Intertwined Quantum Phase Transitions in odd-mass Nb Isotopes

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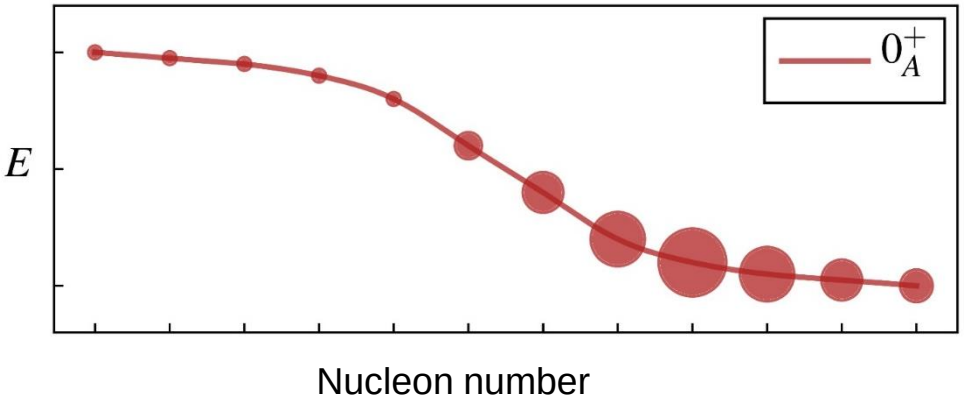
Quantum Phase Transitions (QPTs)

- Type I: Single configuration
  (shape-phase transitions)

$$\hat{H} = (1-\xi)\hat{H}_1 + \xi\hat{H}_2$$

neutron number 90 region: Nd-Sm-Gd



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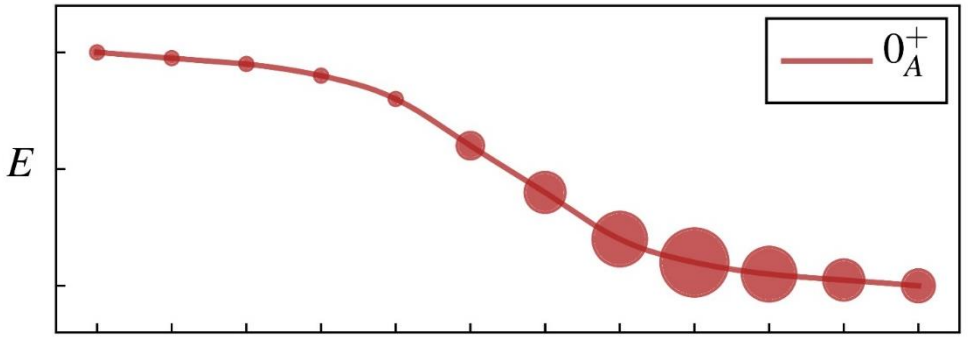
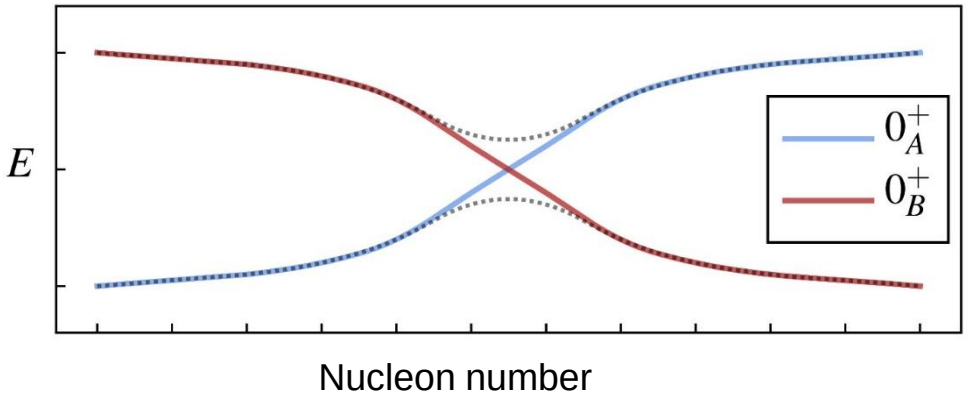


- Type II: Two configurations A, B
  (coexistence normal-intruder states)

$$\hat{H} = \begin{bmatrix} \hat{H}_A(\xi_A) & \hat{W}(\omega) \\ \hat{W}(\omega) & \hat{H}_B(\xi_B) \end{bmatrix}$$

nuclei near shell-closure: Pb-Hg



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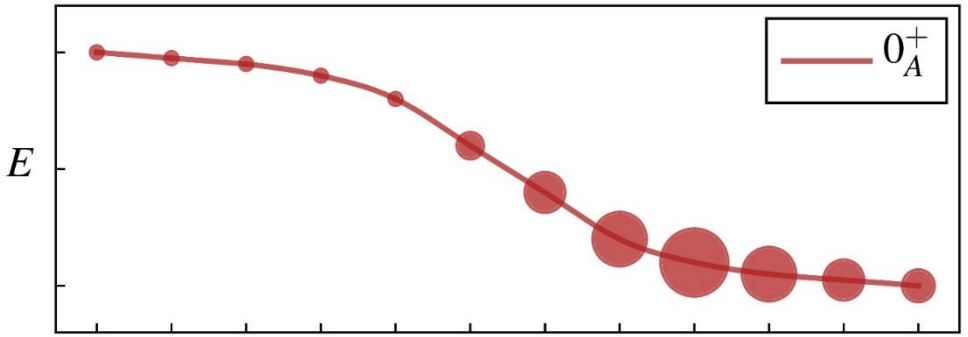

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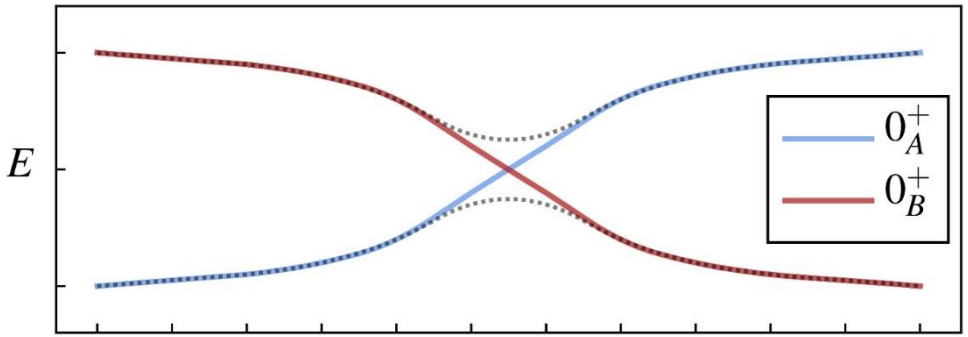
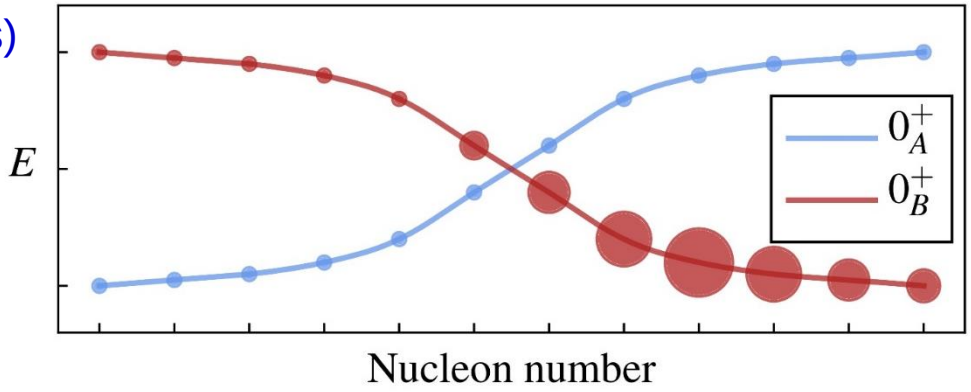


