

Ab-initio description of monopole resonances in light- and medium-mass nuclei

Methods, uses and recent results

Andrea Porro, PhD Student
IRFU, CEA, Université Paris-Saclay

SUPERVISORS

Thomas Duguet
Vittorio Somà

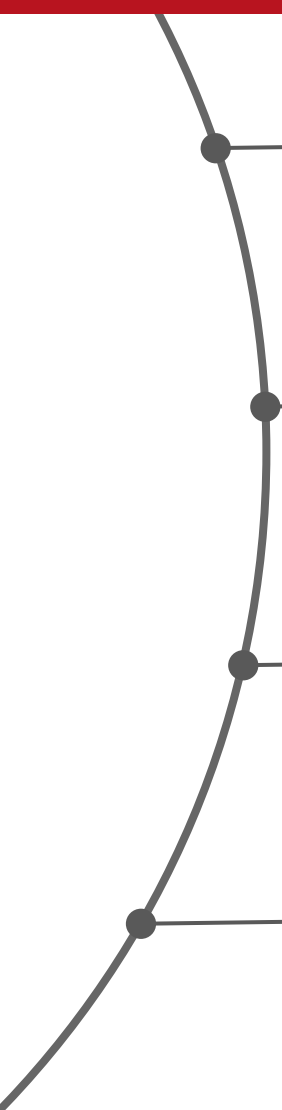
July 21st, 2023

XVII International Symposium on Capture Gamma-Ray
Spectroscopy and Related Topics – CGS17

ILL Grenoble



Outline

- 
- Introduction
 - Formalism
 - Results
 - Conclusions

Outline



Introduction

Ab-initio methods and Giant Resonances

Formalism

Results

Conclusions

Ab initio nuclear structure

Global philosophy

$$H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

Ab initio nuclear structure

Global philosophy

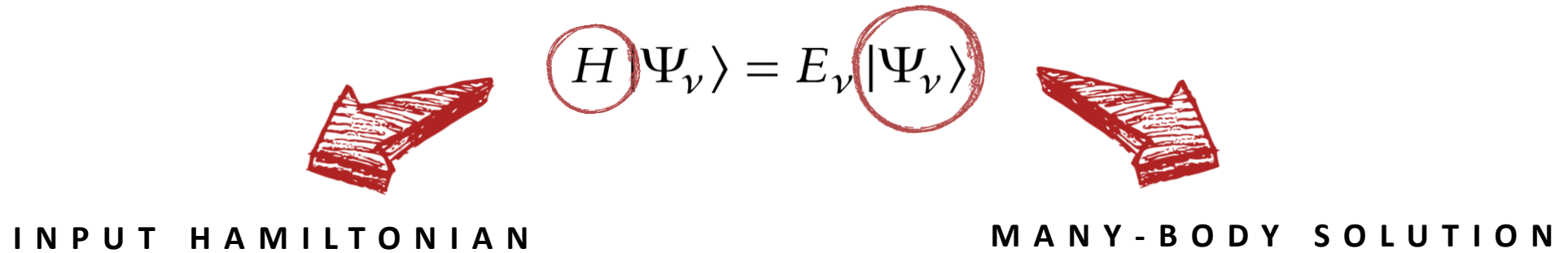


$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

INPUT HAMILTONIAN

Ab initio nuclear structure

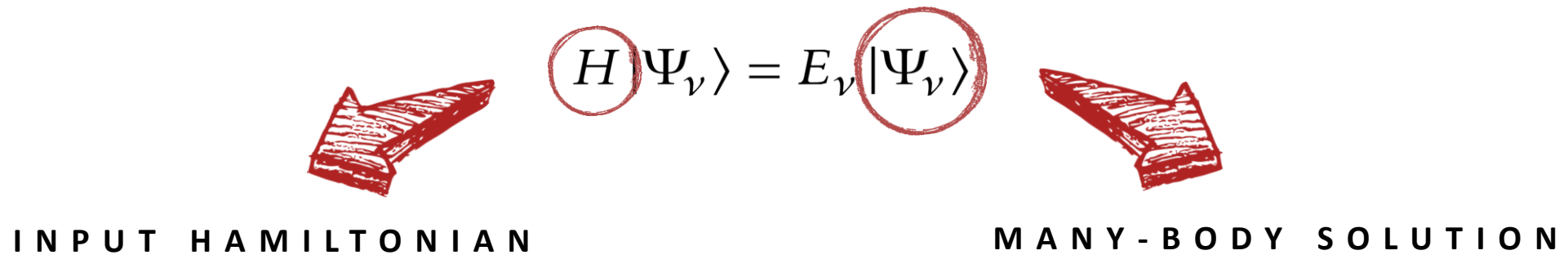
Global philosophy



Ab initio nuclear structure

Global philosophy

The approximate solution must be **systematically improvable** and approach the exact solution in a **well-defined limit**.



Ab initio nuclear structure

Global philosophy

The approximate solution must be **systematically improvable** and approach the exact solution in a **well-defined limit**.

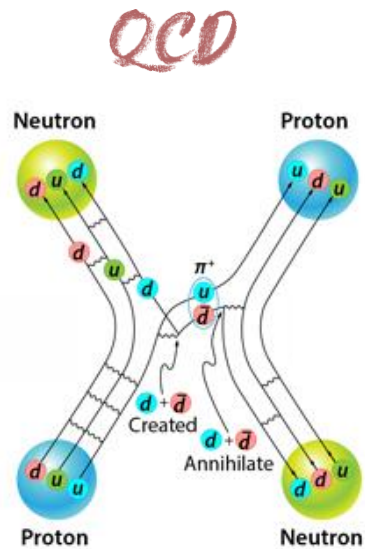
$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$



INPUT HAMILTONIAN



MANY-BODY SOLUTION



Ab initio nuclear structure

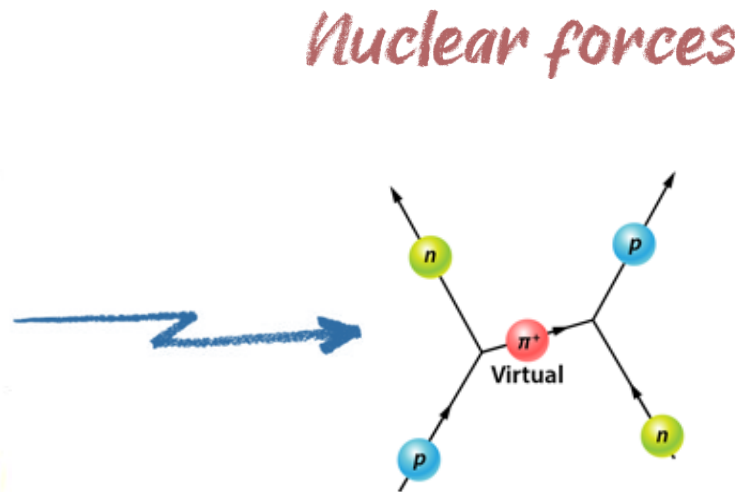
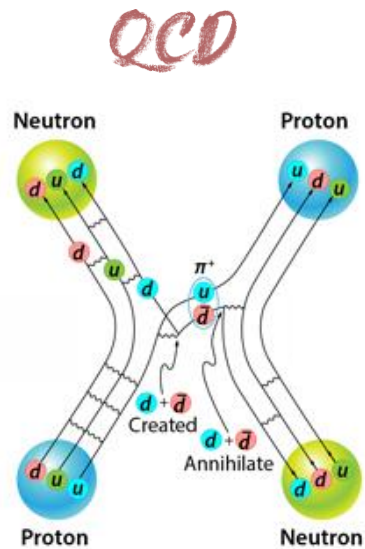
Global philosophy

The approximate solution must be **systematically improvable** and approach the exact solution in a **well-defined limit**.

$$H|\Psi_v\rangle = E_v|\Psi_v\rangle$$

INPUT HAMILTONIAN

MANY-BODY SOLUTION



Ab initio nuclear structure

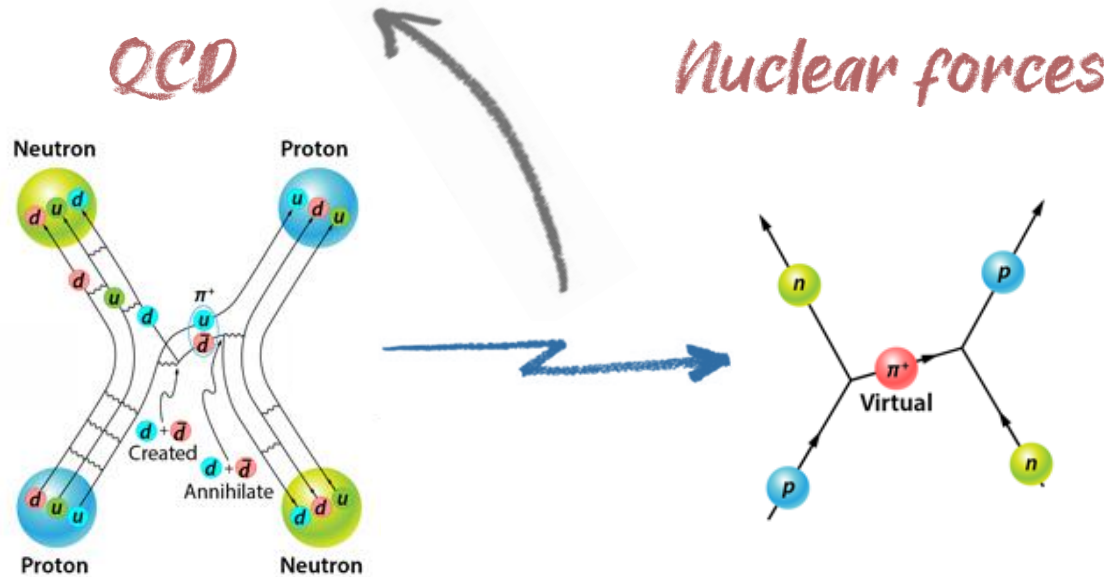
Global philosophy

The approximate solution must be **systematically improvable** and approach the exact solution in a **well-defined limit**.

$$H|\Psi_v\rangle = E_v|\Psi_v\rangle$$

INPUT HAMILTONIAN

MANY-BODY SOLUTION



Ab initio nuclear structure

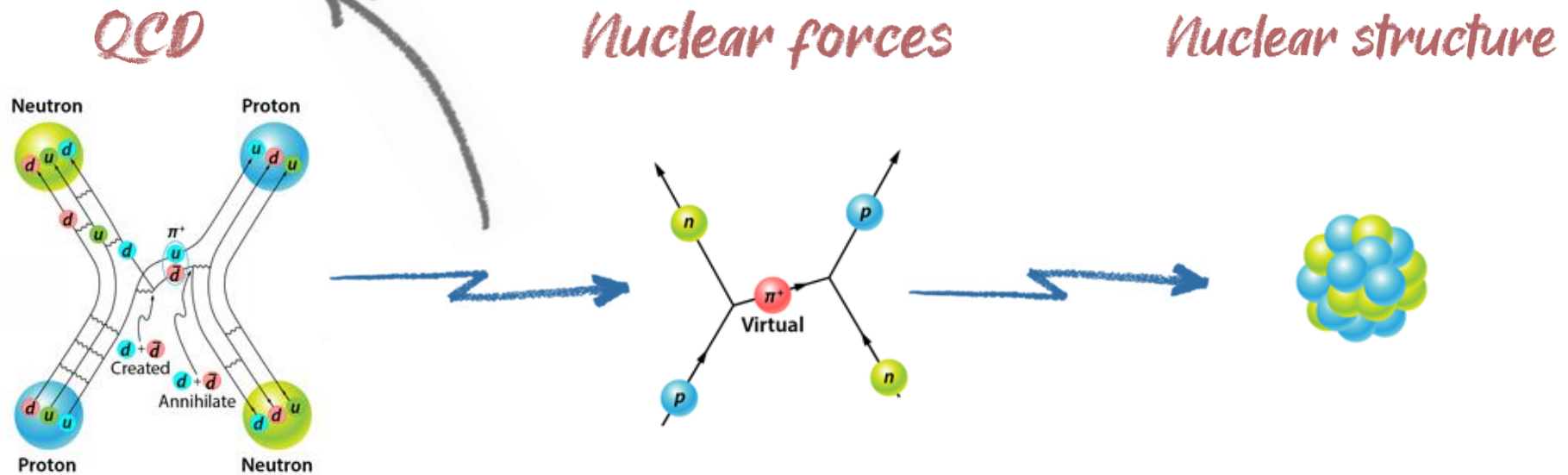
Global philosophy

The approximate solution must be **systematically improvable** and approach the exact solution in a **well-defined limit**.

$$H|\Psi_v\rangle = E_v|\Psi_v\rangle$$

INPUT HAMILTONIAN

MANY-BODY SOLUTION



Ab initio nuclear structure

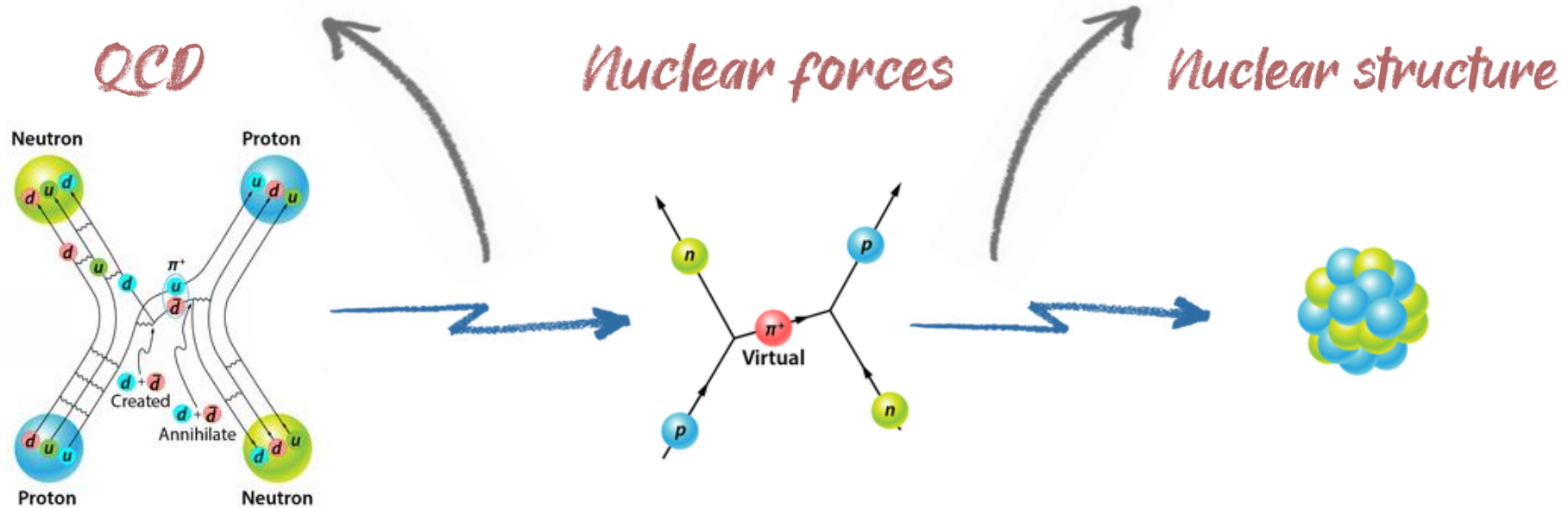
Global philosophy

The approximate solution must be **systematically improvable** and approach the exact solution in a **well-defined limit**.

$$H|\Psi_v\rangle = E_v|\Psi_v\rangle$$

INPUT HAMILTONIAN

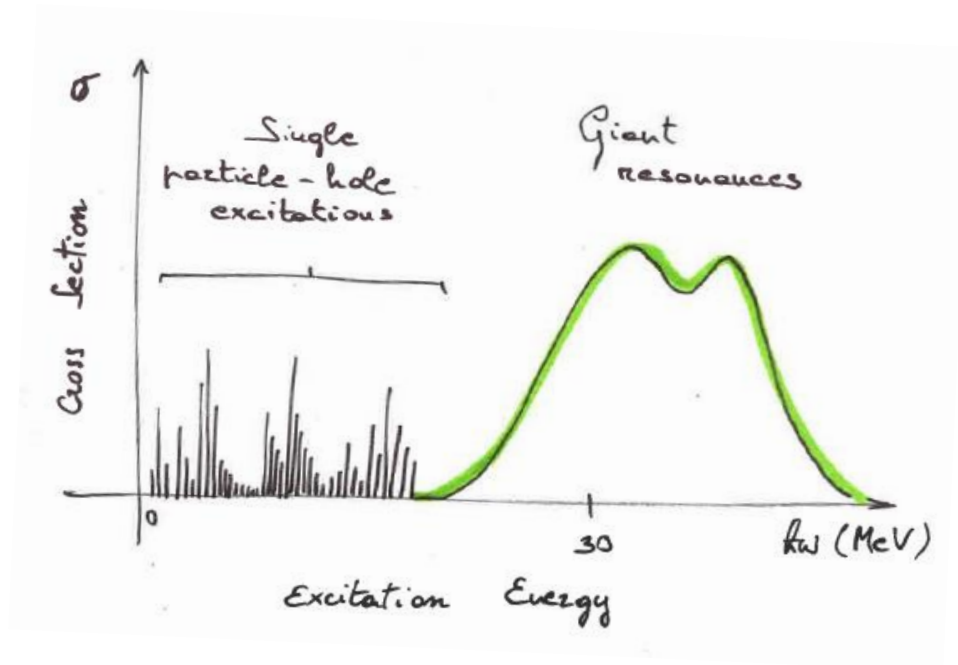
MANY-BODY SOLUTION



Giant Resonances

Dual nature of nucleus

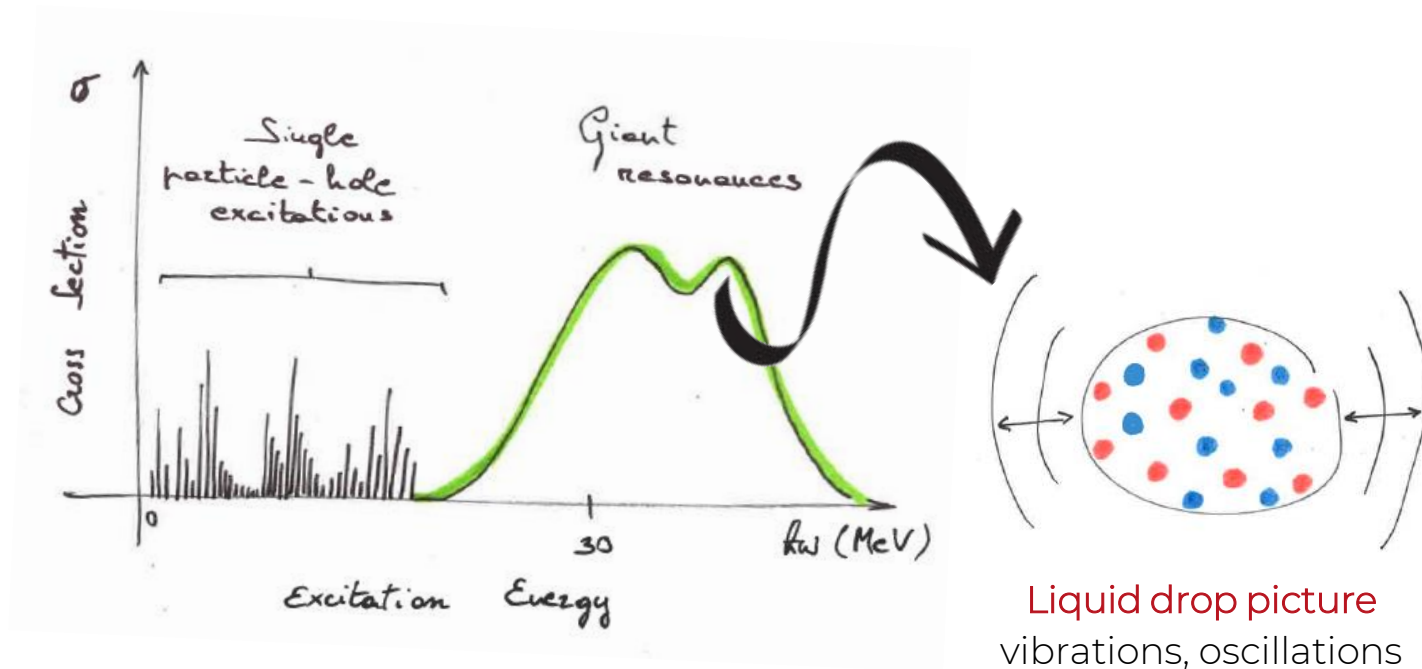
- single-particle features
- collective behaviour



