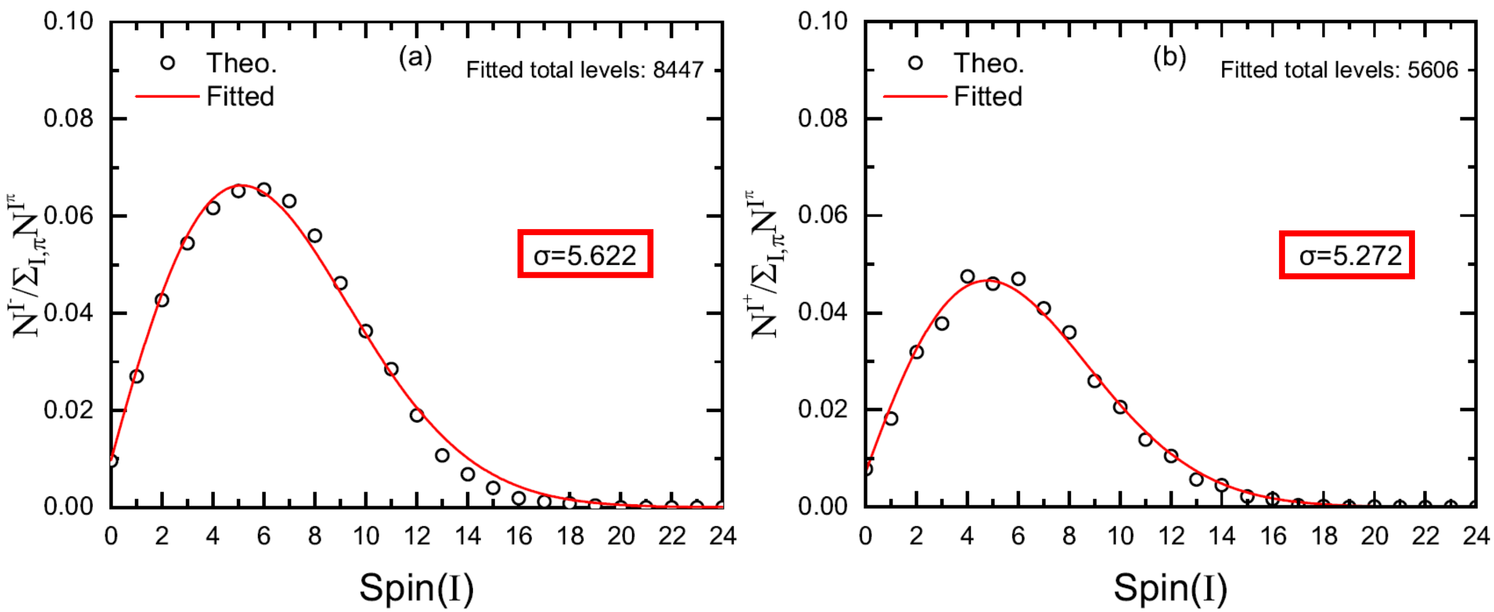
$$\rho(E,I) \approx \rho(E)\frac{2I+1}{2\sqrt{2\pi}\sigma^3}\exp\left\{-\frac{I(I+1)}{2\sigma^2}\right\}$$

T. Ericson, Advances in Physics 9, (1960) 425

Ericson suggested calculation for σ^2:

$$\sigma^2 = \frac{\mathcal{J}T}{\hbar^2}$$

$$\frac{1}{T} = \frac{d}{dE}\ln\rho(E)$$



γ-strength functions

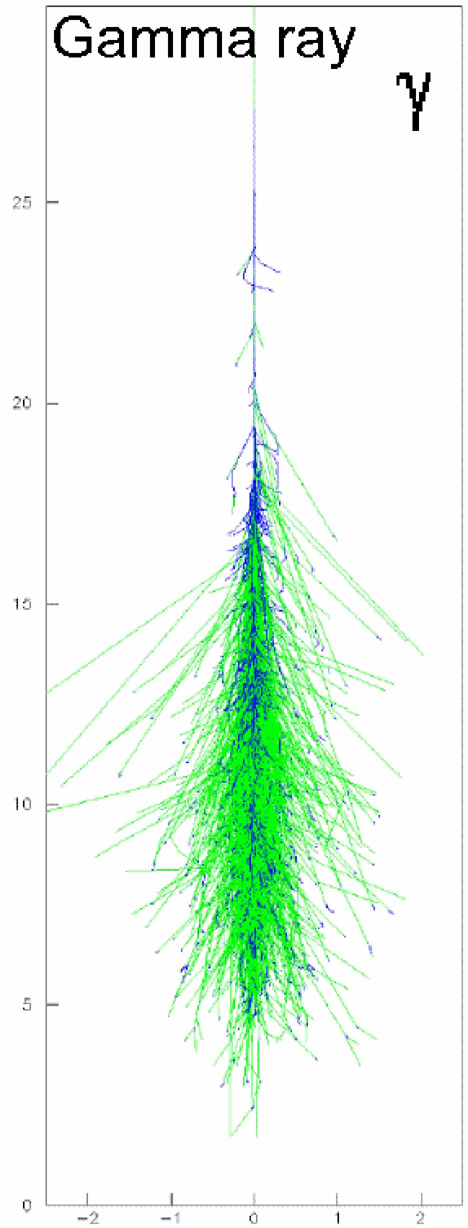
- For γ-strength functions, among all multipolarities, E1, M1, E2 are the main components.
- The Oslo γ-strength functions can be compared.
- The PSM solves many-body eigenvalues equation

$$H\,|\Psi\rangle = E\,|\Psi\rangle$$

and can calculate γ-strength functions directly using our "complete" shell-model wavefunctions.

- We can also treat the relative motion of the proton and neutron bodies.

W. Hauser and H. Feshbach, Phys. Rev. 87, 366 (1952)
M. R. Mumpower et al., Phys. Rev. C 96, 024612 (2017)



Scissors-Mode Vibrations and the Emergence of SU(3) Symmetry from the Projected Deformed Mean Field

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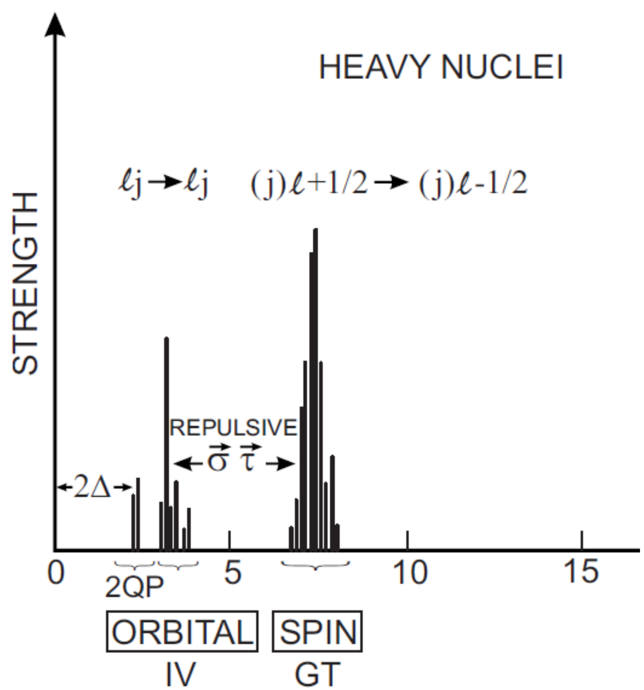
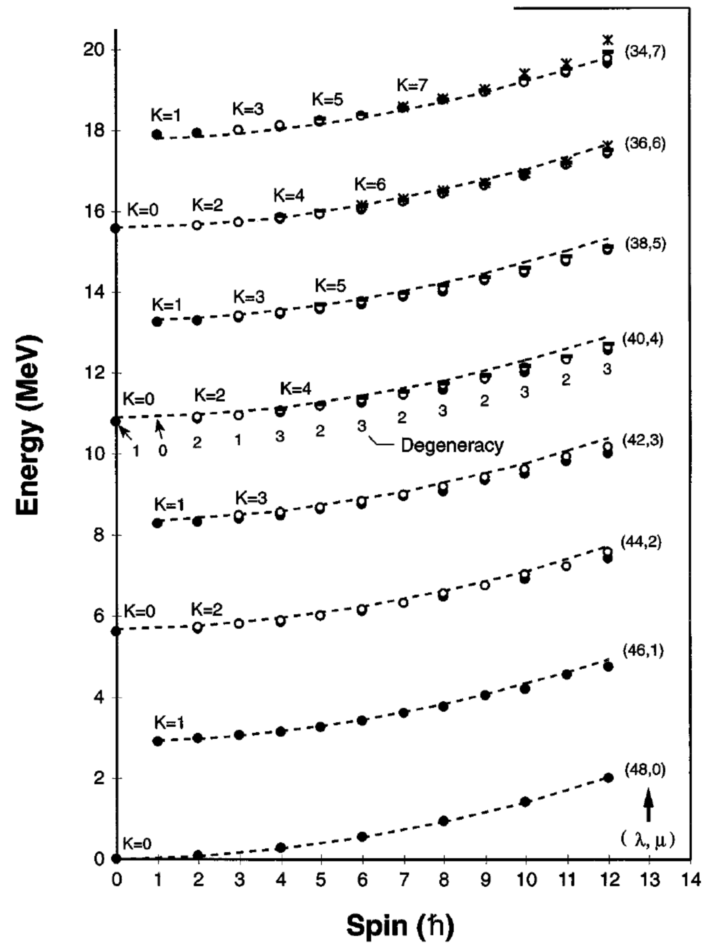
Early shell model description for excited states of scissors mode