- Intertwined quantum phase transitions (IQPTs)

  Type II QPT and Type I QPT coexist
  configuration crossing accompanied by
  pronounced individual shape-evolutions

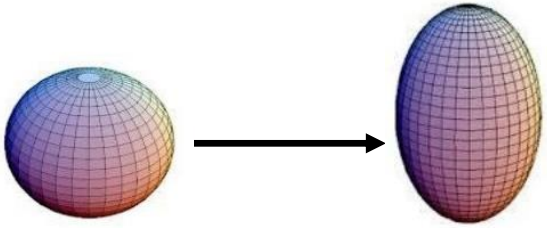
neutron numbr 60 region: Zr, Nb THIS TALK



Type I QPT (shape-phase transition)

- IBM: Dynamical symmetries ↔ phases
- QPT: H(ξ) interpolates between different DS limits

$$H(\xi) = \xi H_{G1} + (1-\xi) H_{G2}$$



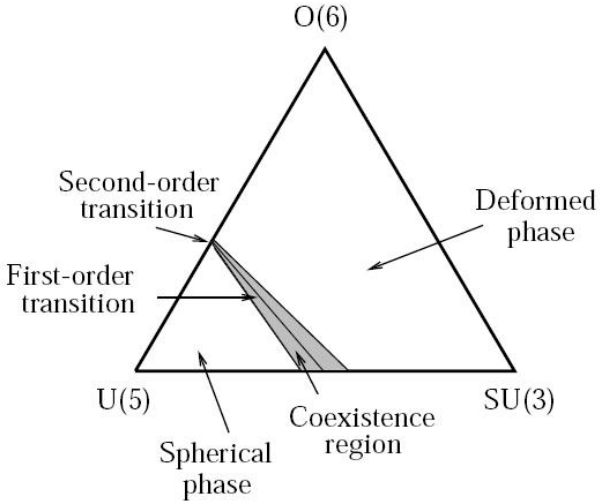
G1 G2

G_i = U(5), SU(3), SO(6) ↔ phases [spherical, deformed: axial, γ-unstable]

- Landau potential $E_N(\beta,\gamma;\xi) = \langle \beta,\gamma;N|\hat{H}|\beta,\gamma;N\rangle$
- Order parameter $\frac{\langle \hat{n}_d \rangle_{0_1^+}}{N} \approx \frac{\beta_{eq}^2}{1+\beta_{eq}^2}$

$$\hat{H}(\epsilon_d,\kappa,\chi) = \epsilon_d \hat{n}_d + \kappa \hat{Q}_\chi \cdot \hat{Q}_\chi$$

$$\hat{Q}_\chi = d^\dagger s + s^\dagger \tilde{d} + \chi (d^\dagger \times \tilde{d})^{(2)}$$



$$E_N(\beta,\gamma;\epsilon_d,\kappa,\chi) = 5\kappa N + \frac{N\beta^2}{1+\beta^2}\left[\epsilon_d + \kappa(\chi^2-4)\right] + \frac{N(N-1)\beta^2}{(1+\beta^2)^2}\kappa\left[4 - 4\bar{\chi}\beta\Gamma + \bar{\chi}^2\beta^2\right]$$

$$\bar{\chi} = \sqrt{\frac{2}{7}}\chi \qquad \Gamma = \cos 3\gamma$$

U(5): $\kappa = 0$

SU(3): $(\epsilon_d = 0, \chi = -\sqrt{7}/2)$

SO(6): $(\epsilon_d = 0, \chi = 0)$

U(5)-SU(3) 1st order

U(5)-SO6) 2nd order

SU(3)-SO(6) crossover

Type II QPT (coexistence near shell closure)

- Multiparticle-multihole intruder excitations across shell gaps
- Interacting boson model with configuration mixing (IBM-CM) [Duval, Barrett, PLB 81]

0p-0h, 2p-2h, 4p-4h,… → [N] ⊕ [N+2] ⊕ [N+4]… normal⊕ intruder states

- Hamiltonian

$$\hat{H} = \begin{bmatrix} \hat{H}_A(\xi_A) & \hat{W}(\omega) \\ \hat{W}(\omega) & \hat{H}_B(\xi_B) \end{bmatrix} = \hat{H}_A^{(N)} + \hat{H}_B^{(N+2)} + \hat{W}$$

$$\hat{H}_A = \epsilon_d^{(A)} \hat{n}_d + \kappa^{(A)} \hat{Q}_\chi \cdot \hat{Q}_\chi$$

$$\hat{H}_B = \epsilon_d^{(B)} \hat{n}_d + \kappa^{(B)} \hat{Q}_\chi \cdot \hat{Q}_\chi + \kappa'^{(B)} \hat{L} \cdot \hat{L} + \Delta_p^{(B)}$$

$$\hat{W} = \omega \left[(d^\dagger \times d^\dagger)^{(0)} + (s^\dagger)^2 + \text{H.c.} \right]$$

$$\hat{Q}_\chi = d^\dagger s + s^\dagger \tilde{d} + \chi (d^\dagger \times \tilde{d})^{(2)}$$

$$\hat{T}(E2) = e^{(A)} \hat{Q}_\chi^{(N)} + e^{(B)} \hat{Q}_\chi^{(N+2)}$$

- Wave functions

$$|\Psi; L\rangle = a|\Psi_A; [N], L\rangle + b|\Psi_B; [N+2], L\rangle$$

normal intruder

- Geometry

$$E(\beta, \gamma) = \begin{bmatrix} E_A(\beta, \gamma; \xi_A) & \Omega(\beta, \gamma; \omega) \\ \Omega(\beta, \gamma; \omega) & E_B(\beta, \gamma; \xi_B) \end{bmatrix}$$