Giant Resonances

Dual nature of nucleus

- single-particle features
- collective behaviour



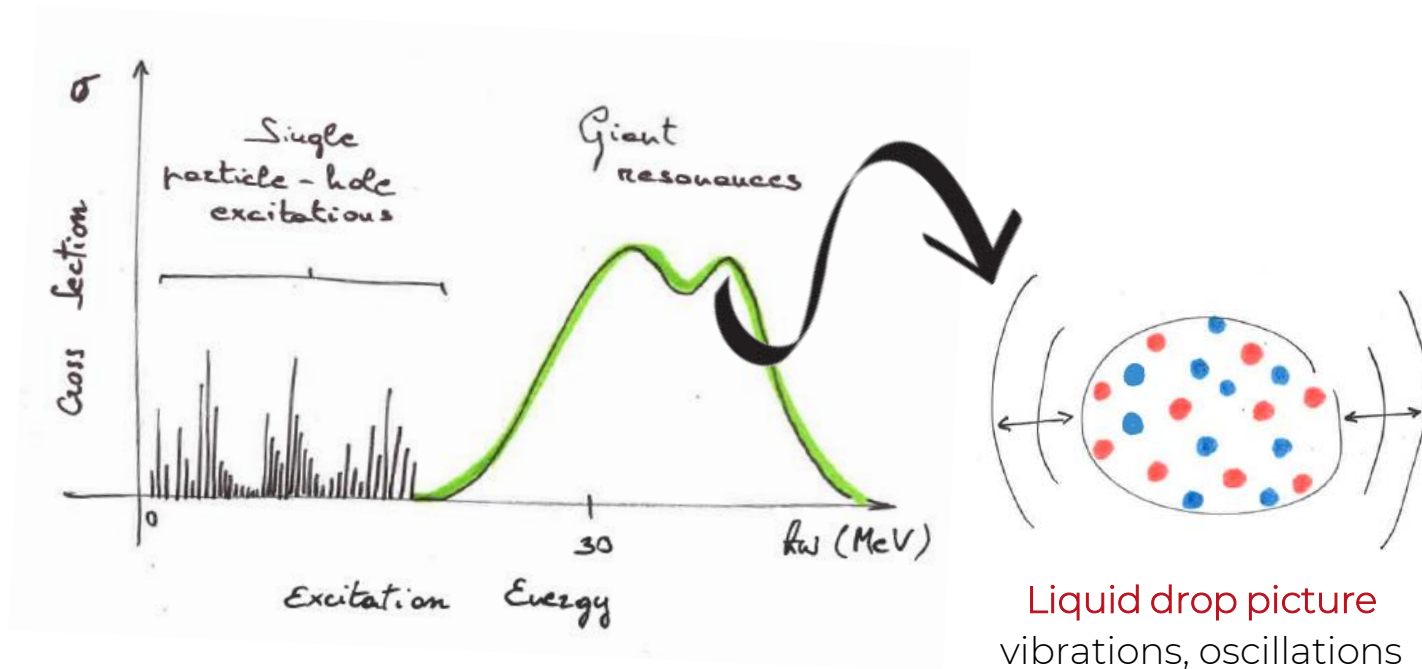
Giant Resonances (GRs)

clearest manifestation of collective motion

Giant Resonances

Dual nature of nucleus

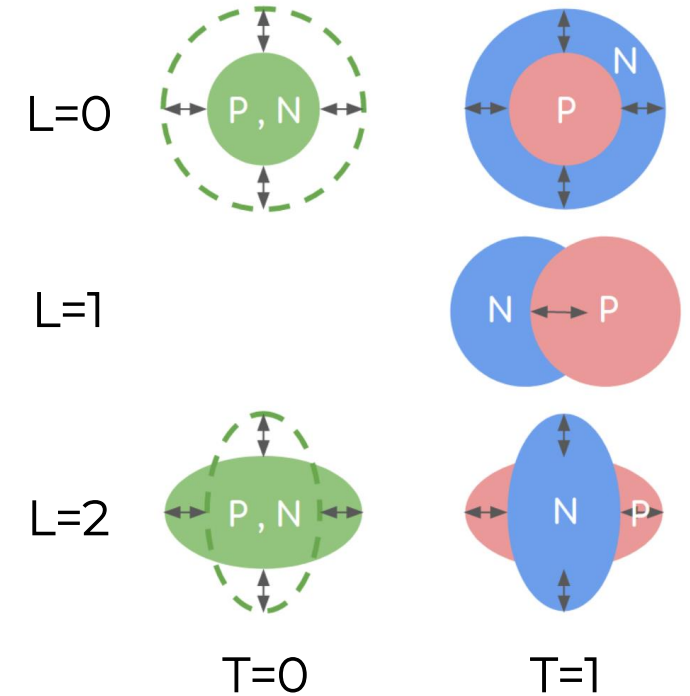
- single-particle features
- collective behaviour



Liquid drop picture
vibrations, oscillations

Giant Resonances (GRs)

clearest manifestation of collective motion



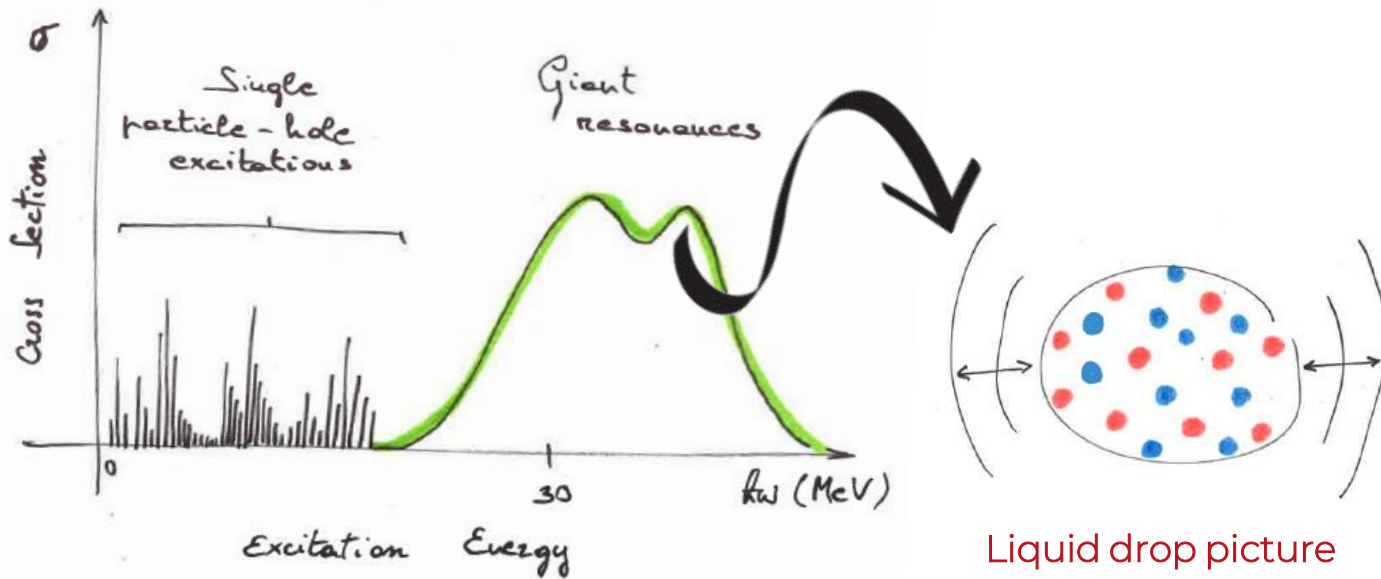
Giant Resonances

Dual nature of nucleus

- single-particle features
- collective behaviour

Compression-mode resonances

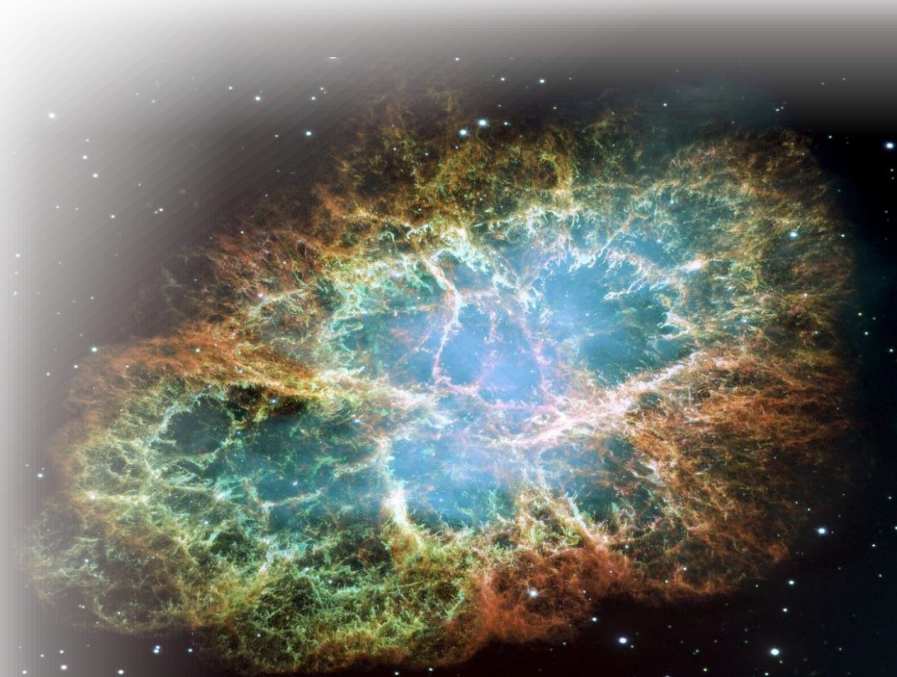
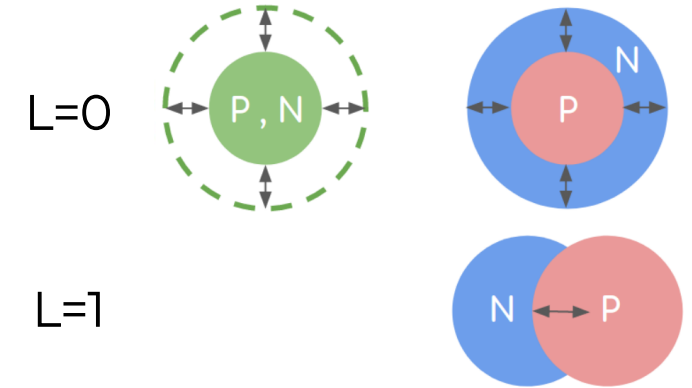
- Incompressibility of nuclear matter K_∞
- Nuclear Equation of State
- Core-collapse supernova explosion



Liquid drop picture
vibrations, oscillations

Giant Resonances (GRs)

clearest manifestation of collective motion



Giant Monopole Resonance

Giant Monopole Resonance

What is it ?

- Collective excitation (breathing mode)
- Involving most if not all the nucleons
- Coherent particle-hole excitations

Giant Monopole Resonance

What is it ?

- Collective excitation (breathing mode)
- Involving most if not all the nucleons
- Coherent particle-hole excitations

Why studying again GMR ?

- Renewed experimental interest
- Investigate new physics

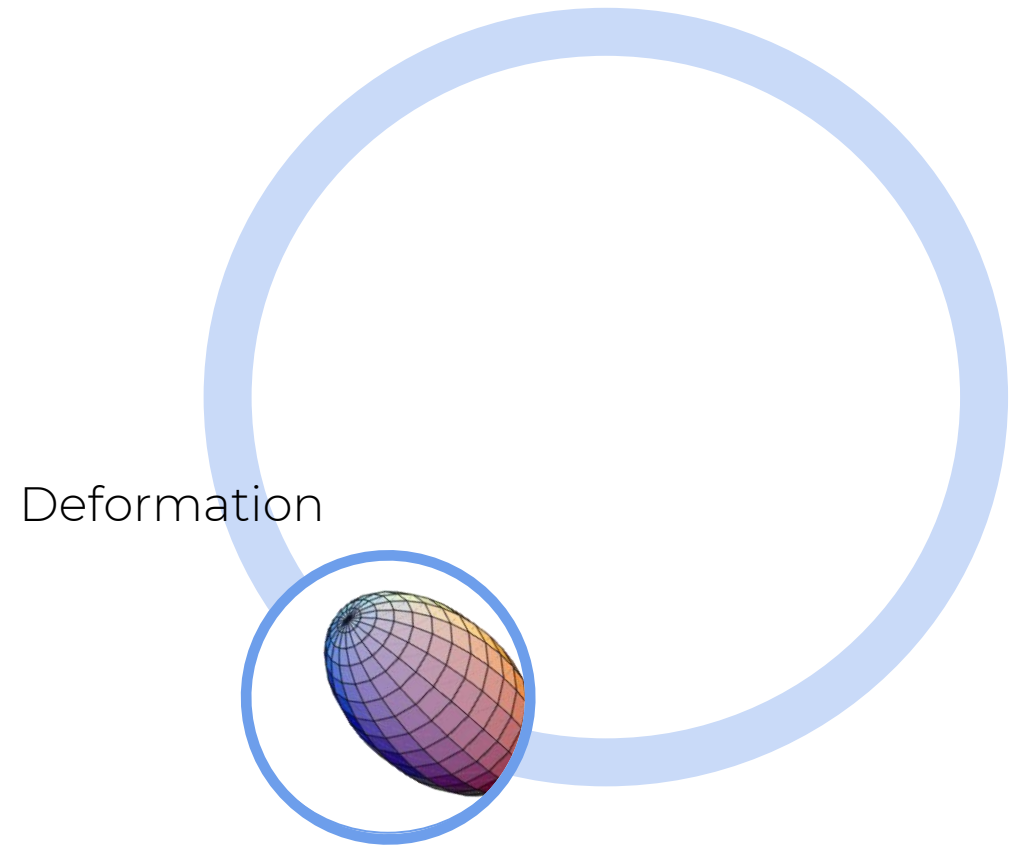
Giant Monopole Resonance

What is it ?

- Collective excitation (breathing mode)
- Involving most if not all the nucleons
- Coherent particle-hole excitations

Why studying again GMR ?

- Renewed experimental interest
- Investigate new physics



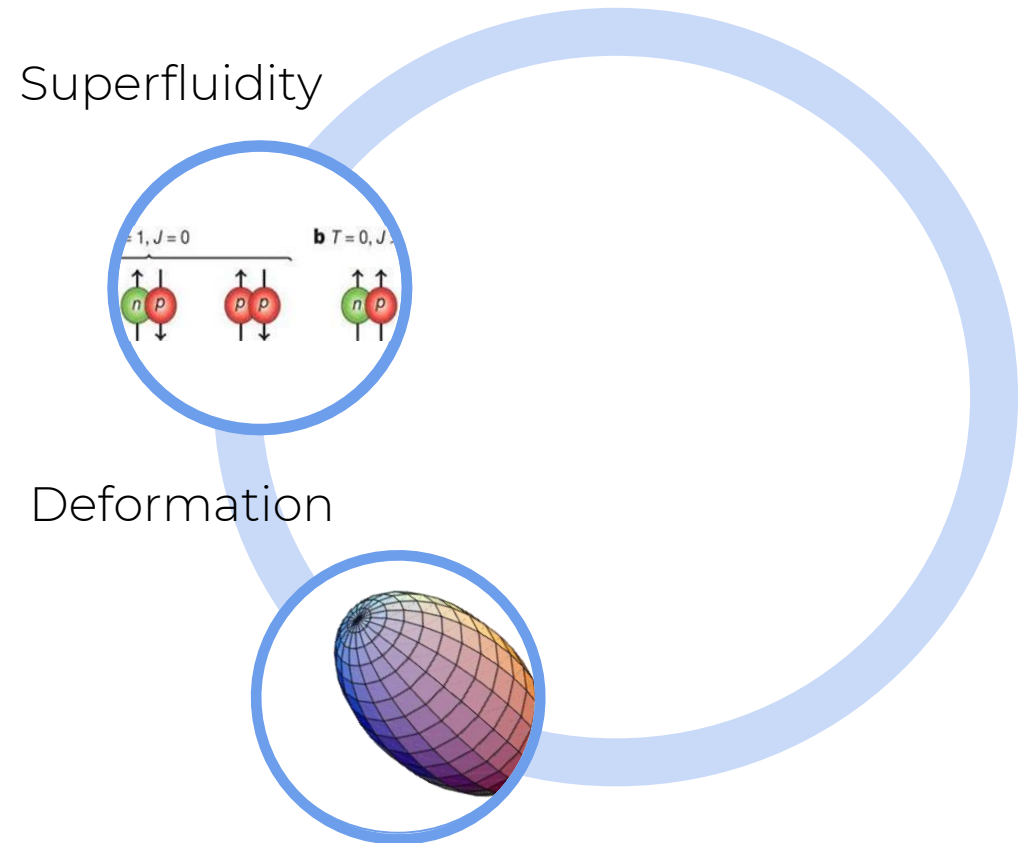
Giant Monopole Resonance

What is it ?

- Collective excitation (breathing mode)
- Involving most if not all the nucleons
- Coherent particle-hole excitations

Why studying again GMR ?