A. Richter, Nucl Phys A 507 (1990) 99c



$\Delta I = 2$ bifurcation as a characteristic feature of scissors rotational bands

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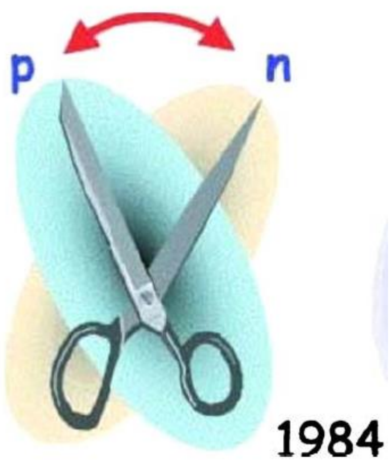
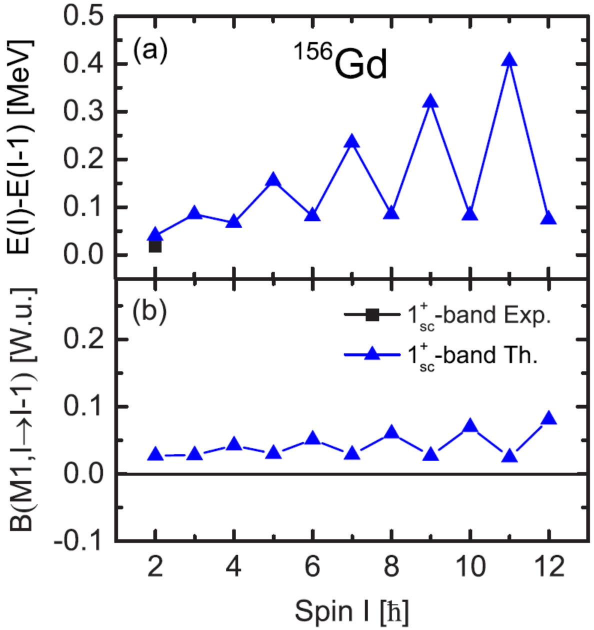
(Dated: September 3, 2021)

We report microscopic many-body calculations indicating that rotational bands based on nuclear scissors vibrations exhibit systematic splitting between neighboring spin states ($\Delta I = 2$ bifurcation) in which the magnitude of the moment of inertia oscillates between states having even and odd spins. This unexpected result is independent of details such as interactions or occupation of particular single-particle orbits. We propose that the puzzling excited state found above the 1^+ scissors state in ^{156}Gd (Phys. Rev. Lett. **118**, 212502 (2017)) is the first evidence of this effect, and predict that bifurcation should appear in all other scissors rotational bands of deformed nuclei, and possibly in other systems exhibiting collective scissors vibrations.