Matrix coherent states

$E_\pm(\beta,\gamma)$ Eigen-potentials

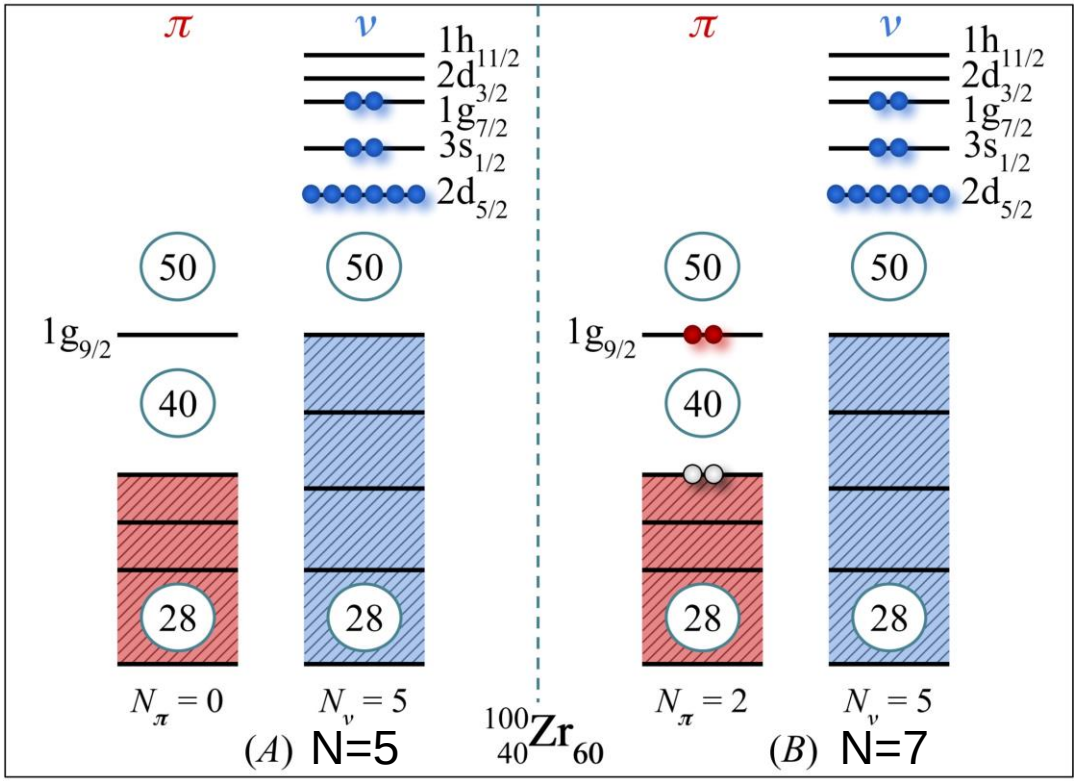
- Order parameters

$$\langle \hat{n}_d \rangle_A \qquad \langle \hat{n}_d \rangle_B \qquad \langle \hat{n}_d \rangle_{0_1^+} = a^2 \langle \hat{n}_d \rangle_A + b^2 \langle \hat{n}_d \rangle_B$$

IBM-CM in the Zr chain

$_{40}$Zr isotopes

Positive parity states



Normal (A) configuration Z=40 subshell closure [N]

Intruder (B) configuration two-proton excitation (2p-2h states) [N+2]

$$|\Psi; L\rangle = a|\Psi_A; [N], L\rangle + b|\Psi_B; [N+2], L\rangle$$

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QPTs in odd-mass Nuclei

- Interacting boson-fermion model with configuration mixing (IBFM-CM)

$$\hat{H} = \hat{H}_{\rm b} + \hat{H}_{\rm f} + \hat{V}_{\rm bf}$$

$$\hat{H}_{\rm b} = \begin{bmatrix} \hat{H}_{\rm b}^{\rm A}(\xi^{(\rm A)}) & \hat{W}_{\rm b}(\omega) \\ \hat{W}_{\rm b}(\omega) & \hat{H}_{\rm b}^{\rm B}(\xi^{(\rm B)}) \end{bmatrix} \qquad \hat{H}_{\rm f} = \begin{bmatrix} \sum_j \epsilon_j^{(\rm A)} \hat{n}_j & 0 \\ 0 & \sum_j \epsilon_j^{(\rm B)} \hat{n}_j \end{bmatrix} \qquad \hat{V}_{\rm bf} = \begin{bmatrix} \hat{V}_{\rm bf}^{\rm A}(\zeta^{(\rm A)}) & \hat{W}_{\rm bf}(\omega_j) \\ \hat{W}_{\rm bf}(\omega_j) & \hat{V}_{\rm bf}^{\rm B}(\zeta^{(\rm B)}) \end{bmatrix}$$

boson (even-even core) — fermion — boson-fermion

A (normal) B (intruder) configurations ; single j-orbits ; normal-intruder mixing: $\hat{W}_{\rm b}$, $\hat{W}_{\rm bf}$

$$V_{\rm bf}^{\rm MON} = \sum_j A_j \, \hat{n}_d \, \hat{n}_j \ , \qquad \text{Monopole}$$

$$\hat{V}_{\rm bf}^{\rm QUAD} = \sum_{jj'} \Gamma_{jj'} \, \hat{Q}_\chi \cdot (a_j^\dagger \tilde{a}_j)^{(2)} \ , \qquad \text{Quadrupole}$$

$$\hat{V}_{\rm bf}^{\rm EXC} = \sum_{j,j'j''} \Lambda_{jj'}^{j''} \sqrt{2j''+1} : [(d^\dagger \tilde{a}_j)^{(j'')} \times (\tilde{d} a_{j'}^\dagger)^{(j'')}]^{(0)} : \qquad \text{Exchange}$$

$$\hat{W}_{\rm bf}(\omega_j) = \sum_j \omega_j \hat{n}_j [(d^\dagger d^\dagger)^{(0)} + (s^\dagger)^2 + {\rm H.c.}] \ . \qquad \text{Mixing}$$

$$|\Psi; J\rangle = \sum_{\alpha,L,j} C_{\alpha,L,j}^{(N,J)} |\Psi_{\rm A}; [N], \alpha, L; j; J\rangle + \sum_{\alpha,L,j} C_{L,j}^{(N+2,J)} |\Psi_{\rm B}; [N+2], \alpha, L; j; J\rangle$$