- Renewed experimental interest
- Investigate new physics



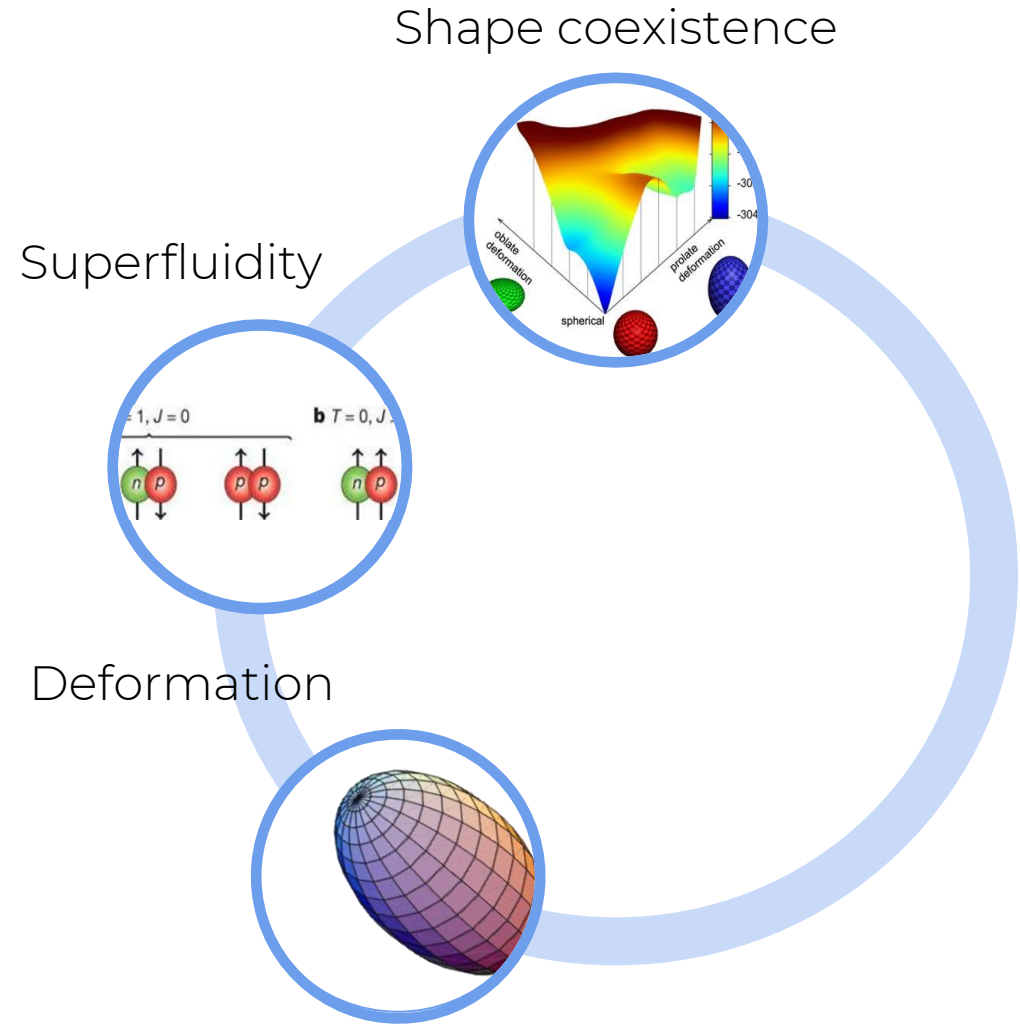
Giant Monopole Resonance

What is it ?

- Collective excitation (breathing mode)
- Involving most if not all the nucleons
- Coherent particle-hole excitations

Why studying again GMR ?

- Renewed experimental interest
- Investigate new physics



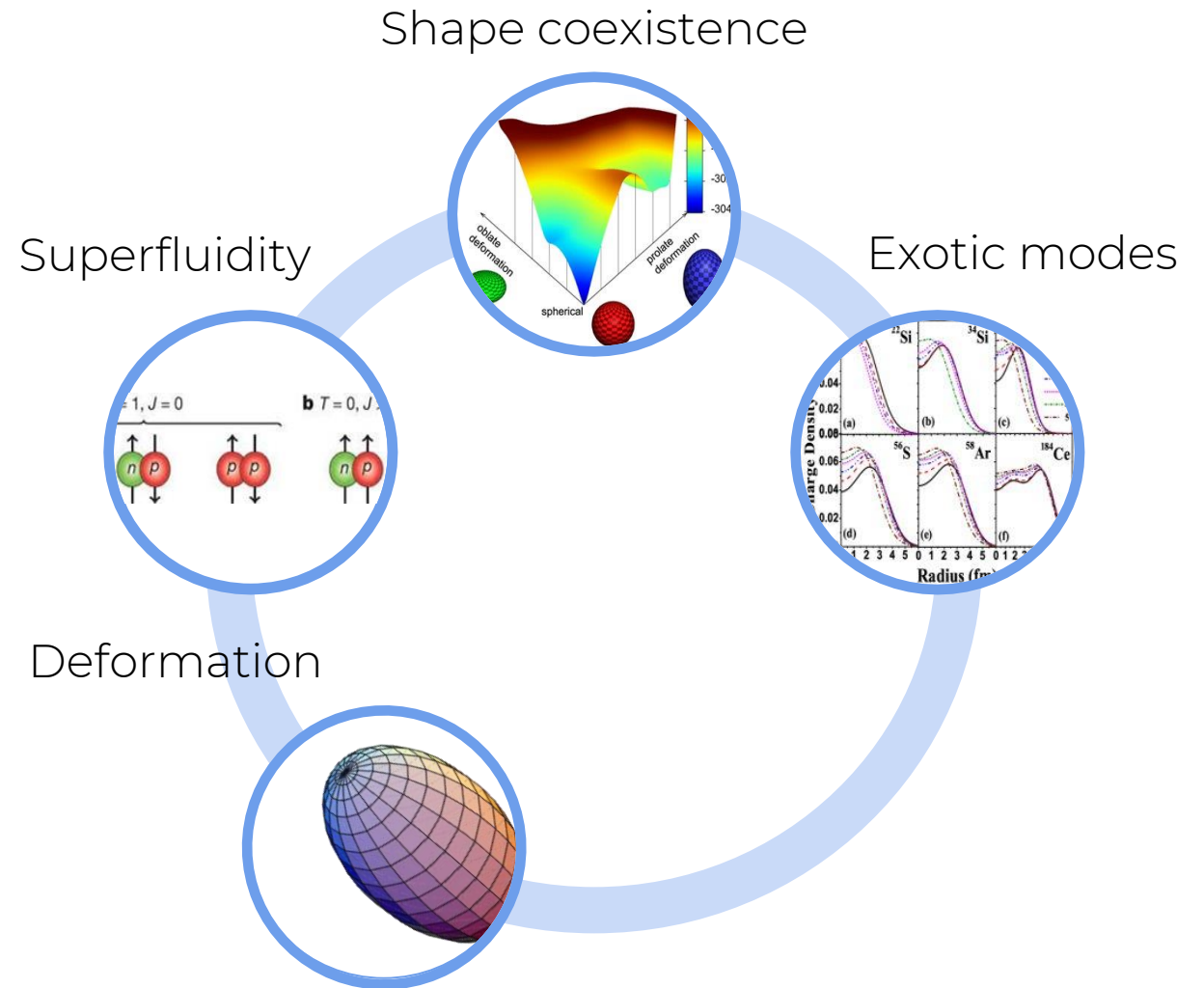
Giant Monopole Resonance

What is it ?

- Collective excitation (breathing mode)
- Involving most if not all the nucleons
- Coherent particle-hole excitations

Why studying again GMR ?

- Renewed experimental interest
- Investigate new physics



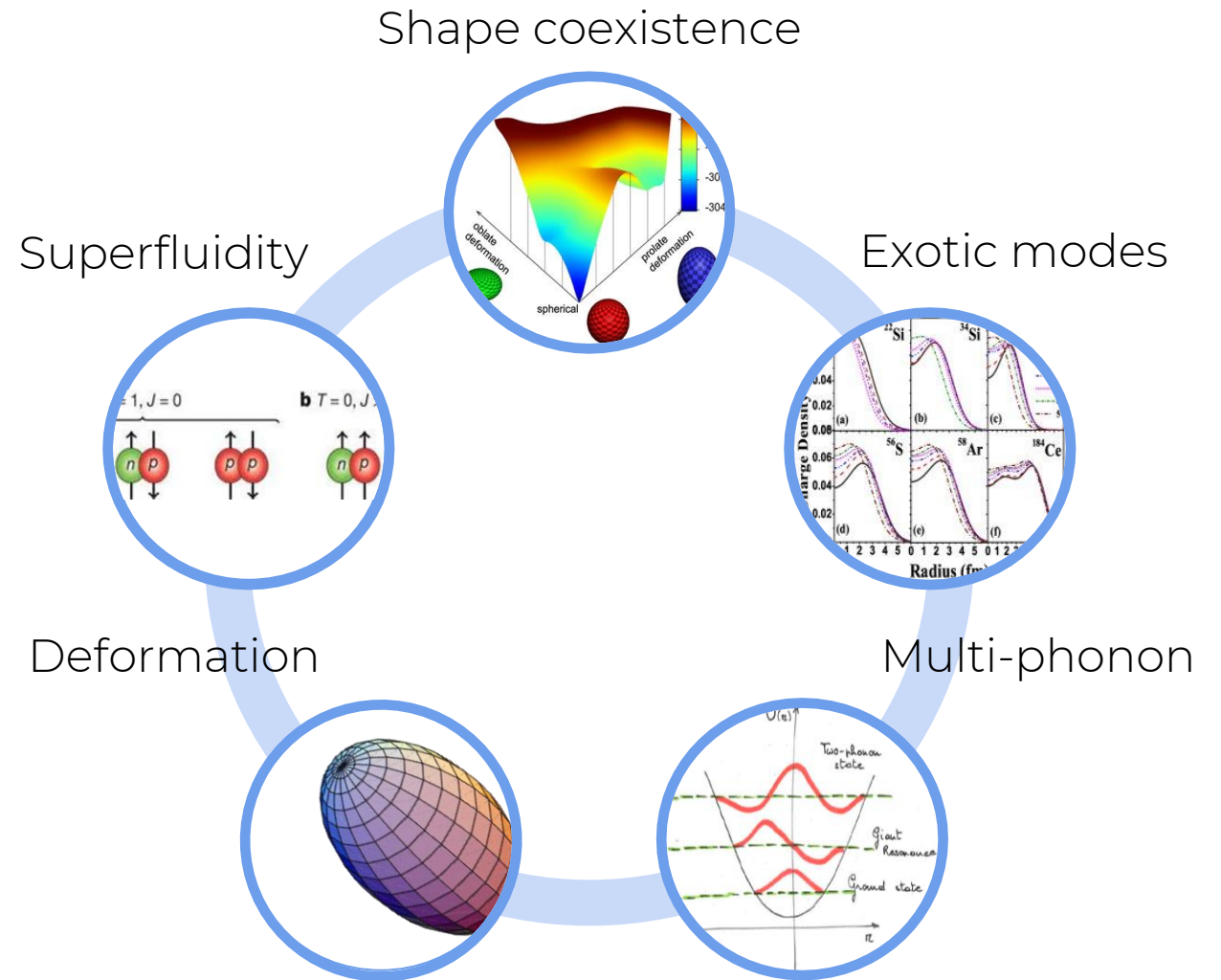
Giant Monopole Resonance

What is it ?

- Collective excitation (breathing mode)
- Involving most if not all the nucleons
- Coherent particle-hole excitations

Why studying again GMR ?

- Renewed experimental interest
- Investigate new physics



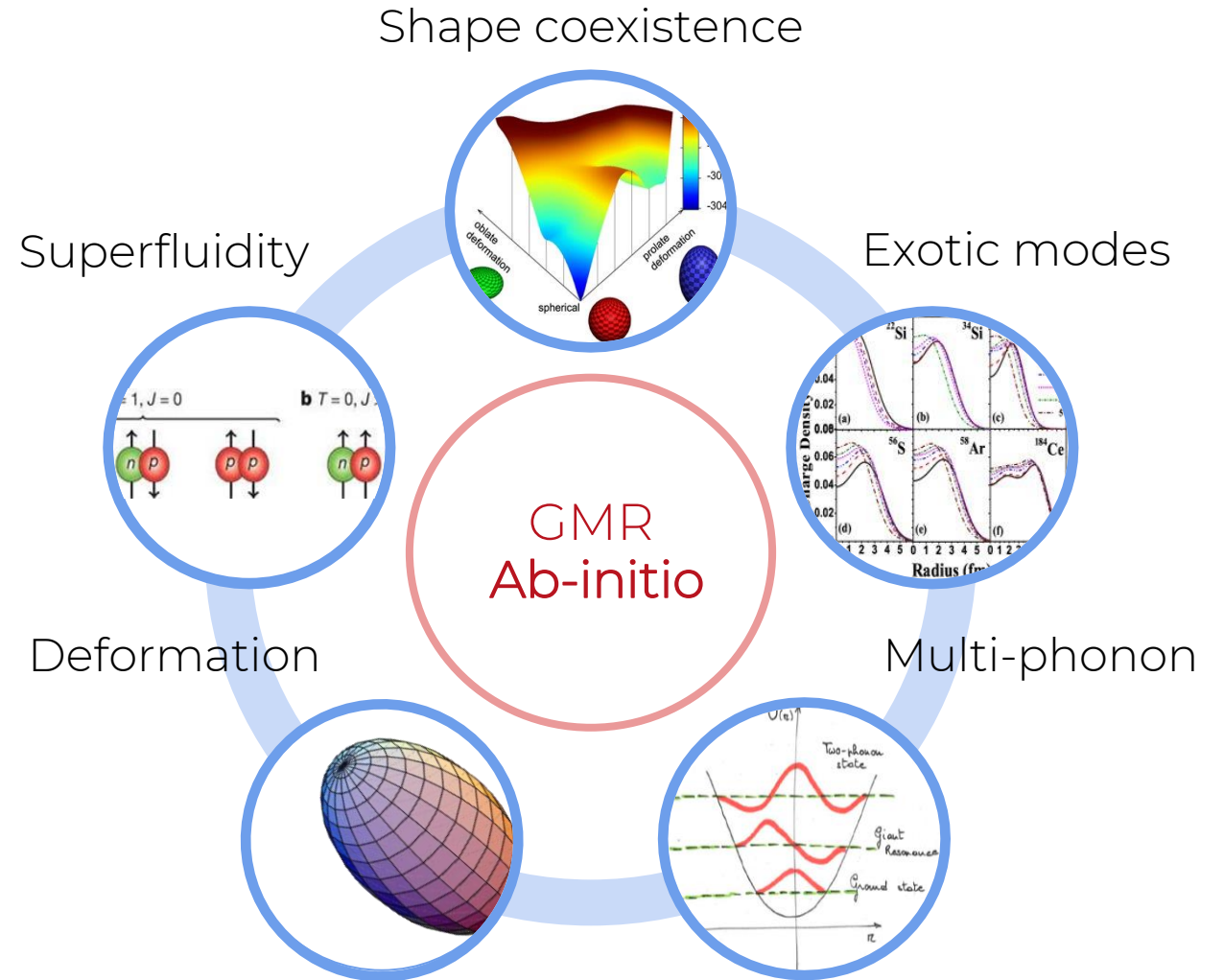
Giant Monopole Resonance

What is it ?

- Collective excitation (breathing mode)
- Involving most if not all the nucleons
- Coherent particle-hole excitations

Why studying again GMR ?

- Renewed experimental interest
- Investigate new physics



Giant Monopole Resonance

What is it ?

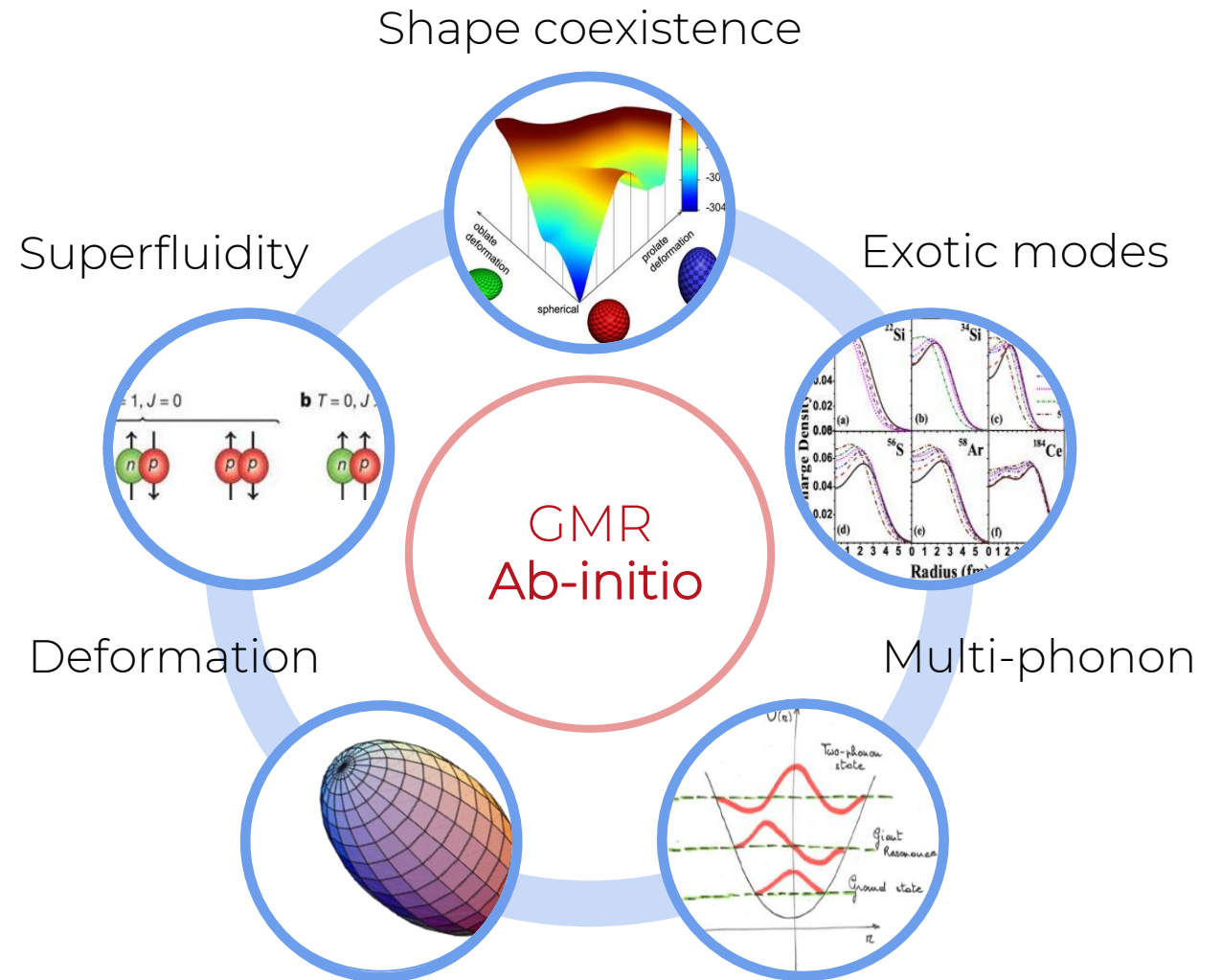
- Collective excitation (breathing mode)
- Involving most if not all the nucleons
- Coherent particle-hole excitations

Why studying again GMR ?