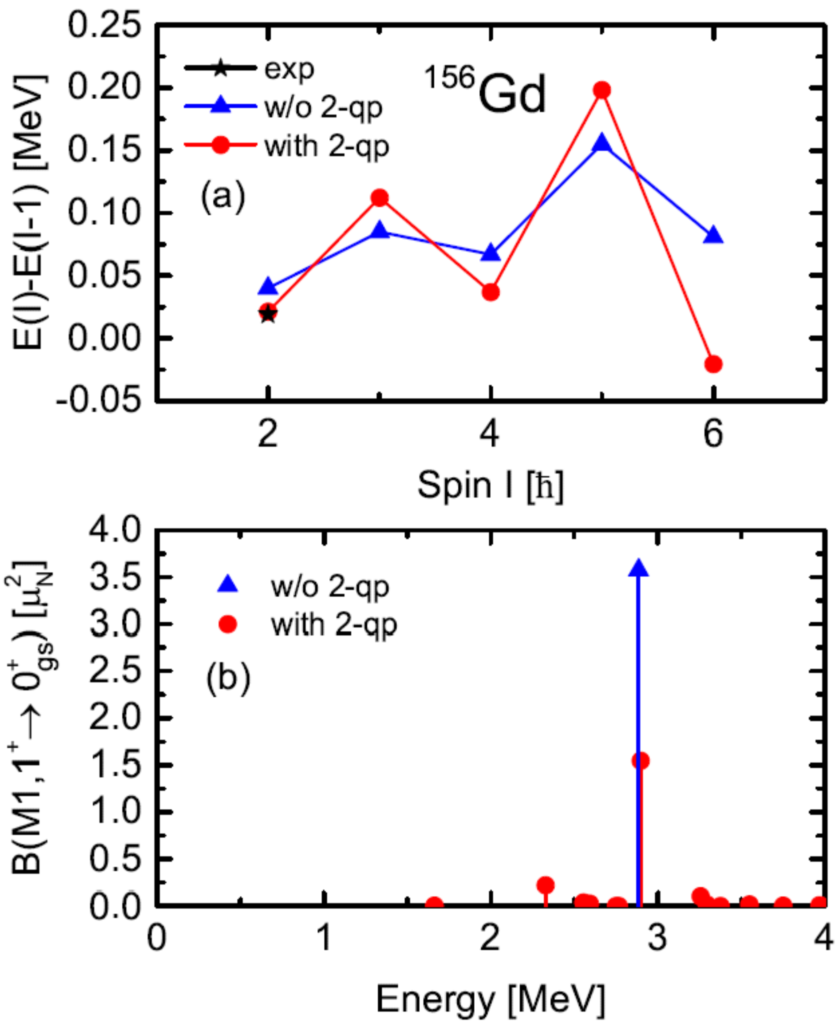




Lv et al., Phys. Rev. Lett., 2022, 129: 042502

Quasiparticle enhancement for the staggering

- Quasiparticle states coupled to the scissors mode state can give rise to **many new states,** contributing to level densities and to the fragmented M1 strength.
- The enhancement of level density and M1 strength at ~3 MeV can modify γ-strength function, may have significant impact on nuclear reactions (for example, in nucleosynthesis study).



Lv et al., Phys. Rev. Lett., 2022, 129: 042502
M. Mumpower and W. Misch, private discussion

Scissors mode and Pygmy mode

- Pygmy dipole resonance (PDR) has been commonly referred to the electric dipole (E1) strength around and below the neutron-separation energy.

E. G. Lanza et al., Prog. Part. Nucl. Phys. 129 (2023) 104006

- E1 strength in heavy nuclei exhibits a local concentration, overlapping with the excitation energy region of the M1 spin-flip resonance.

- Both scissors mode and pygmy mode are related to **relative motion** of proton and neutron systems. From the shell-model point of view, they may have a same origin.

- A unified description using the current projected shell model for both modes and γ-strength function is possible.

Summary

- We intended to produce a “complete” set of nuclear levels using the shell-model concept.
- Guided by experimental evidences, we build the many-body configuration space using multi-quasiparticle states from broken pairs.
- Known low-excitation levels can be described quantitatively. Structure effects are emphasized in level density with different excitation energies.
- Spin- and parity-dependence can be precisely obtained.
- γ-strength functions including the Scissors mode and Pygmy mode contributions can be described consistently.
- The method can be applied to arbitrarily heavy, deformed nuclei.
- Although our model reproduces the general behavior of NLD predicted by free Fermi gas model of Hans Bethe, the wavefunctions are a set of strongly correlated many-body states.

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- Long-Jun Wang (Southwest University, China)