normal — intruder

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IBFM-CM in the Nb chain

$_{41}$Nb isotopes

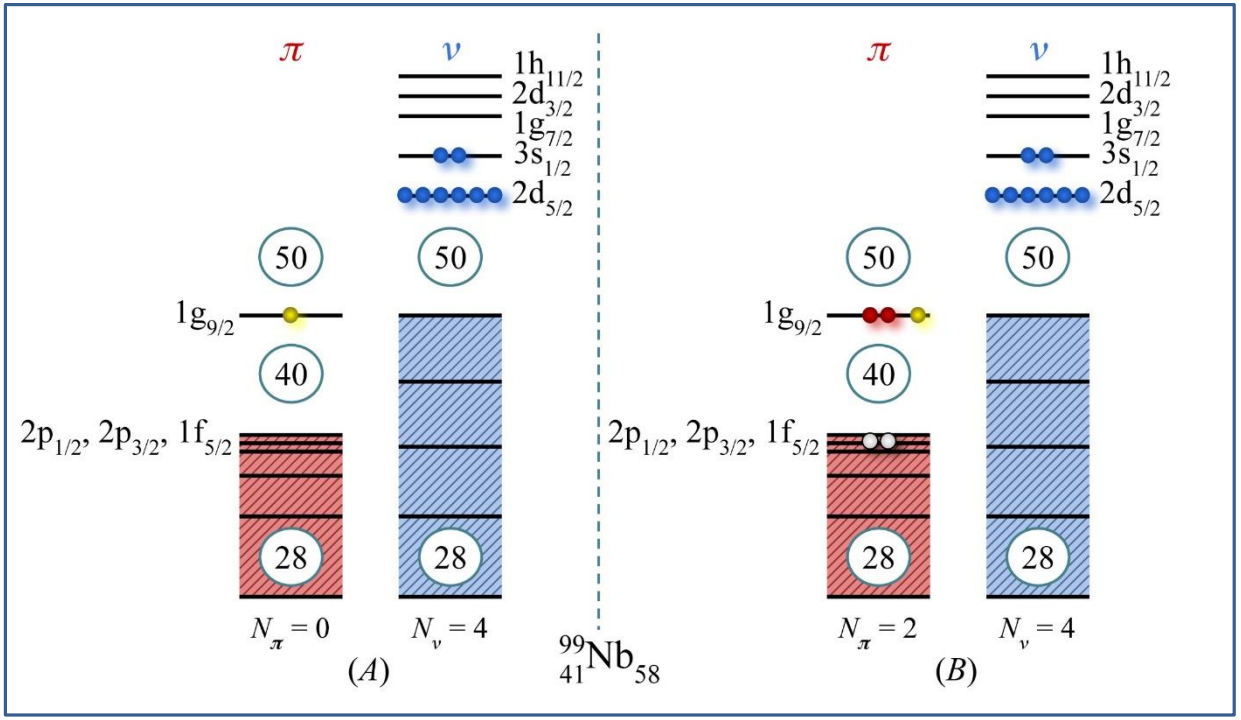
Positive parity states

($_{40}$Zr core) + single-j

$\left([N] \oplus [N+2]\right) \otimes \pi(1g_{9/2})$

↓ ↓

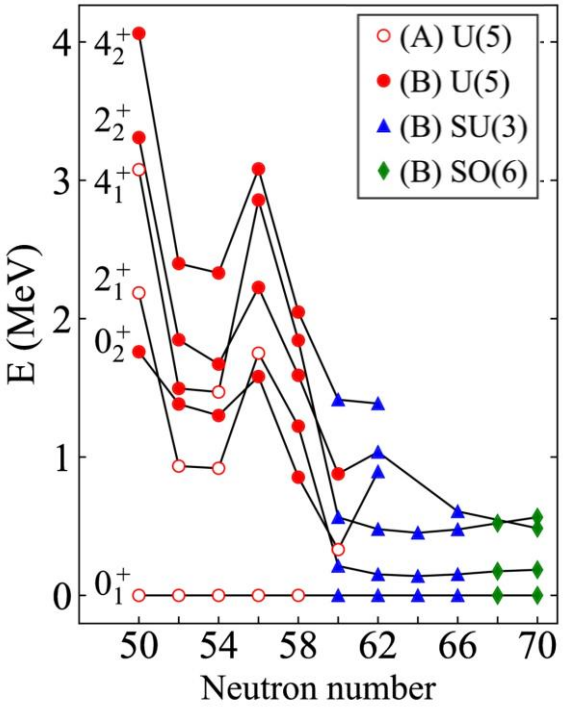
normal intruder



$$|\Psi; J\rangle = \sum_{\alpha,L} C^{(N,J)}_{\alpha,L,j} |\Psi_{\rm A}; [N], \alpha, L; j; J\rangle + \sum_{\alpha,L} C^{(N+2,J)}_{L,j} |\Psi_{\rm B}; [N+2], \alpha, L; j; J\rangle$$

$$a^2 = \sum_{\alpha,L} |C^{(N,J)}_{\alpha,L,j}|^2 \quad , \quad b^2 = \sum_{\alpha,L} |C^{(N+2,J)}_{\alpha,L,j}|^2 \qquad a^2 + b^2 = 1$$

Zr



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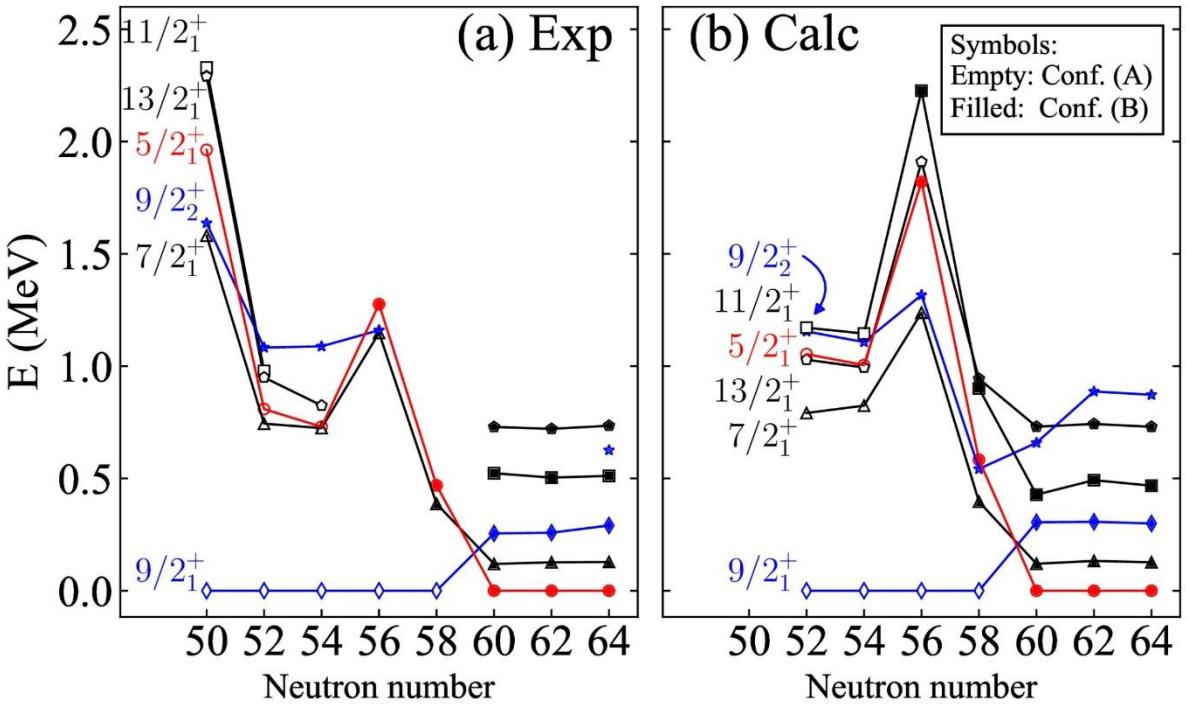


n=50-56: config. (A) spherical (seniority-like) $R^{(A)}_{4/2} \sim 1.6$
config. (B) weakly-deformed $R^{(B)}_{4/2} \sim 2.3$
rise in energy at n=56 due to ν(2d5/2) subshell clousure

From n=58: pronounced drop in energy for states of config. (B)

n=60: two configurations exchange role ⇒ Type II QPT
config. (B) at critical point of U(5)-SU(3) Type I QPT