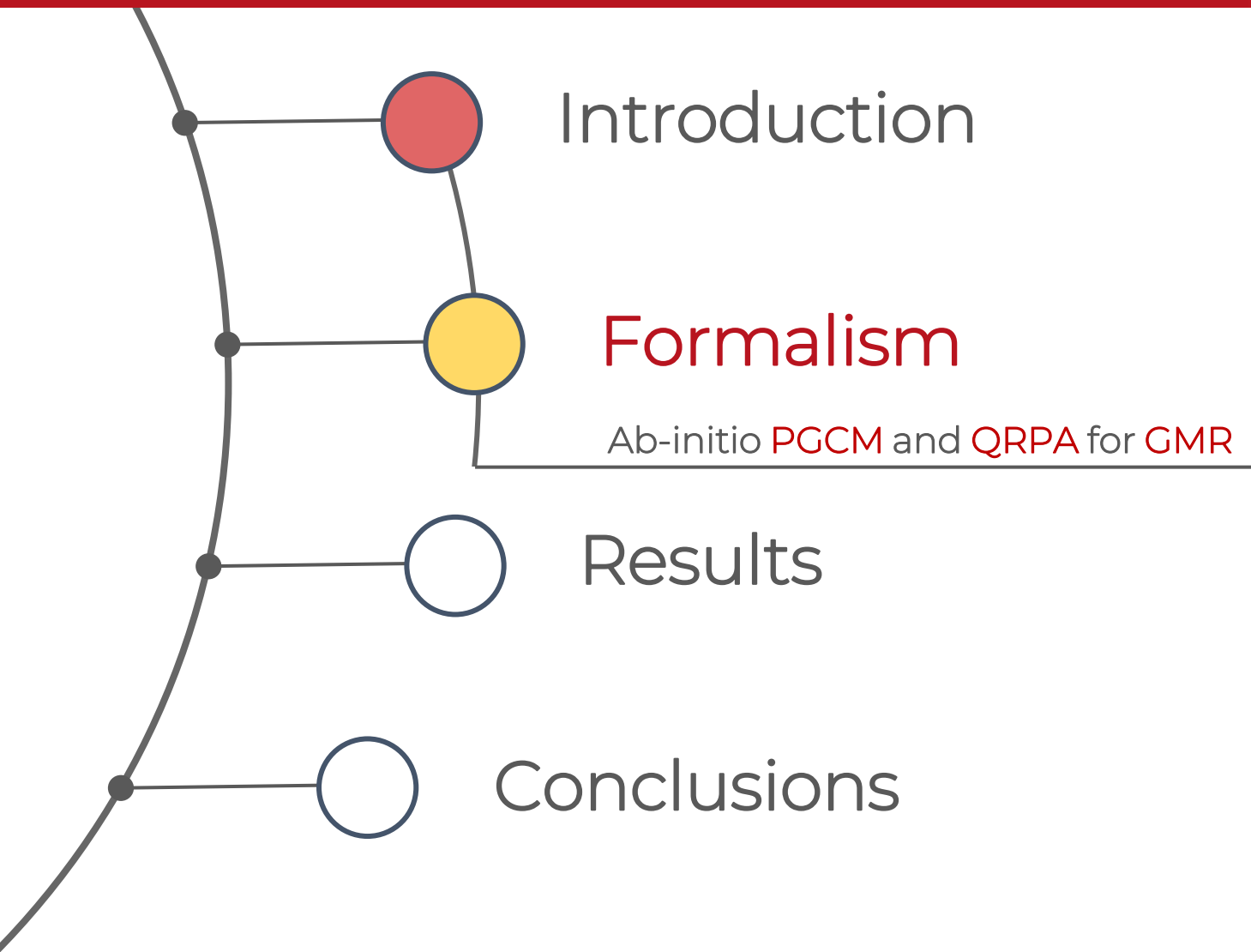
- Renewed experimental interest
- Investigate new physics

Much is still to be understood !

- No systematic studies (EDF as well)
 - Very generic numerical codes needed
- Ab-initio description still seminal



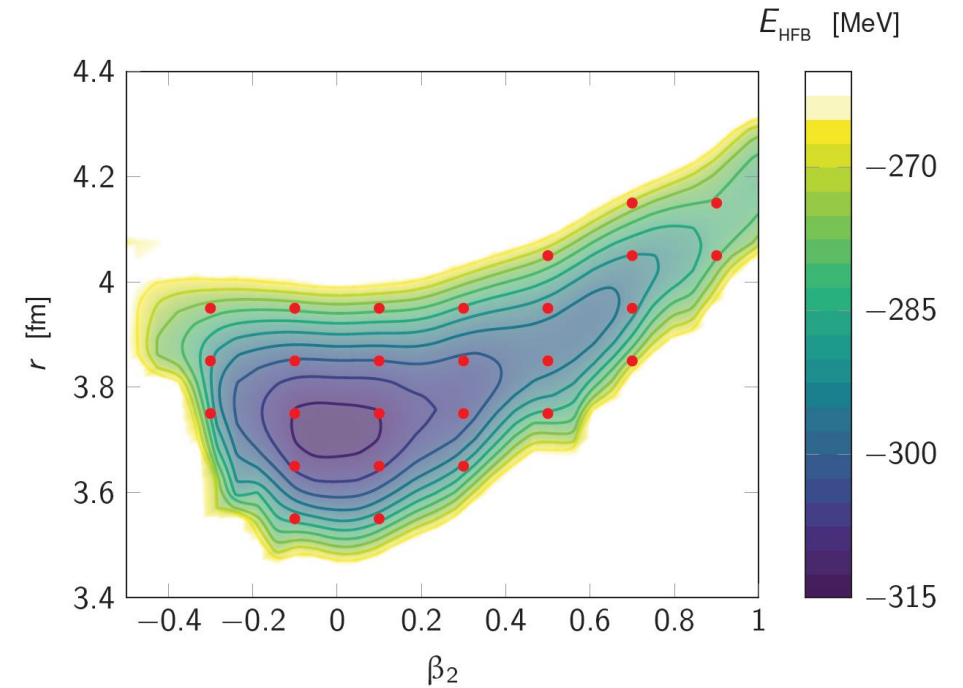
Outline



Projected Generator Coordinate Method

Schrödinger equation

$$H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

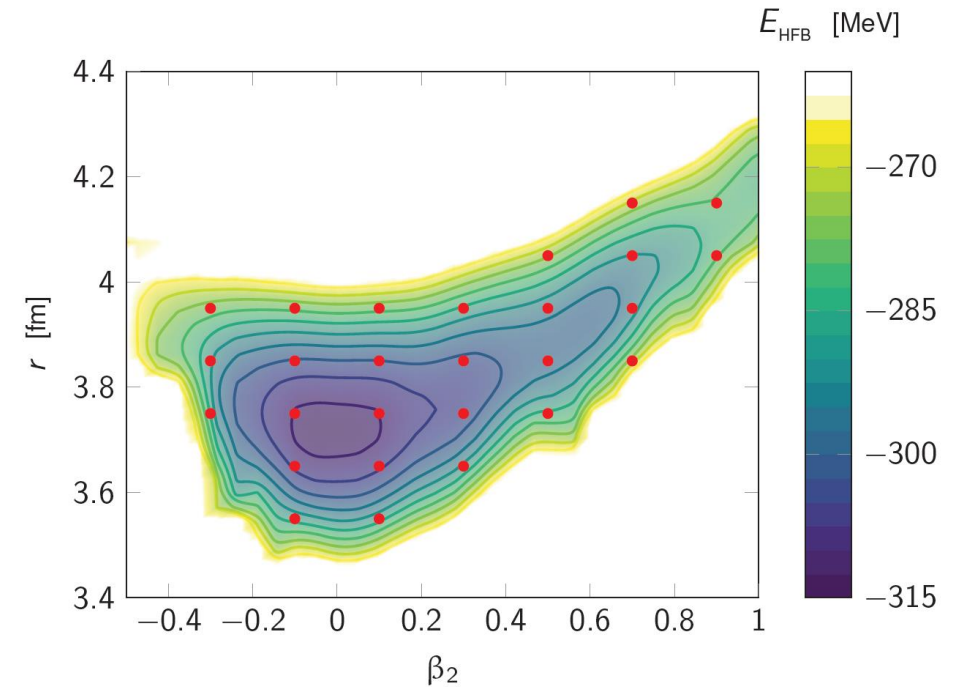


Projected Generator Coordinate Method

Schrödinger equation

$$H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

Open-shell systems



Projected Generator Coordinate Method

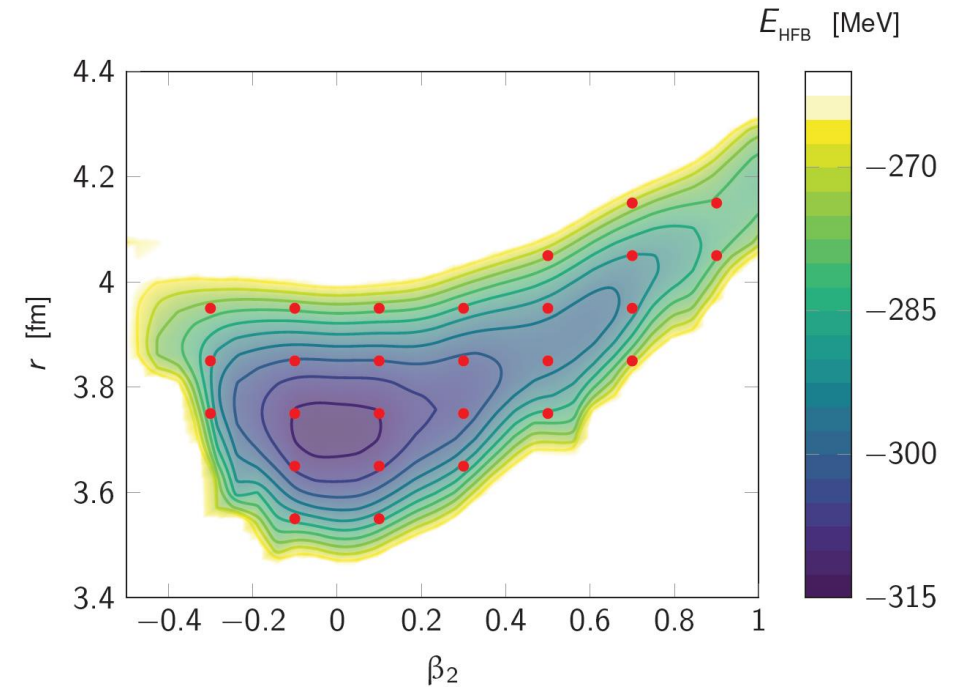
Schrödinger equation

$$H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

Open-shell systems



Strong **static correlations**



Projected Generator Coordinate Method

Schrödinger equation

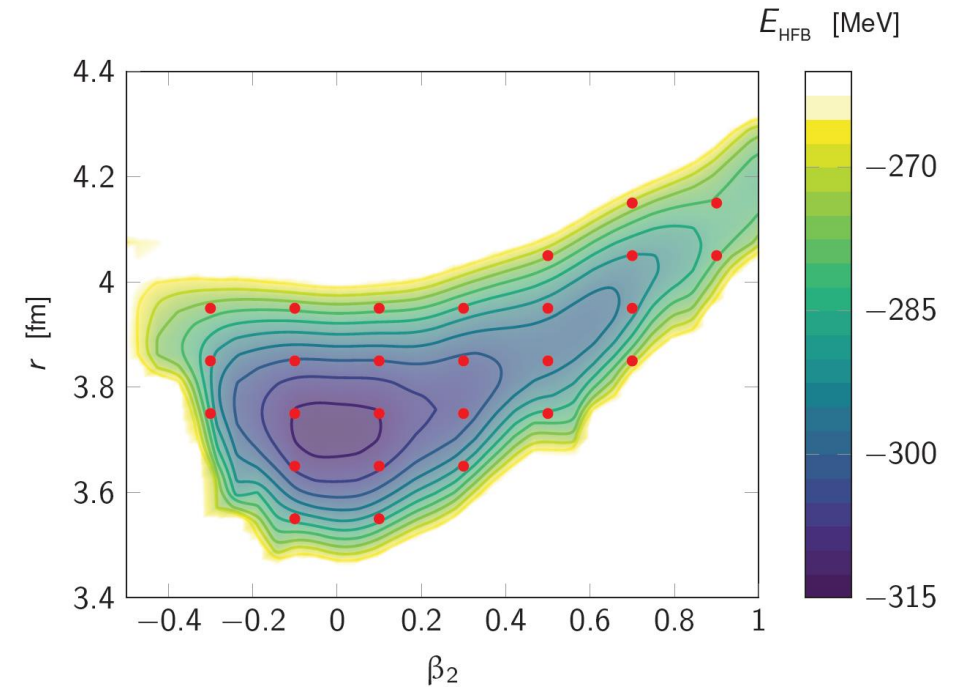
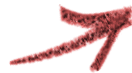
$$H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

Open-shell systems

Symmetry-breaking reference states



Strong static correlations



Projected Generator Coordinate Method

Schrödinger equation

$$H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

Open-shell systems

Symmetry-breaking reference states

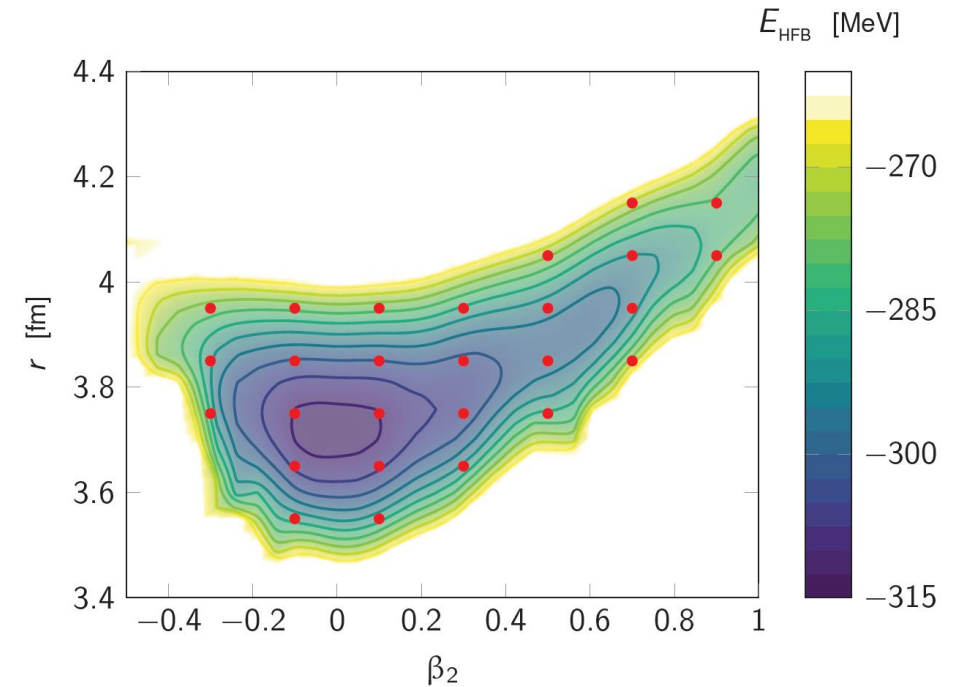


Strong static correlations



1 Constrained HFB solutions

$$|\Phi(r^2, \beta_2)\rangle$$



Projected Generator Coordinate Method

Schrödinger equation

$$H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

Open-shell systems

Symmetry-breaking reference states

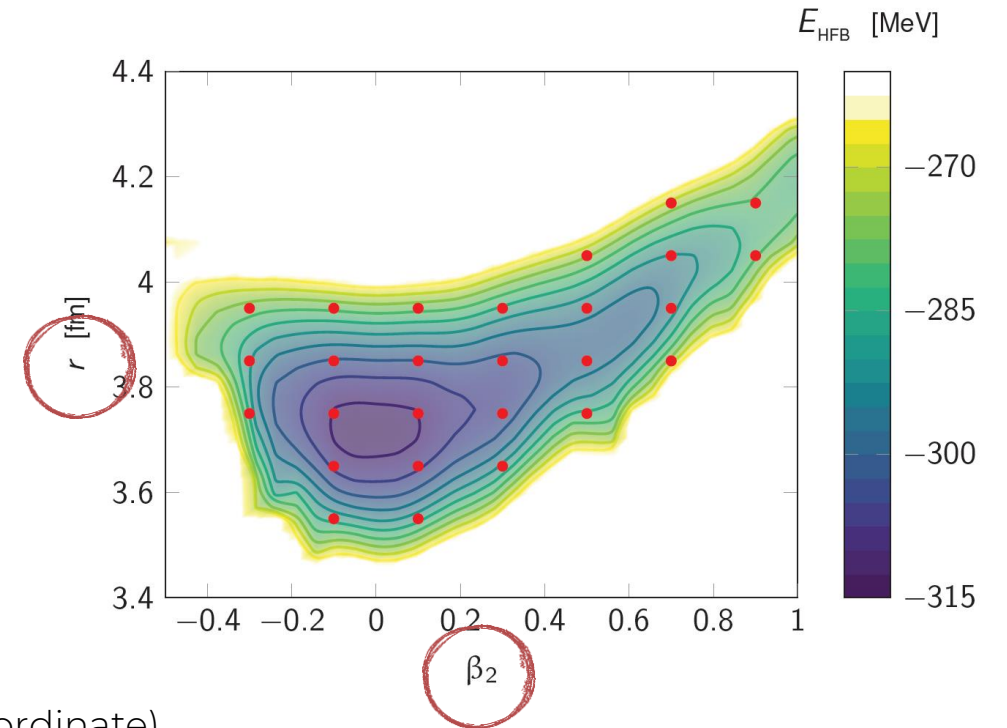
Strong static correlations

1 Constrained HFB solutions

$$|\Phi(r^2, \beta_2)\rangle$$

Generator coordinates

(can be in principle whatever coordinate)



Projected Generator Coordinate Method

Schrödinger equation

$$H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

Open-shell systems

Symmetry-breaking reference states

Strong static correlations

1 Constrained HFB solutions

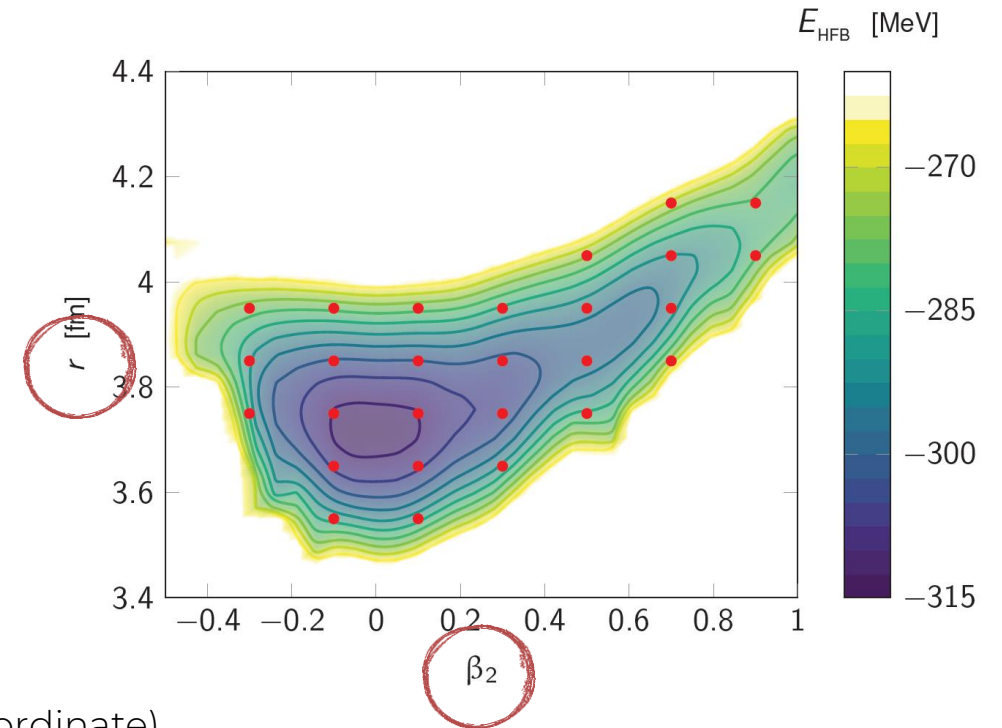
$$|\Phi(r^2, \beta_2)\rangle$$

2 PGCM Ansatz

Generator coordinates

(can be in principle whatever coordinate)

$$|\Psi_\nu\rangle = \sum_{r^2, \beta_2} f_\nu(r^2, \beta_2) |\Phi(r^2, \beta_2)\rangle$$