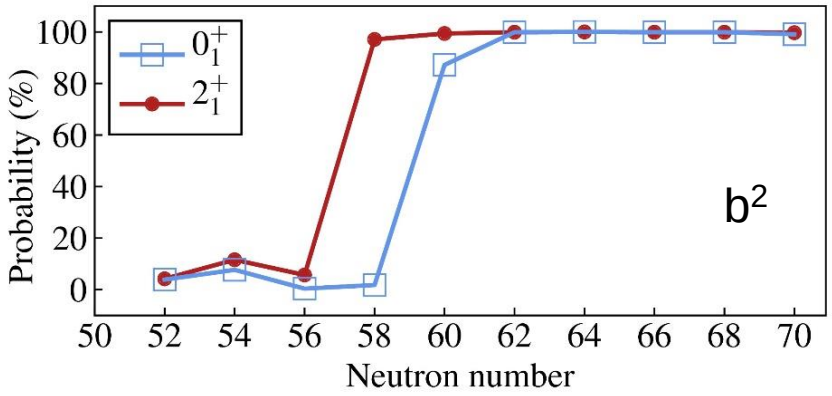
g.s. changes from $9/2^+$ to $5/2^+$

n>60: config. (B) strongly deformed [SU(3)] ^{104}Zr: $R^{(B)}_{4/2} = 3.24$

$K=5/2^+$ band develops
$J=5/2^+, 7/2^+, 9/2^+, 11/2^+, 13/2^+$

n=66: g.s. becomes γ-unstable (or triaxial) SU(3) → SO(6) crossover

Zr



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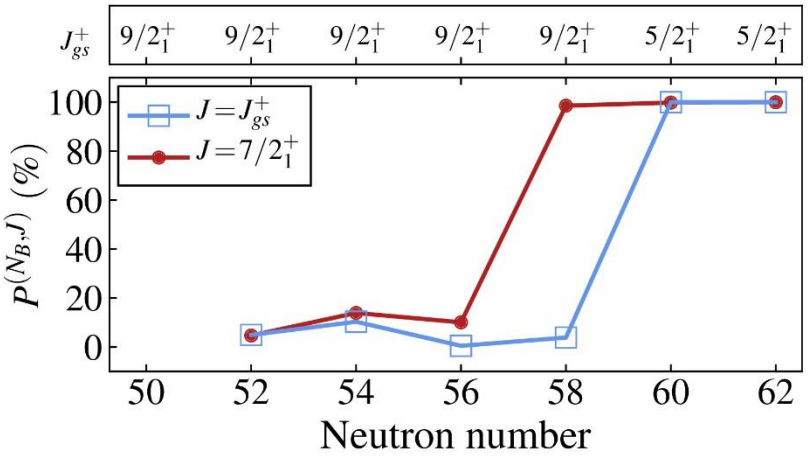
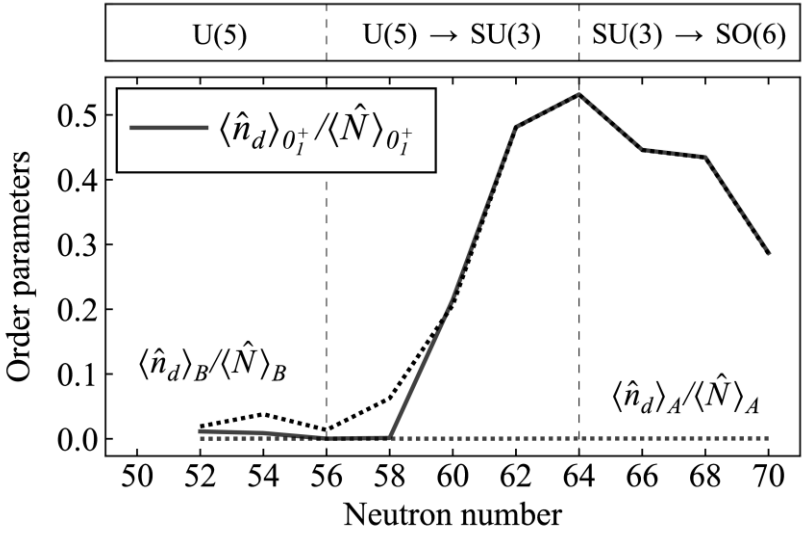


- Abrupt crossing of the two configurations (Type II QPT)



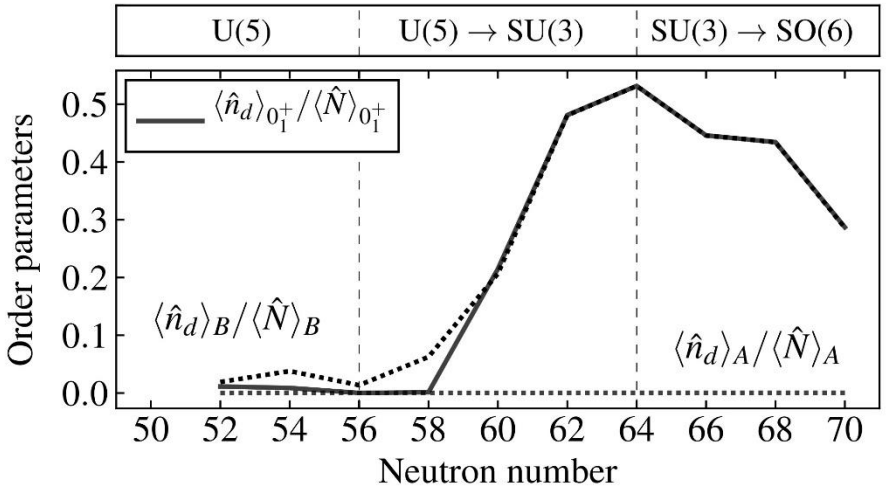
- Gradual spherical to deformed transition (Type I QPT)
  within the intruder (B) configuration

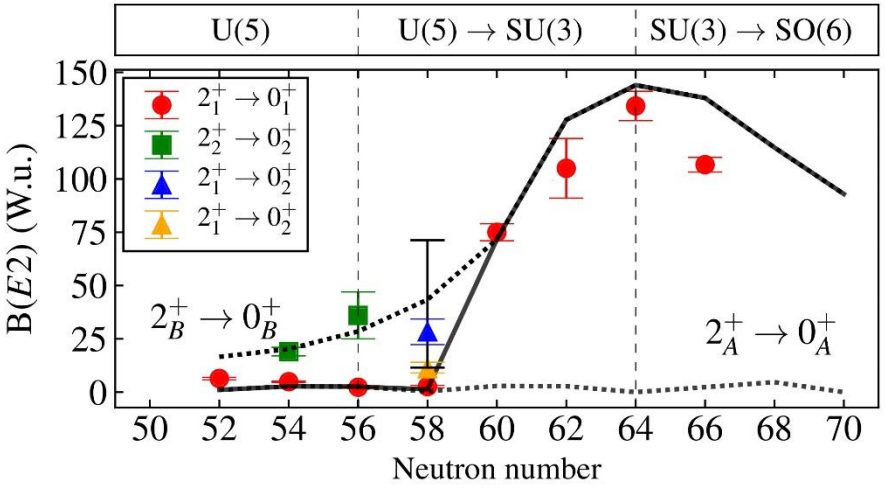
Transition from weak to strong coupling

Coexisting Type I QPT and Type II QPT
⇒ Intertwined QPTs (IQPTs)

B(E2) values and quadrupole moments

Zr





$2_A \to 0_A$ coincides with $2_1 \to 0_1$ for n=52-58

$2_B \to 0_B$ coincides with $2_2 \to 0_2$ for n=52-56

$2_1 \to 0_2$ n=58

$2_1 \to 0_1$ n=60-64

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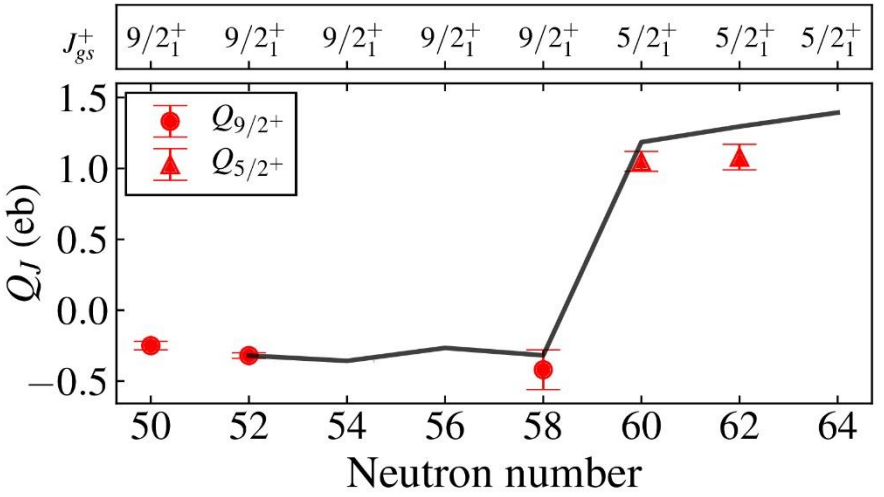
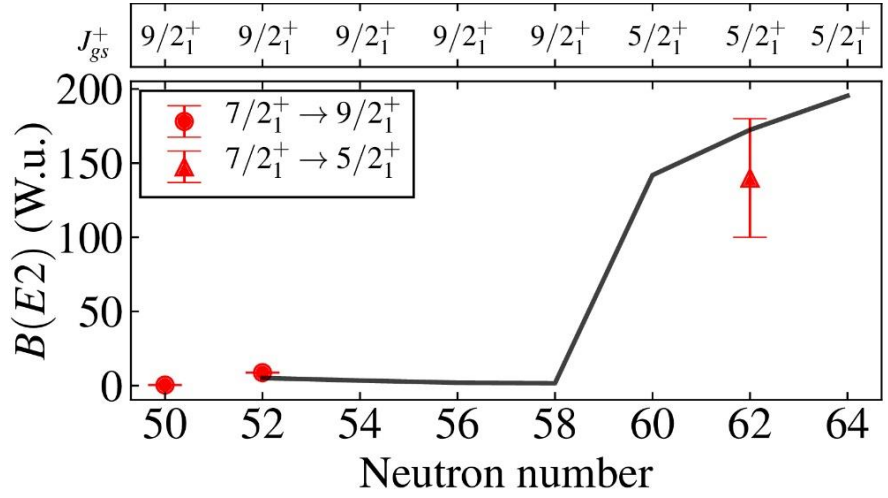


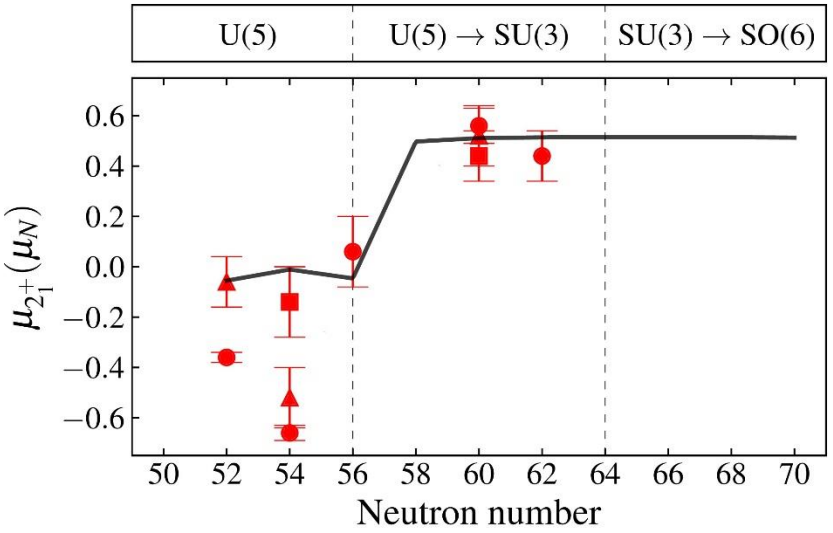


B(E2) and Q(J_{gs}) related to deformation (order param)

n=60: Type II QPT (jump from A to B configuration)

Magnetic moments

Zr



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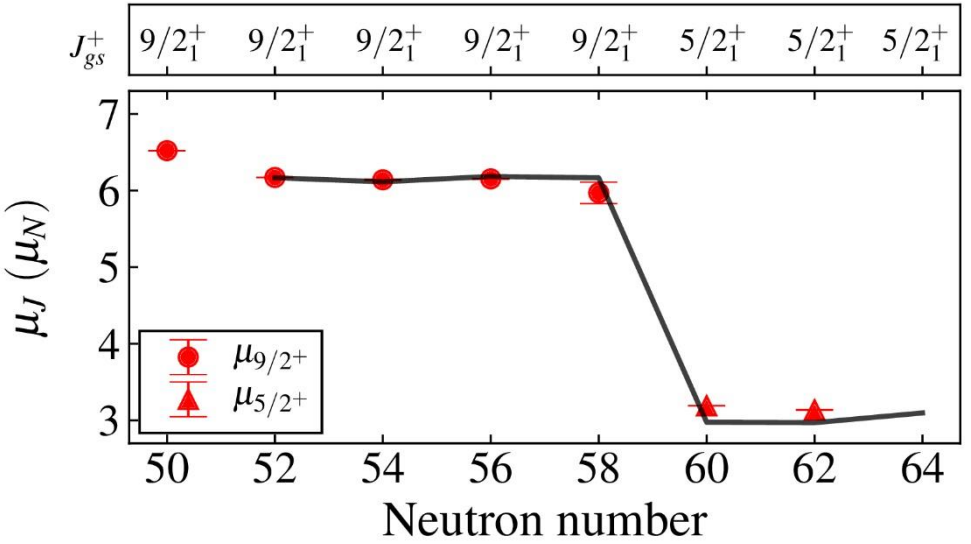


n = 52-56: μ_2 close to zero (or negative)
single-particle structures

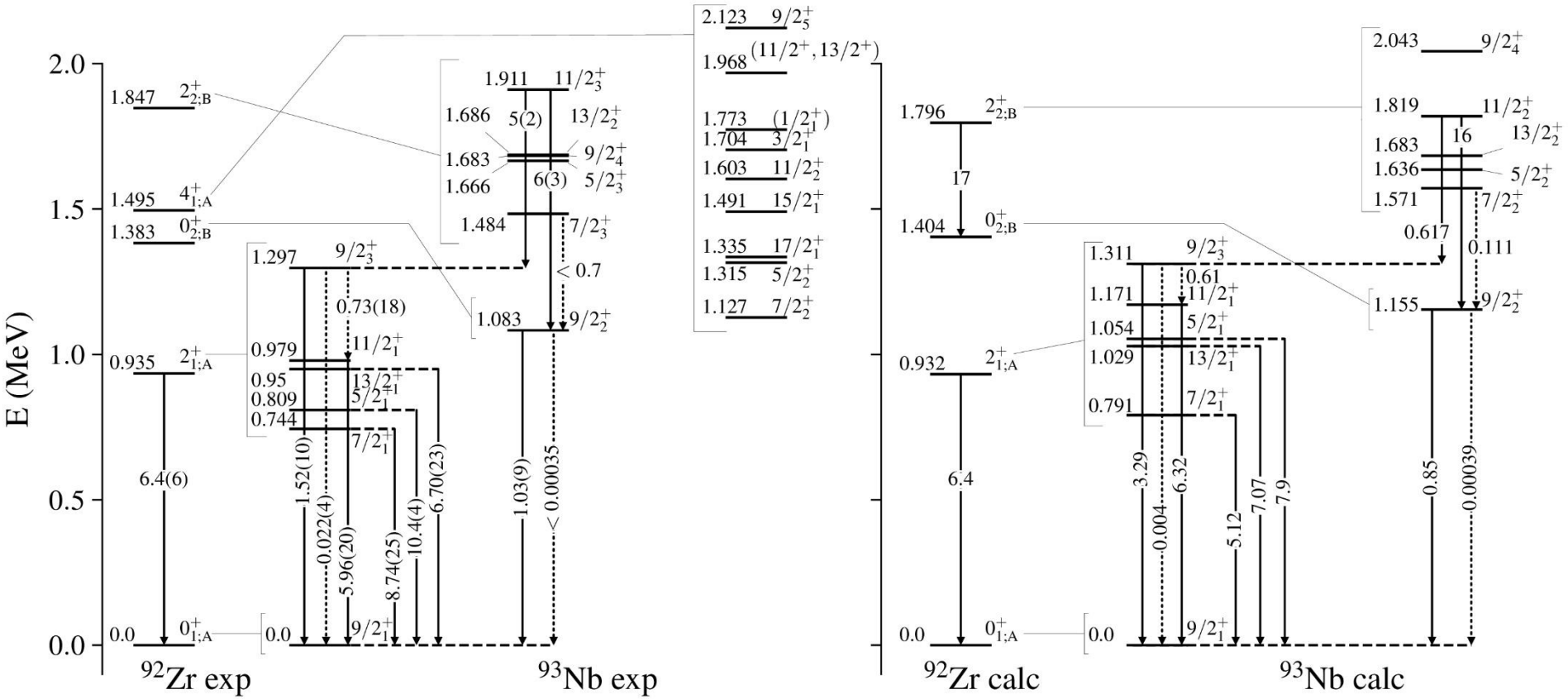
n = 58-70: μ_2 positive, close to $+0.5\mu_N$
collective structures $g_2 \approx Z/A$