Projected Generator Coordinate Method

Schrödinger equation

$$H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

Open-shell systems

Symmetry-breaking reference states

Strong static correlations

1 Constrained HFB solutions

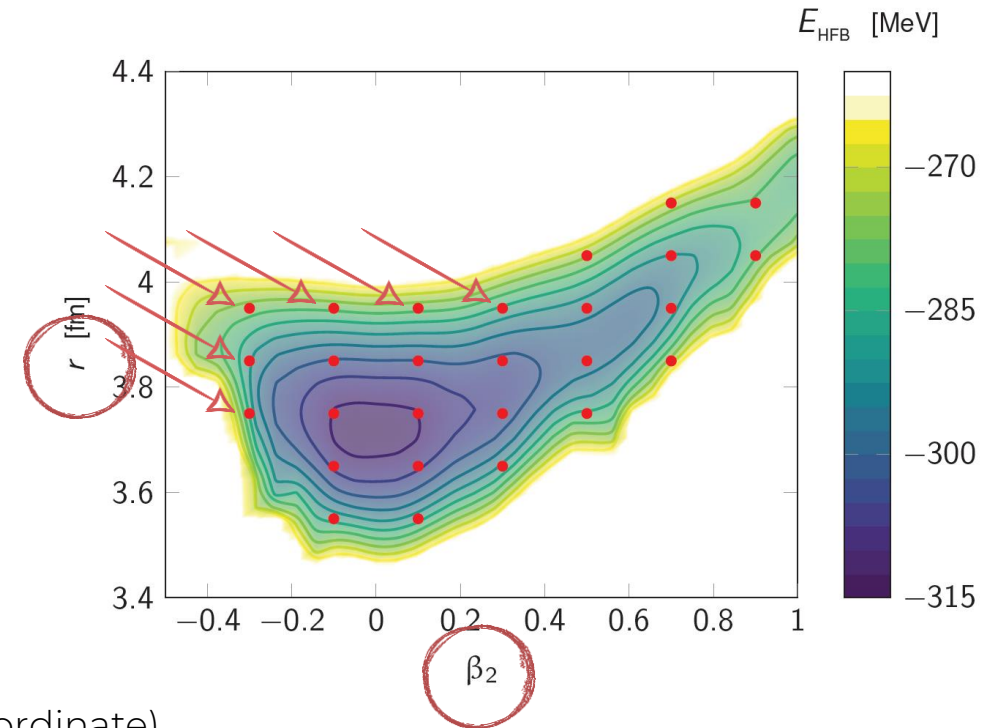
$$|\Phi(r^2, \beta_2)\rangle$$

Generator coordinates

(can be in principle whatever coordinate)

2 PGCM Ansatz

$$|\Psi_\nu\rangle = \sum_{r^2, \beta_2} f_\nu(r^2, \beta_2) |\Phi(r^2, \beta_2)\rangle$$



Projected Generator Coordinate Method

Schrödinger equation

$$H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

Open-shell systems

Symmetry-breaking reference states

Strong static correlations

1 Constrained HFB solutions

$$|\Phi(r^2, \beta_2)\rangle$$

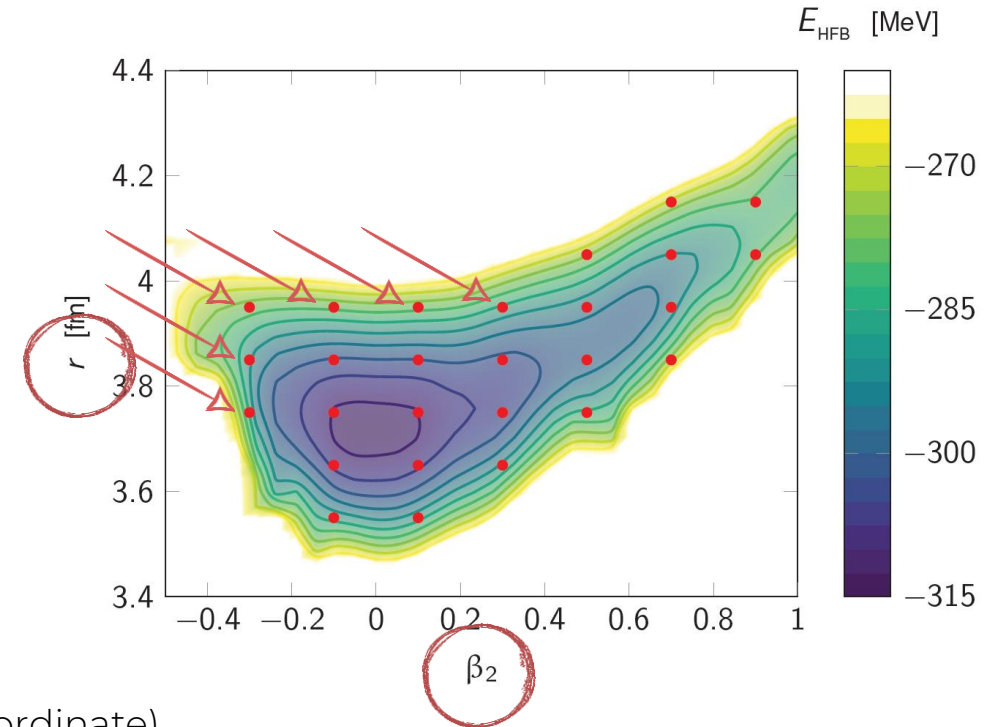
Generator coordinates

(can be in principle whatever coordinate)

2 PGCM Ansatz

$$|\Psi_\nu\rangle = \sum_{r^2, \beta_2} f_\nu(r^2, \beta_2) |\Phi(r^2, \beta_2)\rangle$$

Linear coefficients



Projected Generator Coordinate Method

Schrödinger equation

$$H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

Open-shell systems

Symmetry-breaking reference states

Strong static correlations

1 Constrained HFB solutions

$$|\Phi(r^2, \beta_2)\rangle$$

Generator coordinates

(can be in principle whatever coordinate)

2 PGCM Ansatz

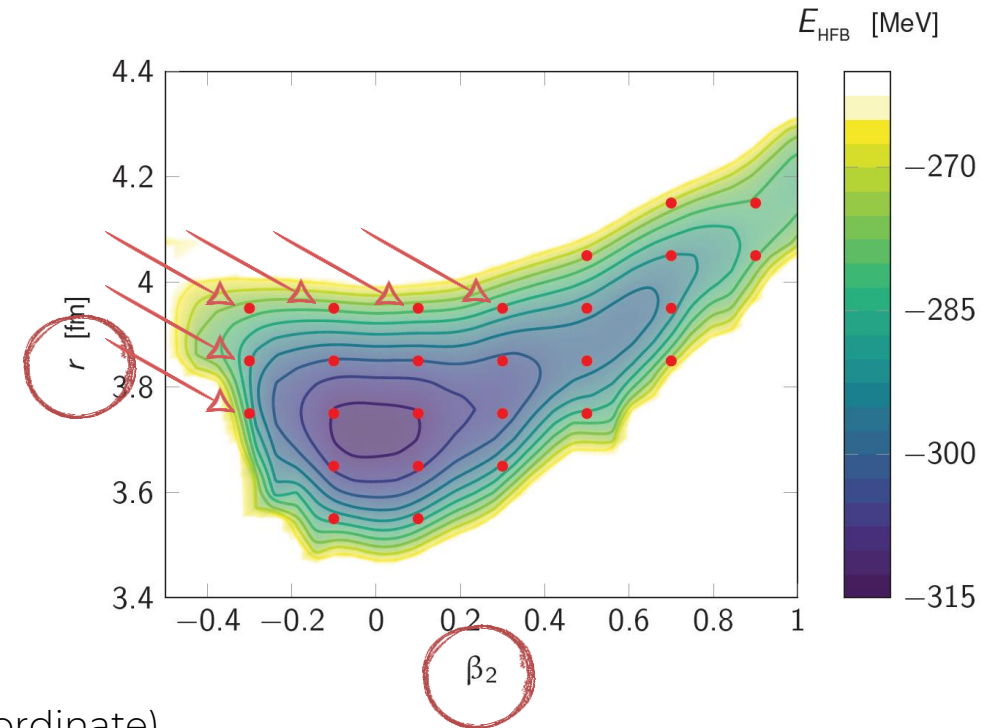
$$|\Psi_\nu\rangle = \sum_{r^2, \beta_2} f_\nu(r^2, \beta_2) |\Phi(r^2, \beta_2)\rangle$$

3 HWG Equation

Variational method

$$\delta \frac{\langle \Psi_\nu | H | \Psi_\nu \rangle}{\langle \Psi_\nu | \Psi_\nu \rangle} = 0$$

Linear coefficients



Projected Generator Coordinate Method

Schrödinger equation

$$H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$$

Open-shell systems

Symmetry-breaking reference states

Strong static correlations

1 Constrained HFB solutions

$$|\Phi(r^2, \beta_2)\rangle$$

Generator coordinates

(can be in principle whatever coordinate)

2 PGCM Ansatz

$$|\Psi_\nu\rangle = \sum_{r^2, \beta_2} f_\nu(r^2, \beta_2) |\Phi(r^2, \beta_2)\rangle$$

Linear coefficients

3 HWG Equation

Variational method

$$\delta \frac{\langle \Psi_\nu | H | \Psi_\nu \rangle}{\langle \Psi_\nu | \Psi_\nu \rangle} = 0$$

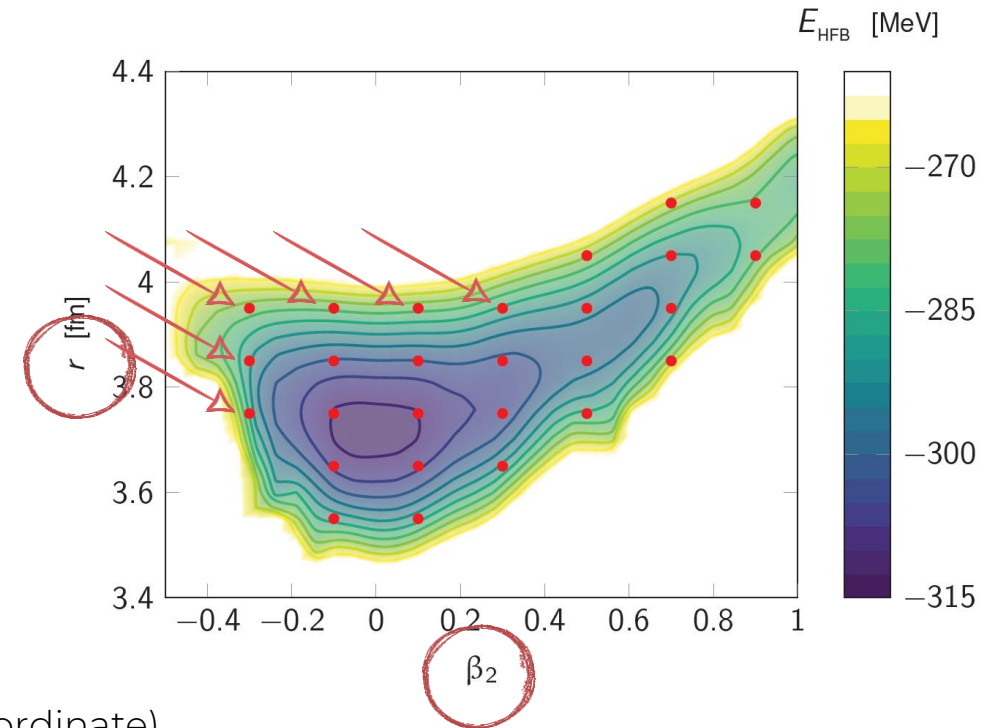
Schrödinger-like equation

$$\sum_q \left[\mathcal{H}(p, q) - E_\nu \mathcal{N}(p, q) \right] f_\nu(q) = 0$$

Kernels evaluation

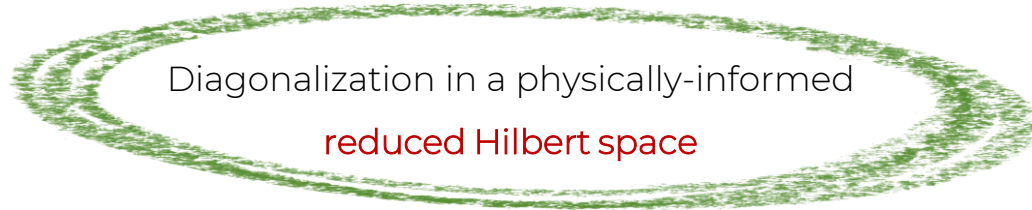
$$\mathcal{H}(p, q) \equiv \langle \Phi(p) | H | \Phi(q) \rangle$$

$$\mathcal{N}(p, q) \equiv \langle \Phi(p) | \Phi(q) \rangle$$



Projected Generator Coordinate Method

Schrödinger equation $H |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle$



1 Constrained HFB solutions

$$|\Phi(r^2, \beta_2)\rangle$$

Generator coordinates
(can be in principle whatever coordinate)

2 PGCM Ansatz

$$|\Psi_\nu\rangle = \sum_{r^2, \beta_2} f_\nu(r^2, \beta_2) |\Phi(r^2, \beta_2)\rangle$$

Linear coefficients

3 HWG Equation

Variational method

$$\delta \frac{\langle \Psi_\nu | H | \Psi_\nu \rangle}{\langle \Psi_\nu | \Psi_\nu \rangle} = 0$$

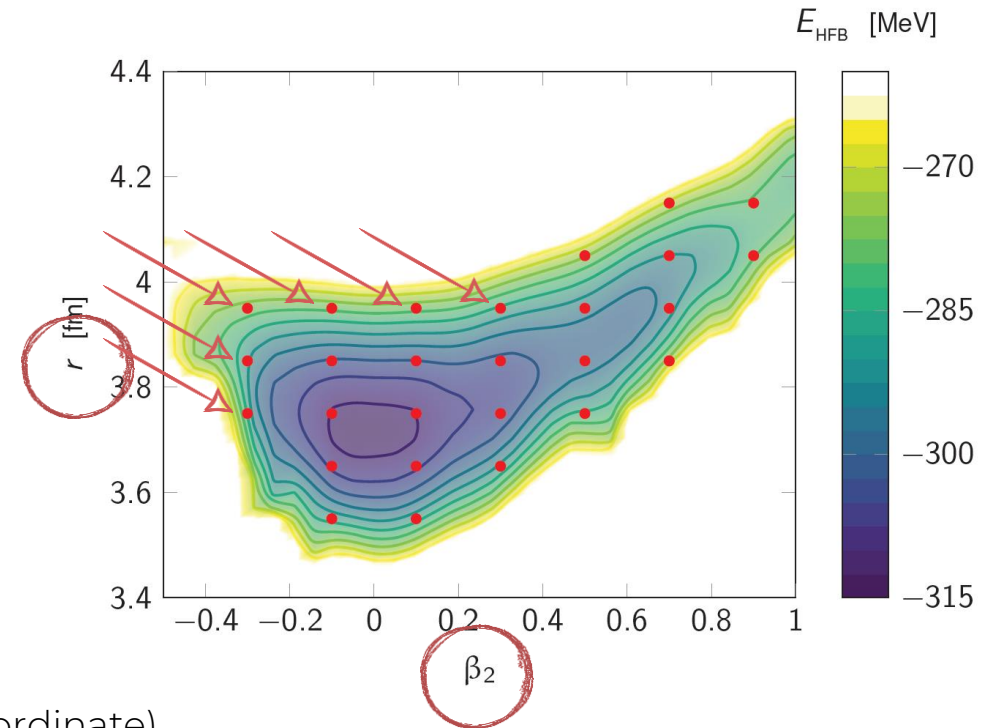
Schrödinger-like equation

$$\sum_q \left[\mathcal{H}(p, q) - E_\nu \mathcal{N}(p, q) \right] f_\nu(q) = 0$$

Kernels evaluation

$$\mathcal{H}(p, q) \equiv \langle \Phi(p) | H | \Phi(q) \rangle$$

$$\mathcal{N}(p, q) \equiv \langle \Phi(p) | \Phi(q) \rangle$$

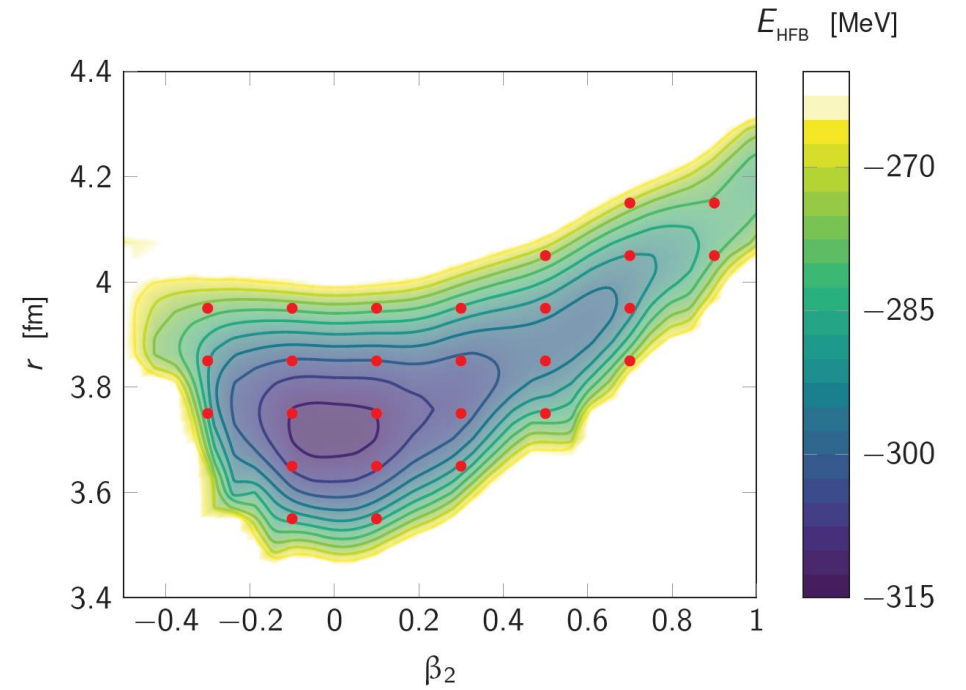


(Q)RPA from GCM

Thouless theorem

(q can be whatever coordinate)

$$|\Phi(q)\rangle = \langle \Phi(q_{min}) | \Phi(q) \rangle e^{\mathbf{Z}(q, q_{min})} |\Phi(q_{min})\rangle$$



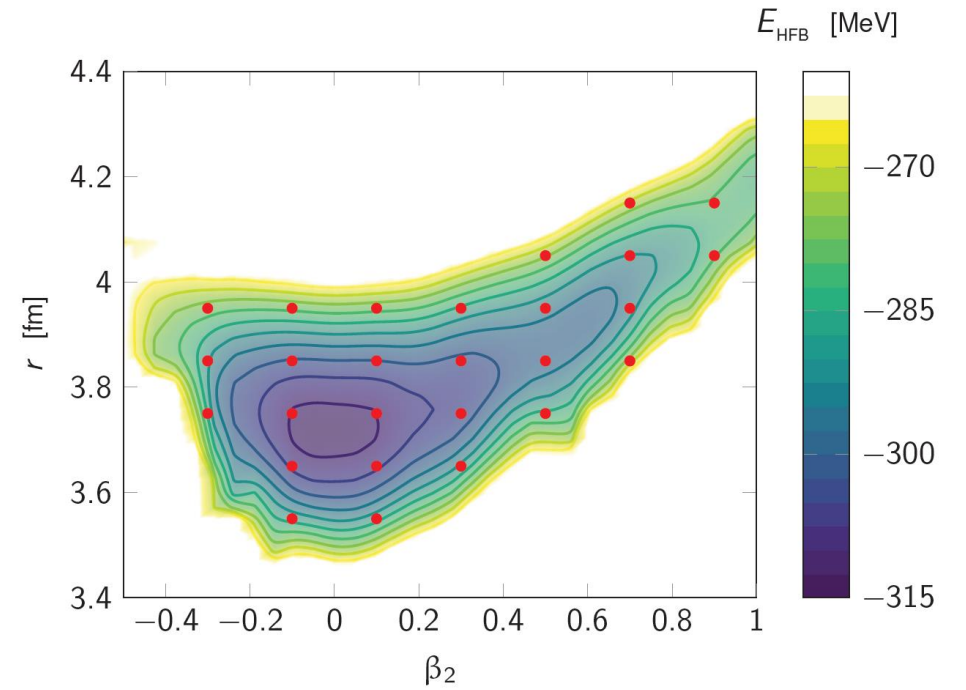
(Q)RPA from GCM

Thouless theorem

(q can be whatever coordinate)

$$|\Phi(q)\rangle = \langle \Phi(q_{min}) | \Phi(q) \rangle e^{\mathbf{Z}(q, q_{min})} |\Phi(q_{min})\rangle$$

Non-unitary transformation



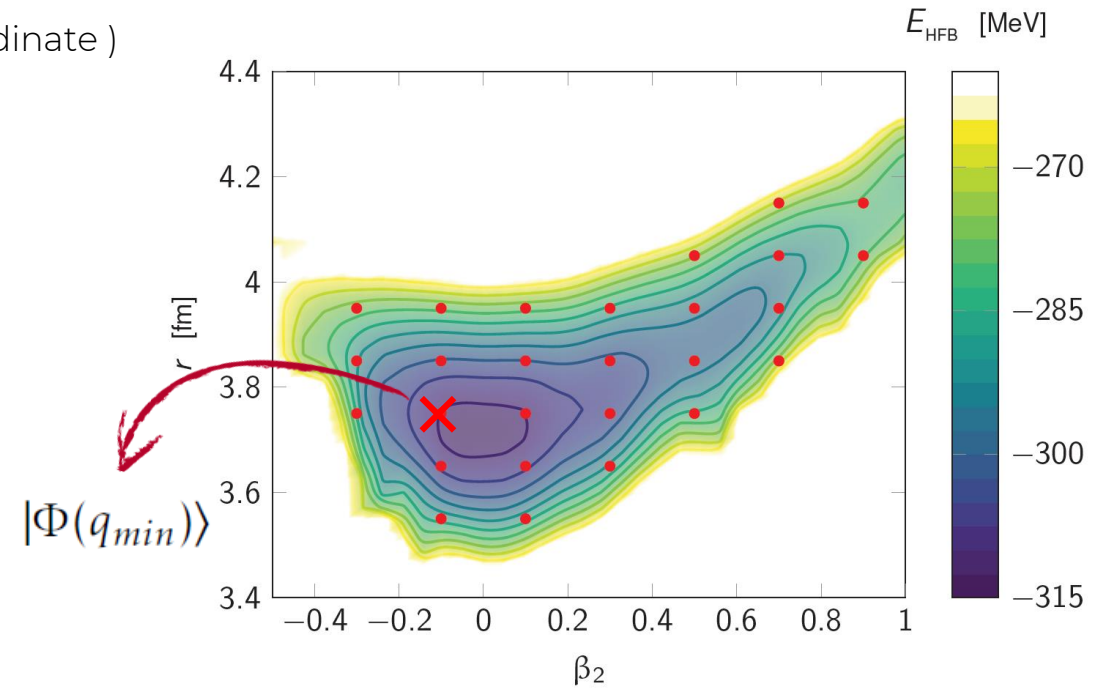
(Q)RPA from GCM

Thouless theorem

(q can be whatever coordinate)

$$|\Phi(q)\rangle = \langle\Phi(q_{min})|\Phi(q)\rangle e^{\mathbf{Z}(q,q_{min})} |\Phi(q_{min})\rangle$$

Non-unitary transformation



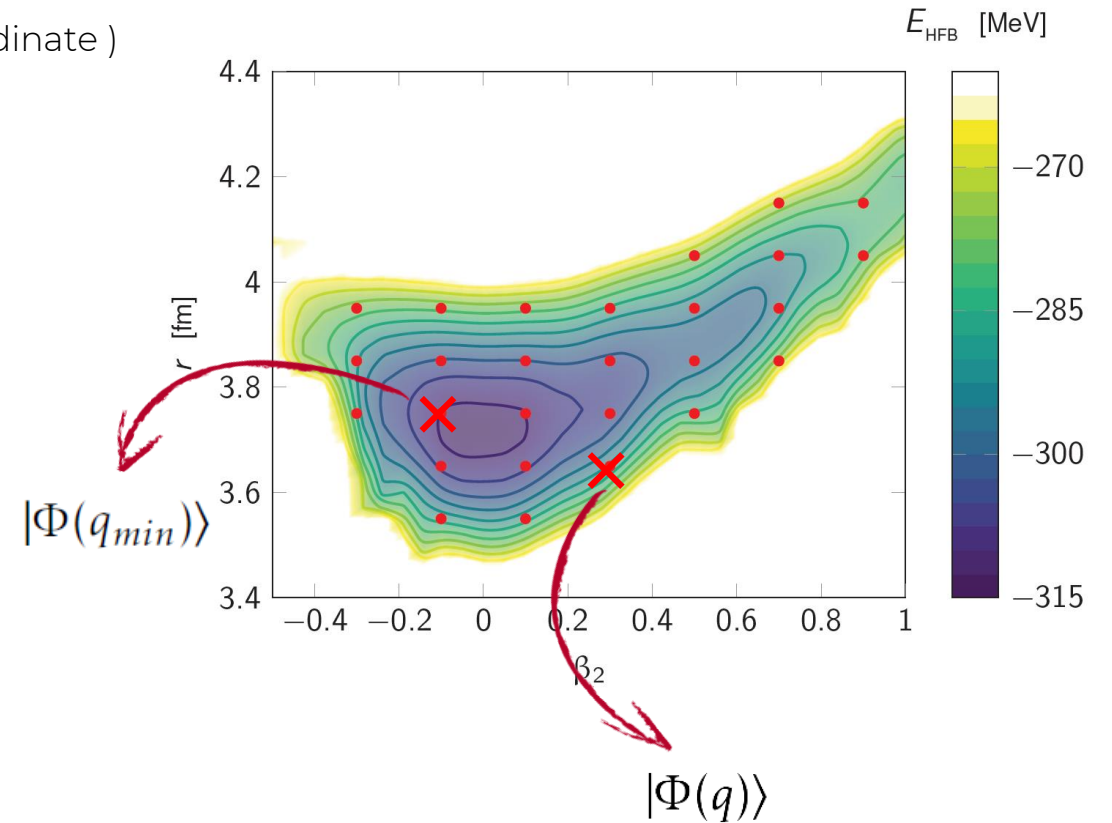
(Q)RPA from GCM

Thouless theorem

(q can be whatever coordinate)

$$|\Phi(q)\rangle = \langle\Phi(q_{min})|\Phi(q)\rangle e^{\mathbf{Z}(q,q_{min})} |\Phi(q_{min})\rangle$$

Non-unitary transformation



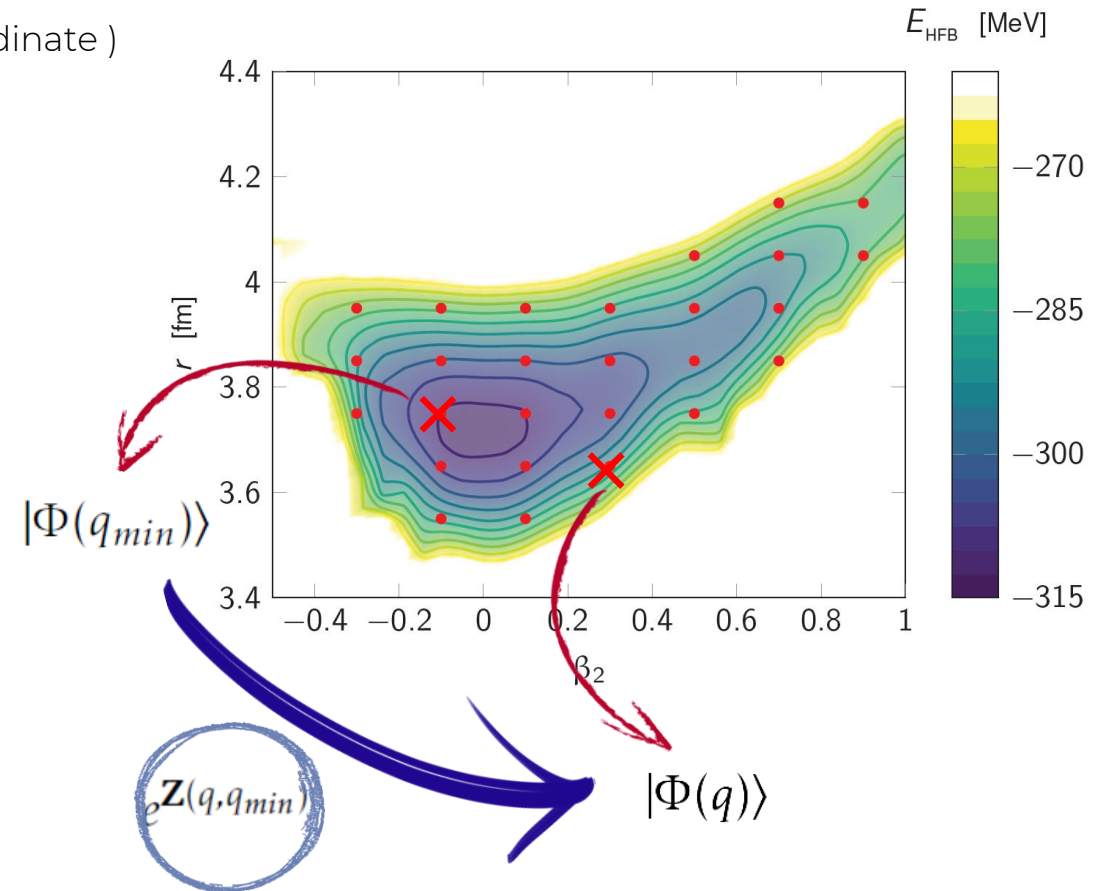
(Q)RPA from GCM

Thouless theorem

(q can be whatever coordinate)

$$|\Phi(q)\rangle = \langle \Phi(q_{min}) | \Phi(q) \rangle e^{\mathbf{Z}(q, q_{min})} |\Phi(q_{min})\rangle$$

Non-unitary transformation



(Q)RPA from GCM

Thouless theorem

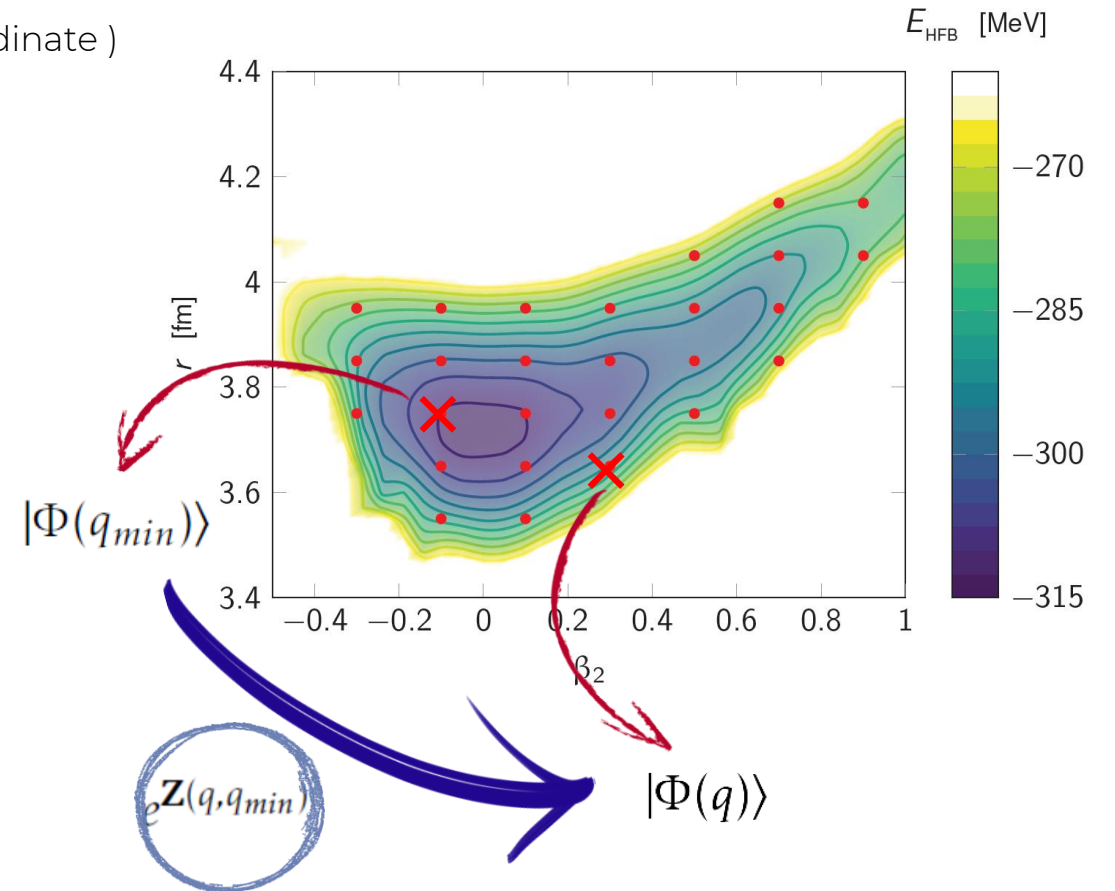
(q can be whatever coordinate)

$$|\Phi(q)\rangle = \langle\Phi(q_{min})|\Phi(q)\rangle e^{\mathbf{Z}(q,q_{min})} |\Phi(q_{min})\rangle$$

Non-unitary transformation

HWG Equation

$$\delta \frac{\langle\Psi_v|H|\Psi_v\rangle}{\langle\Psi_v|\Psi_v\rangle} = 0$$



(Q)RPA from GCM

Thouless theorem

(q can be whatever coordinate)

$$|\Phi(q)\rangle = \langle \Phi(q_{min}) | \Phi(q) \rangle e^{\mathbf{Z}(q, q_{min})} |\Phi(q_{min})\rangle$$

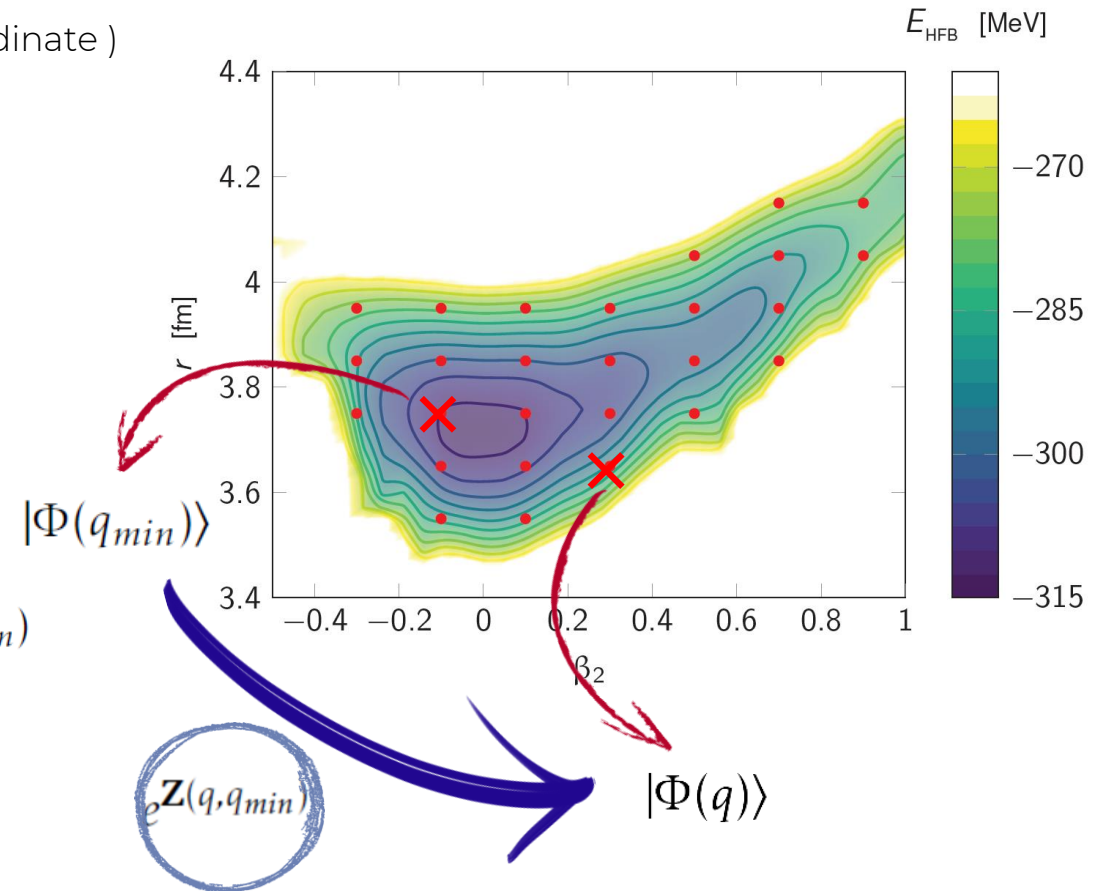
Non-unitary transformation

HWG Equation

$$\delta \frac{\langle \Psi_v | H | \Psi_v \rangle}{\langle \Psi_v | \Psi_v \rangle} = 0$$

Solve with two approximations:

- Quasi-Boson approximation (QBA)
- Expand to the quadratic level in $\mathbf{Z}(q, q_{min})$
 - Harmonic approximation



(Q)RPA from GCM

Thouless theorem

(q can be whatever coordinate)

$$|\Phi(q)\rangle = \langle \Phi(q_{min}) | \Phi(q) \rangle e^{\mathbf{Z}(q, q_{min})} |\Phi(q_{min})\rangle$$

Non-unitary transformation

HWG Equation

$$\delta \frac{\langle \Psi_v | H | \Psi_v \rangle}{\langle \Psi_v | \Psi_v \rangle} = 0$$