μ_J constant value for n = 52-58

μ_J a drop at n=60, which persists for n=60-64

Constant value for each range of n numbers, in line with weak mixing of a Type II QPT

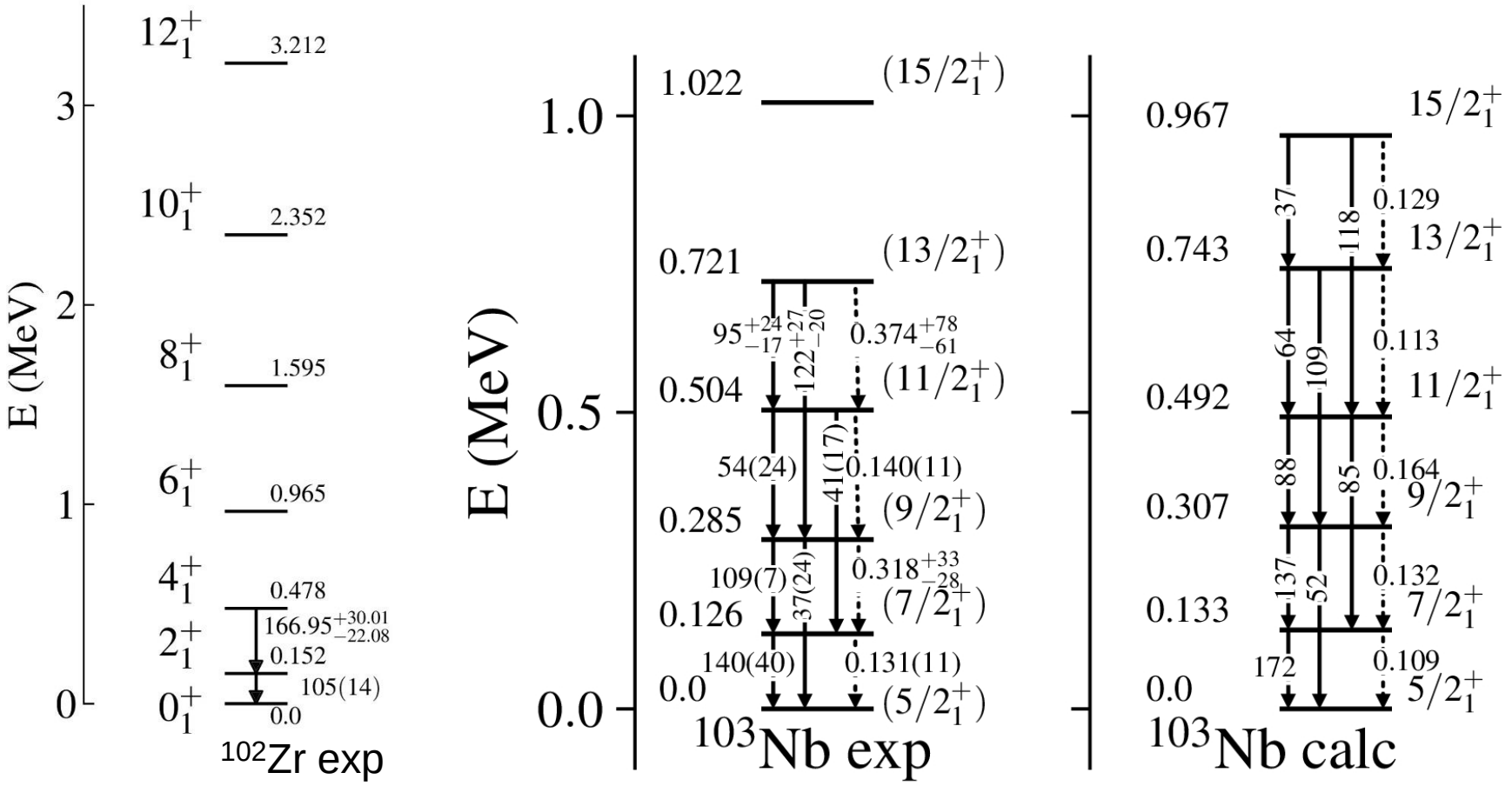
Type I QPT within the intruder B configuration



^{93}Nb Weak coupling

$[(L=0^{+}_{1;A}) \otimes \pi(1g_{9/2})]$ J = $9/2^{+}$
$[(L=2^{+}_{1;A}) \otimes \pi(1g_{9/2})]$ J = $5/2^{+}$, $7/2^{+}$, $9/2^{+}$, $11/2^{+}$,$13/2^{+}$
$[(L=4^{+}_{1;A}) \otimes \pi(1g_{9/2})]$ J = $1/2^{+}$, $3/2^{+}$, $5/2^{+}$, $7/2^{+}$, $9/2^{+}$, $11/2^{+}$, $13/2^{+}$, $15/2^{+}$

$[(L=0^{+}_{1;B}) \otimes \pi(1g_{9/2})]$ J = $9/2^{+}$
$[(L=2^{+}_{1;B}) \otimes \pi(1g_{9/2})]$ J = $5/2^{+}$, $7/2^{+}$, $9/2^{+}$, $11/2^{+}$,$13/2^{+}$

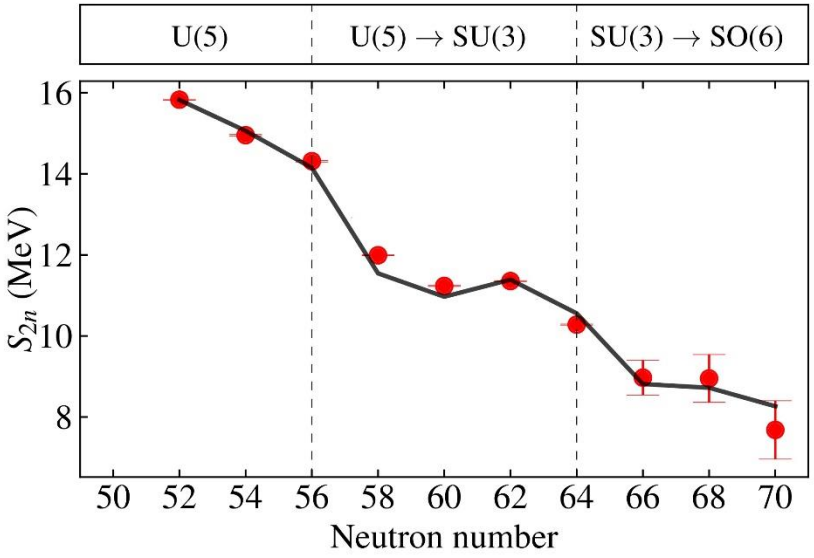


^{103}Nb Strong coupling $5/2^+[422]$

$[\,(L=0^+_{1;B}\,,\,2^+_{1;B}\,,\,4^+_{1;B}\,,\,6^+_{1;B}\,\ldots\,)\otimes\pi(1g_{9/2})\,]\,J$