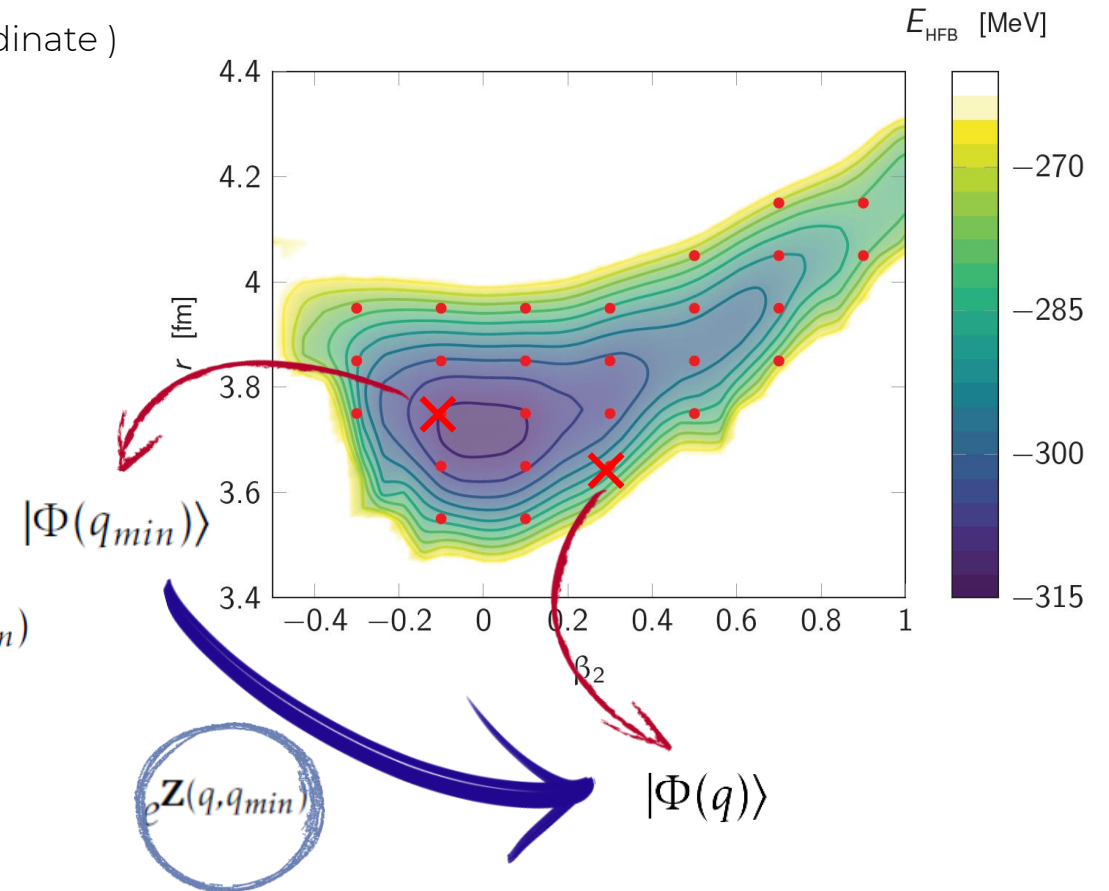
Solve with two approximations:

- Quasi-Boson approximation (QBA)
- Expand to the quadratic level in $\mathbf{Z}(q, q_{min})$
→ Harmonic approximation



No coordinates dependency!

All coordinates are explored
(differently from PGCM)



(Q)RPA from GCM

Thouless theorem

(q can be whatever coordinate)

$$|\Phi(q)\rangle = \langle \Phi(q_{min}) | \Phi(q) \rangle e^{\mathbf{Z}(q, q_{min})} |\Phi(q_{min})\rangle$$

Non-unitary transformation

HWG Equation

$$\delta \frac{\langle \Psi_v | H | \Psi_v \rangle}{\langle \Psi_v | \Psi_v \rangle} = 0$$

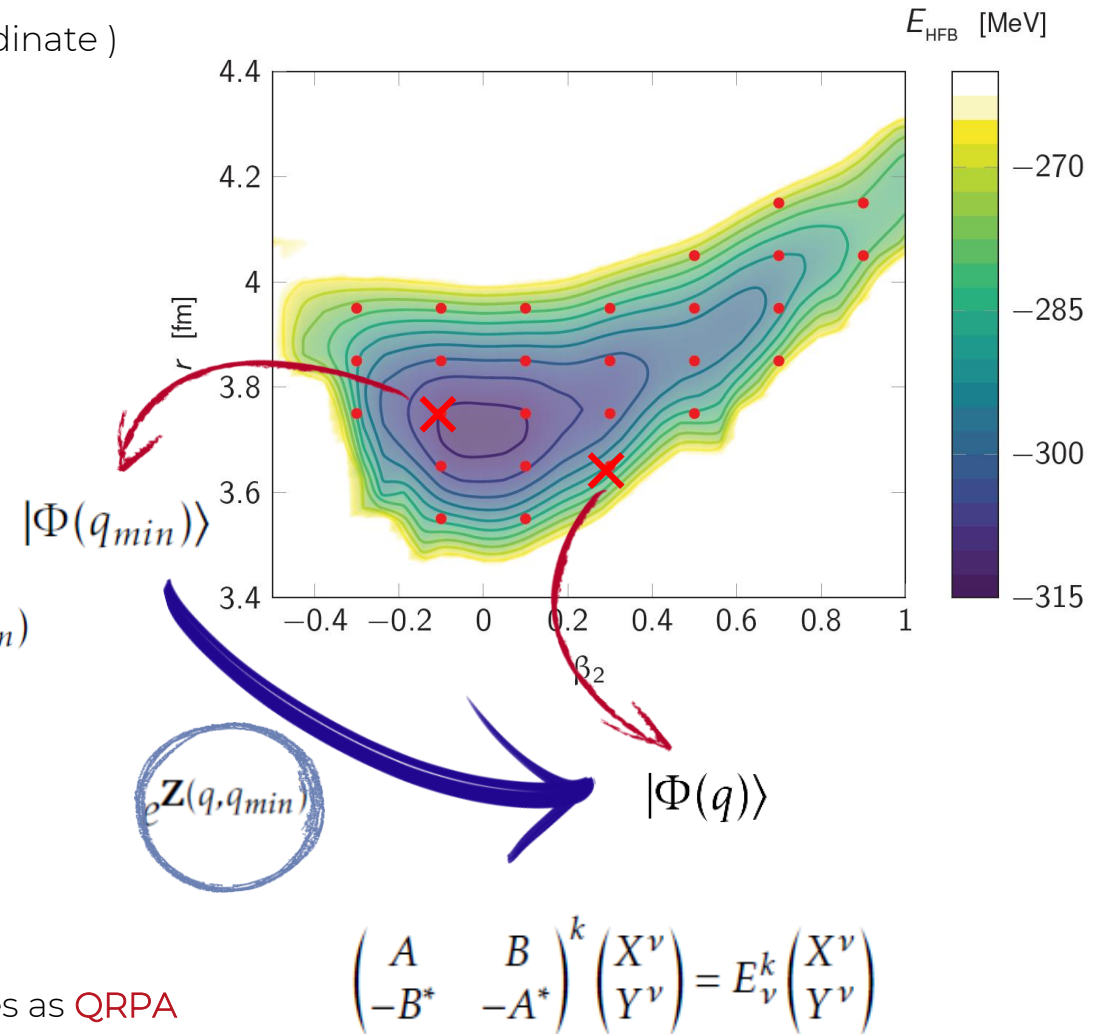
Solve with two approximations:

- Quasi-Boson approximation (QBA)
- Expand to the quadratic level in $\mathbf{Z}(q, q_{min})$
→ Harmonic approximation

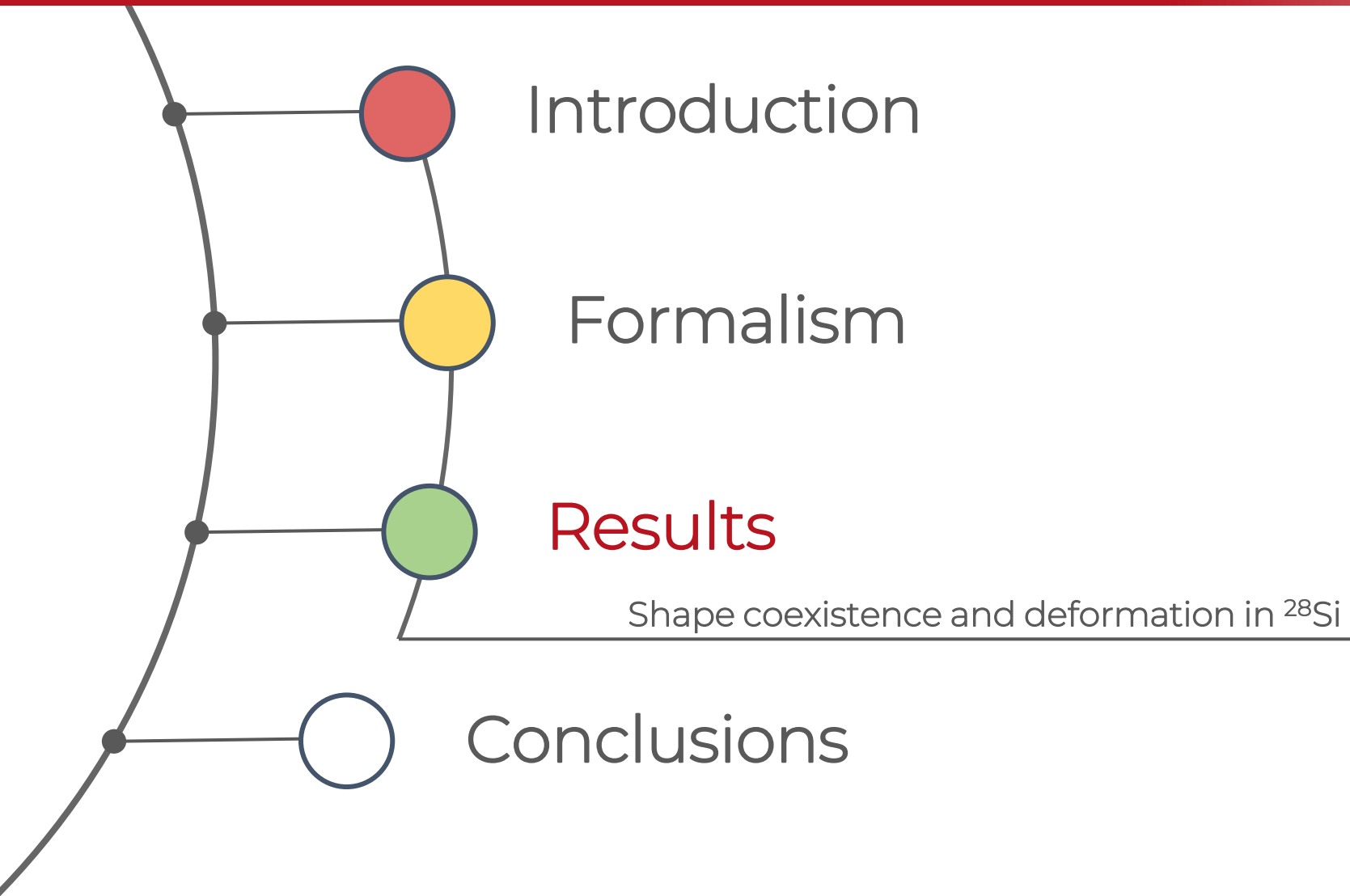


No coordinates dependency!

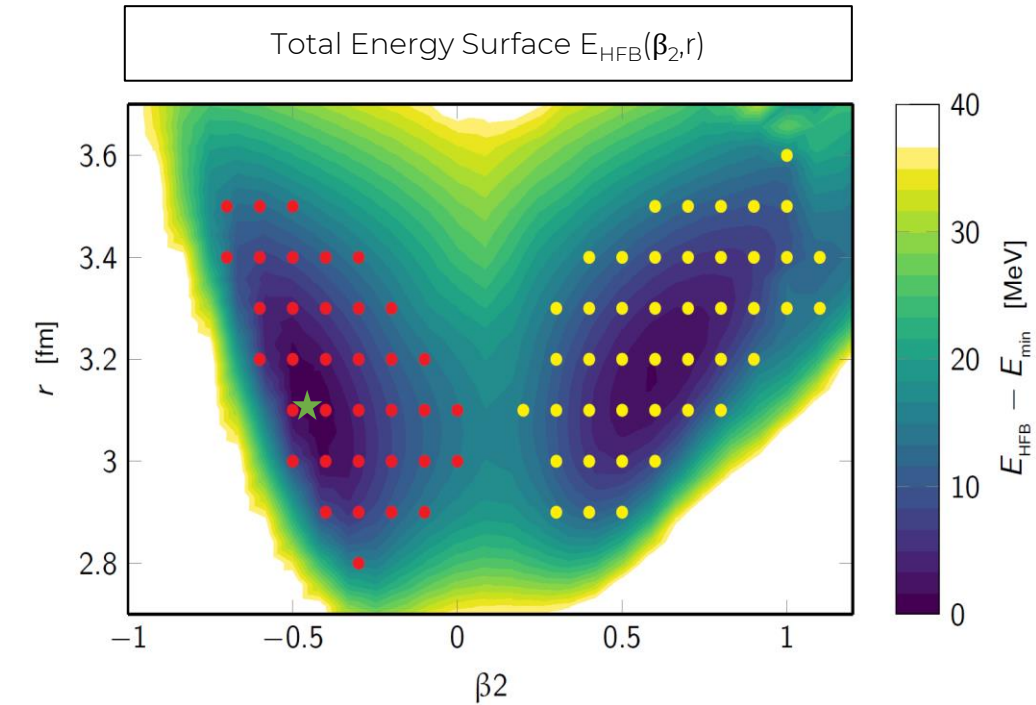
All coordinates are explored
(differently from PGCM)



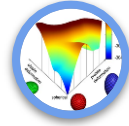
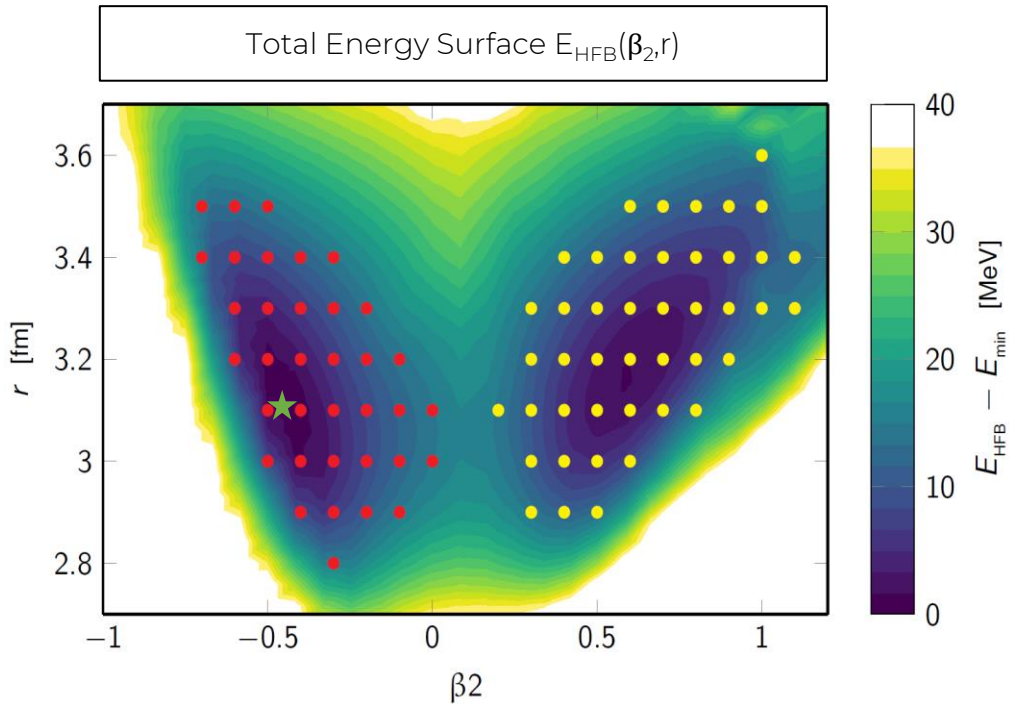
Outline



Shape coexistence effects in ^{28}Si

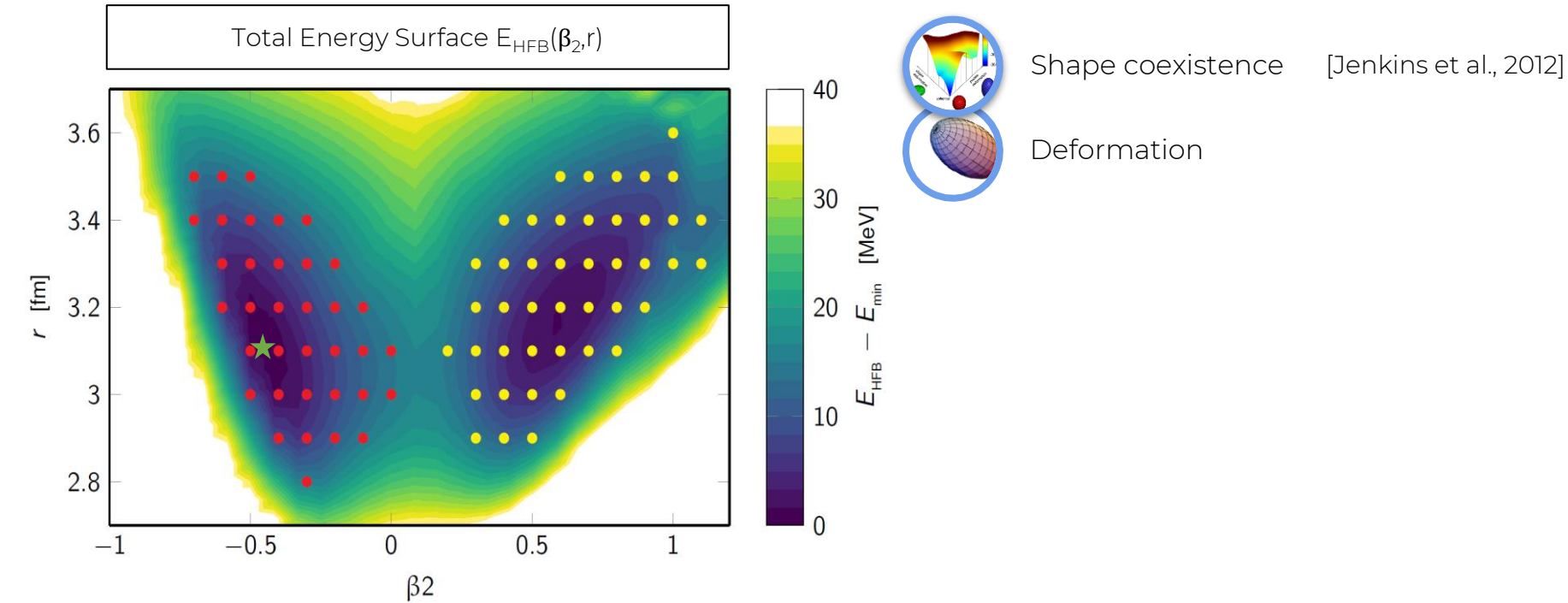


Shape coexistence effects in ^{28}Si

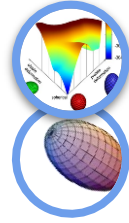
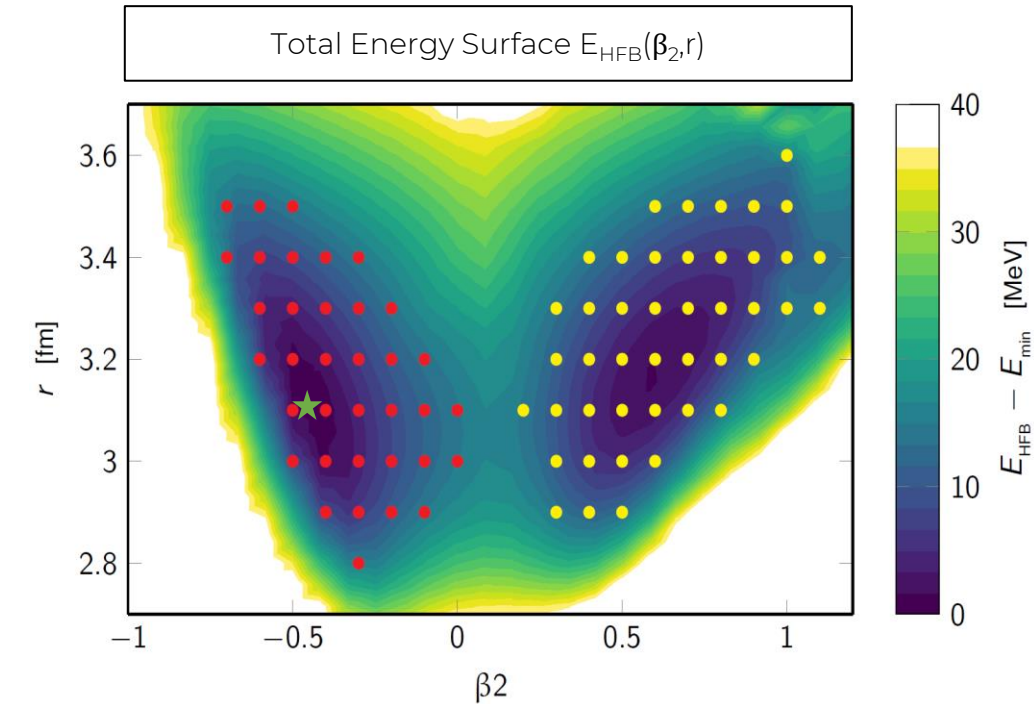