Two-neutron separation energies

Zr



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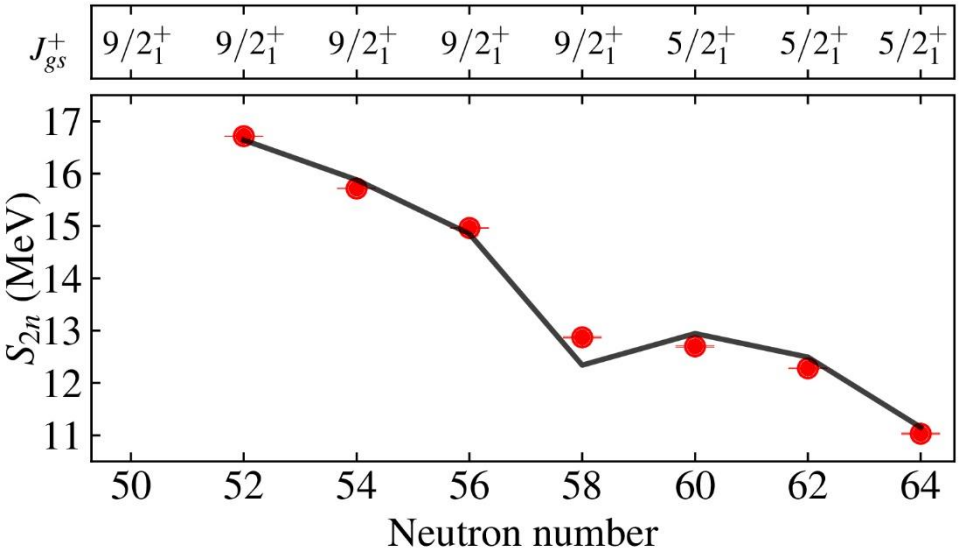


n=52-56: S_{2n} straight line, g.s. spherical (seniority-like)

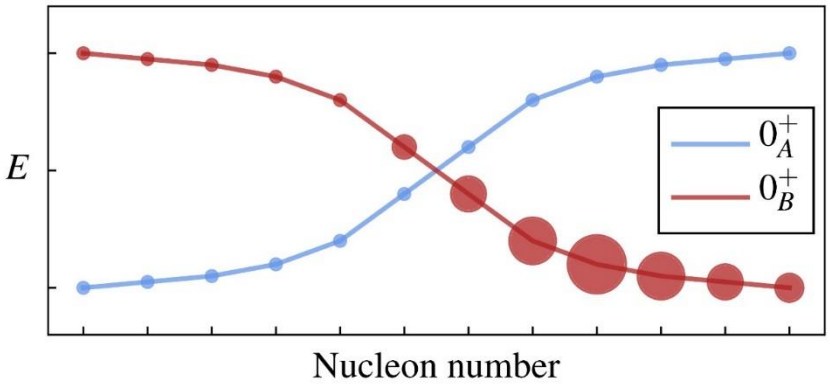
After 56: S_{2n} first goes down (due to sub-shell closure at 56)
then flattens $\Rightarrow$ 1st order U(5) $\rightarrow$ SU(3) QPT

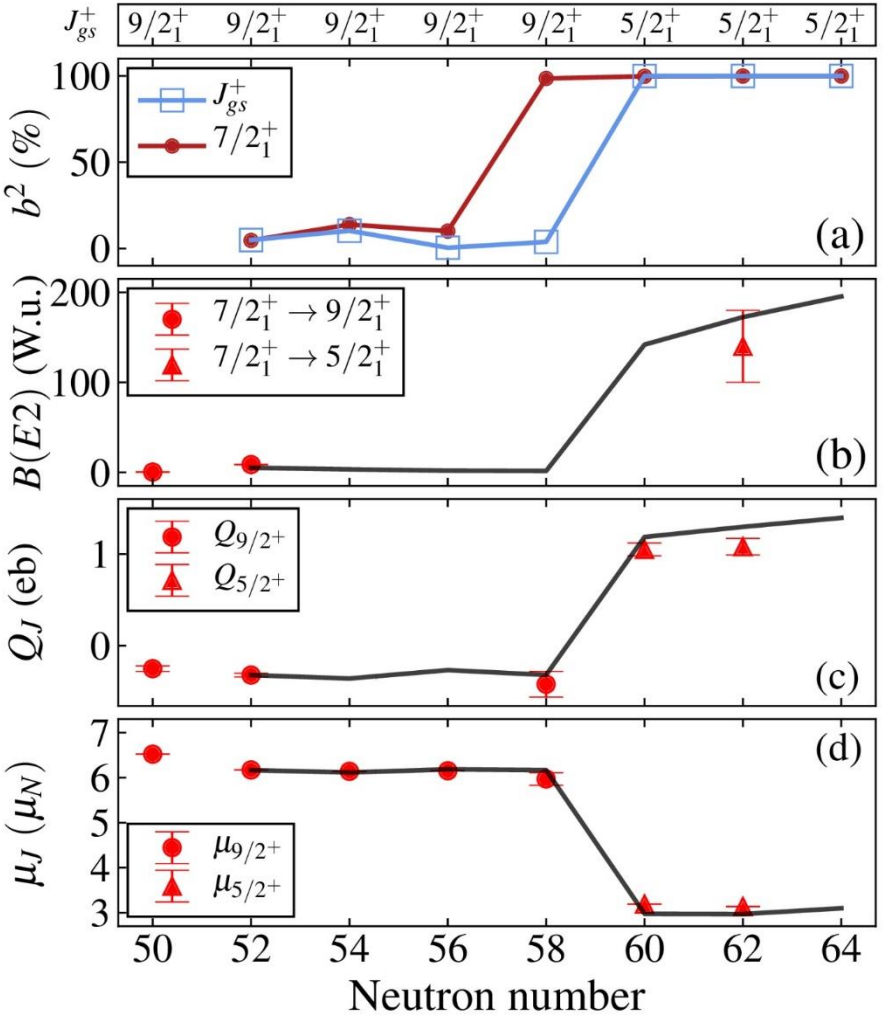
After 62: S_{2n} goes down (increasing deformation)

n=66-70: S_{2n} flattens (SU(3) $\rightarrow$ SO(6) crossover)

Concluding remarks

- Intertwined Quantum Phase Transitions (IQPTs)





- IBFM-CM framework
  Quantitative description of configuration mixing and related QPTs in odd-mass nuclei

- Nb isotopes
  Zr core + single-j fermion
  (Normal A configuration) $\otimes$ $\pi(1g_{9/2})$
  (Intruder B configuration) $\otimes$ $\pi(1g_{9/2})$

- Detailed analysis discloses
  a Type II QPT (abrupt crossing of normal and intruder states) accompanied by
  a Type I QPT (gradual shape evolution and transition from weak to strong coupling within the Intruder configuration), thus demonstrating IQPTs in odd-mass nuclei

- The observed IQPTs in odd-A Nb isotopes echo the IQPTs previously found in the adjacent even-even Zr isotopes

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Thank you