Shape coexistence [Jenkins et al., 2012]

Shape coexistence effects in ^{28}Si



Shape coexistence effects in ^{28}Si



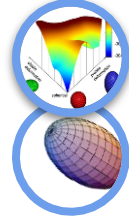
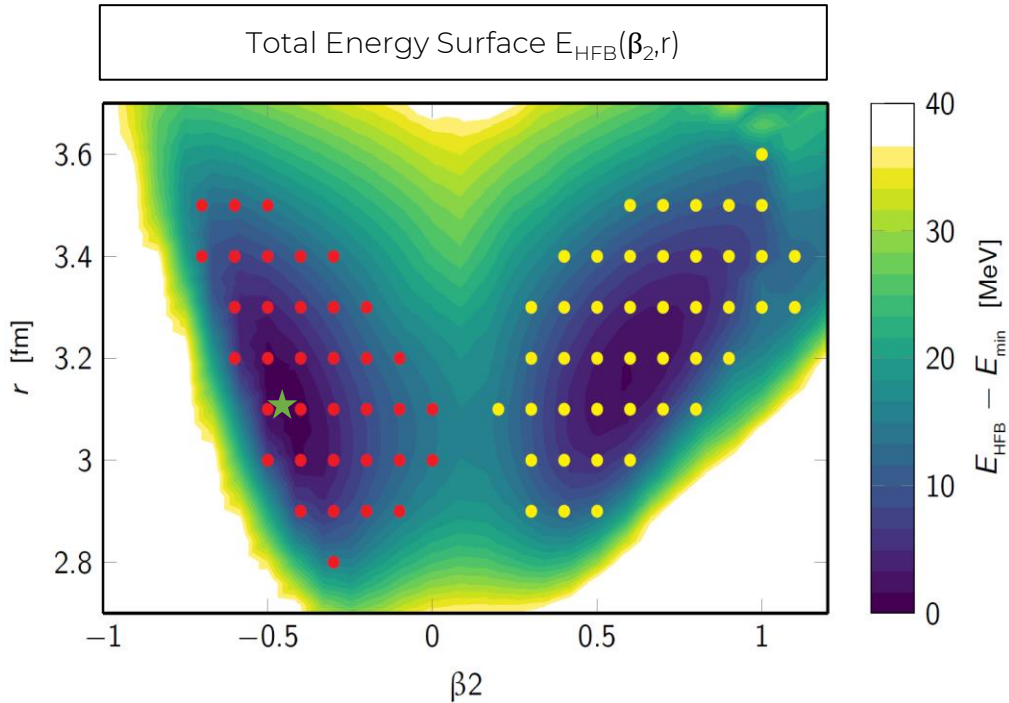
Shape coexistence

[Jenkins et al., 2012]

Deformation

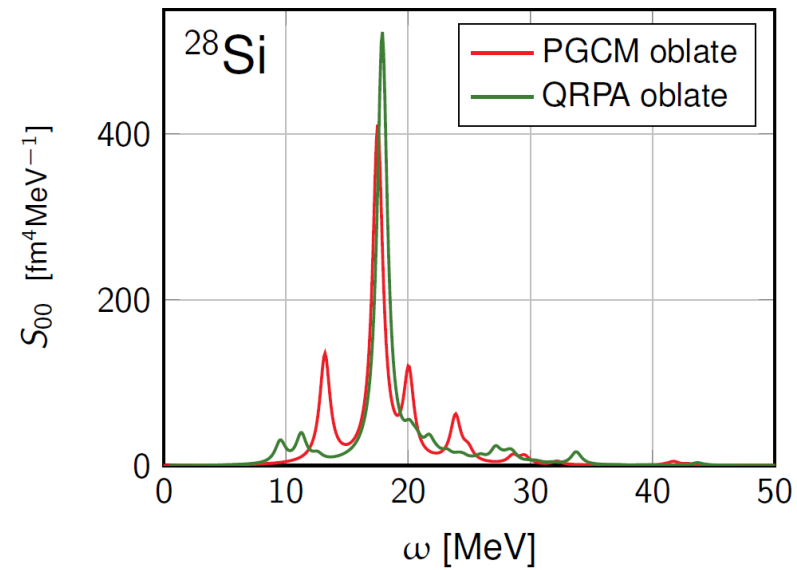
- Oblate minimum and prolate-shape isomer

Shape coexistence effects in ^{28}Si



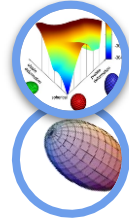
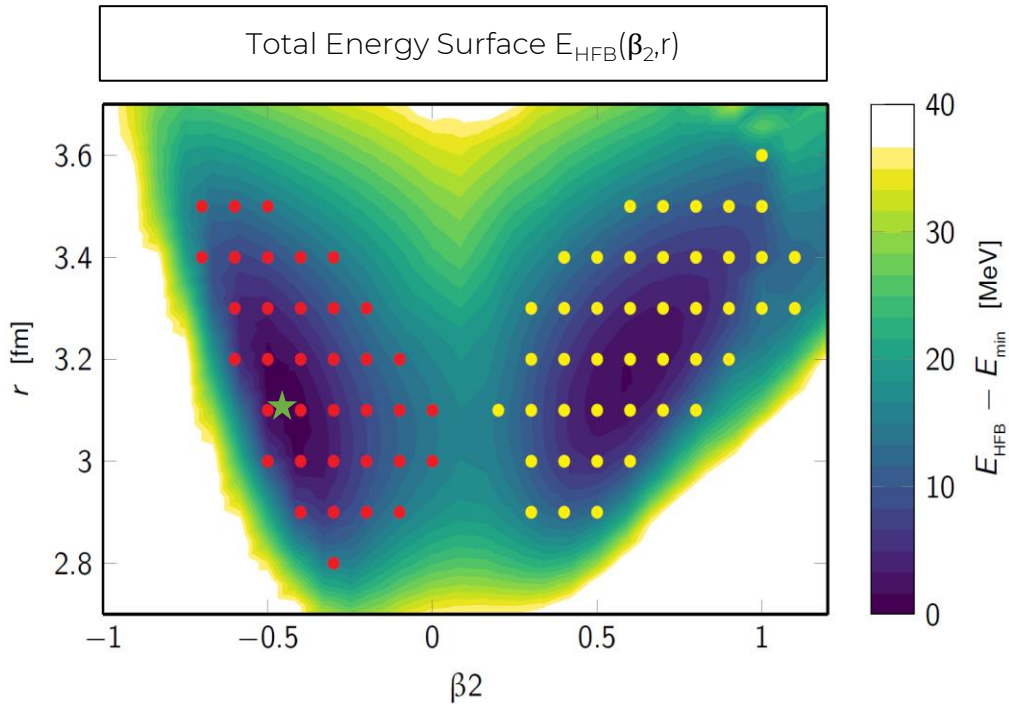
Shape coexistence [Jenkins et al., 2012]

Deformation



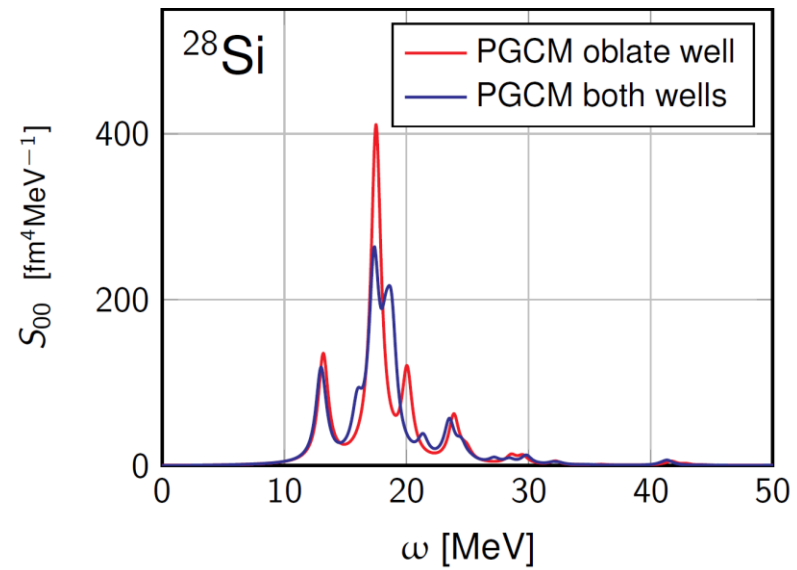
- Oblate minimum and prolate-shape isomer
- Qualitatively similar results QRPA/PGCM
 - QRPA response less fragmented

Shape coexistence effects in ^{28}Si



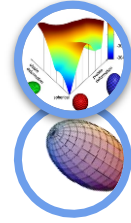
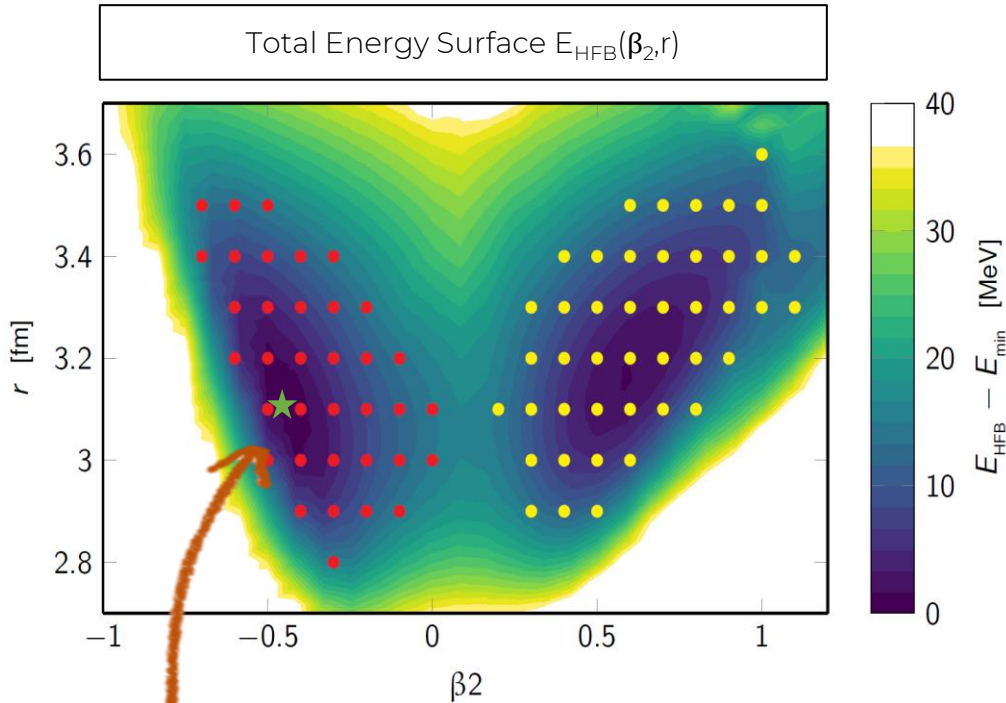
Shape coexistence [Jenkins et al., 2012]

Deformation



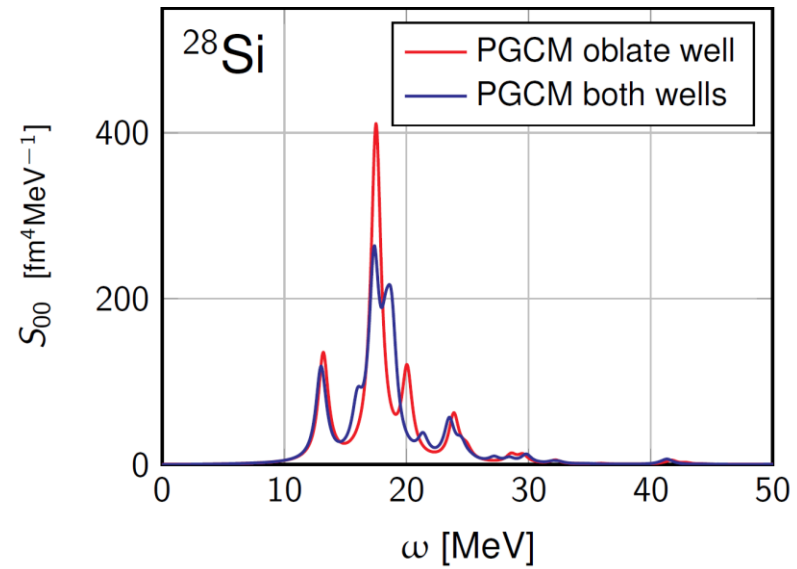
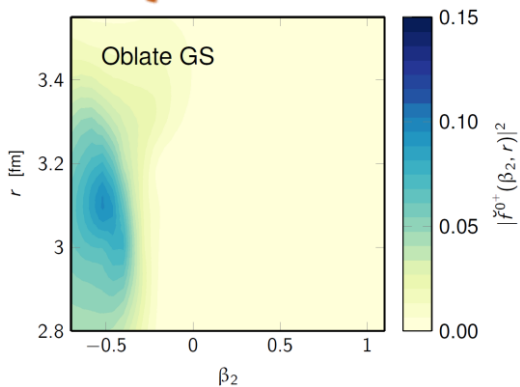
- Oblate minimum and prolate-shape isomer
- Qualitatively similar results QRPA/PGCM
 - QRPA response less fragmented
- Proper study of shape coexistence in PGCM
 - Shape coexistence but no shape mixing

Shape coexistence effects in ^{28}Si



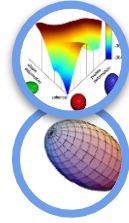
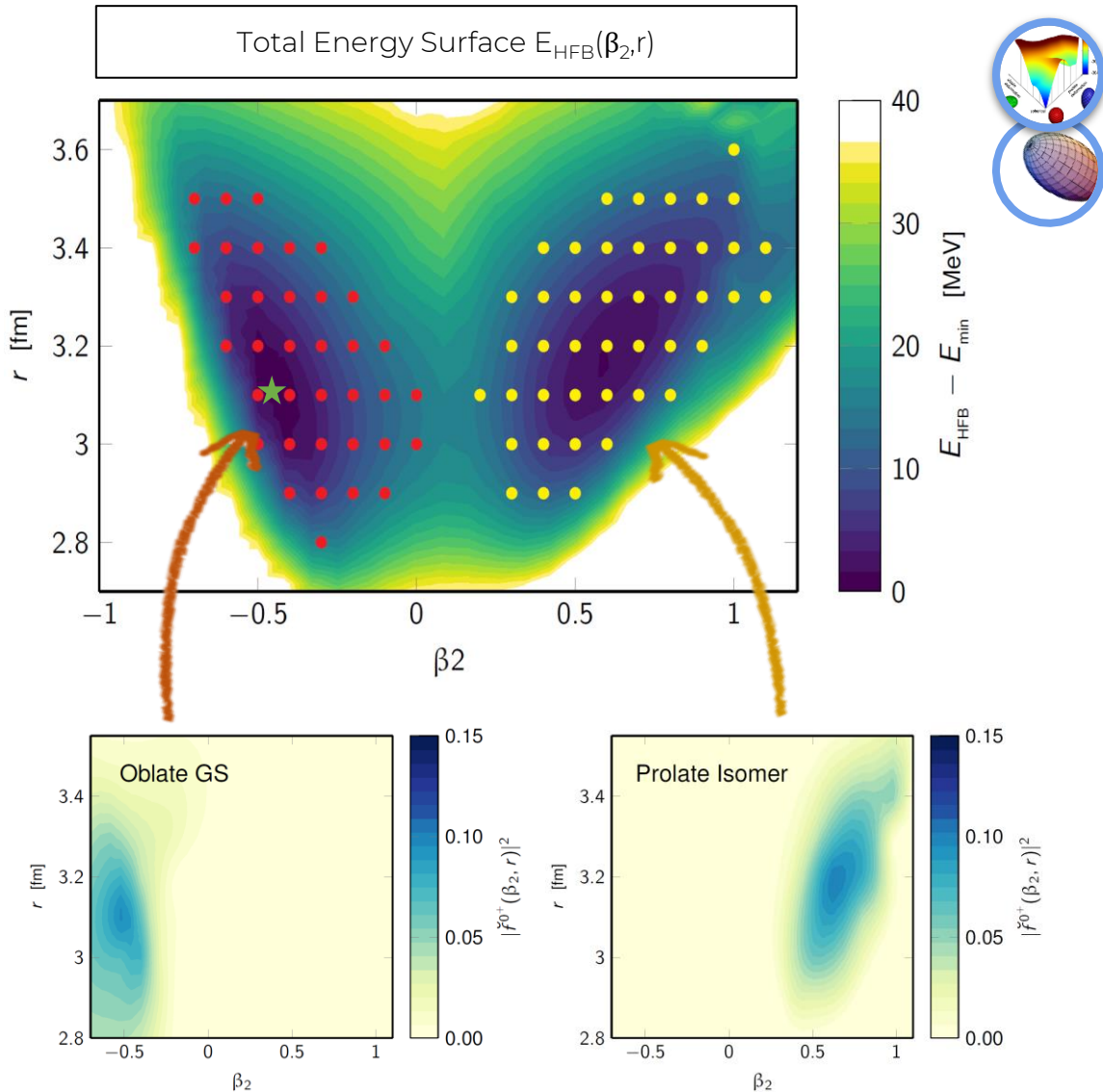
Shape coexistence [Jenkins et al., 2012]

Deformation



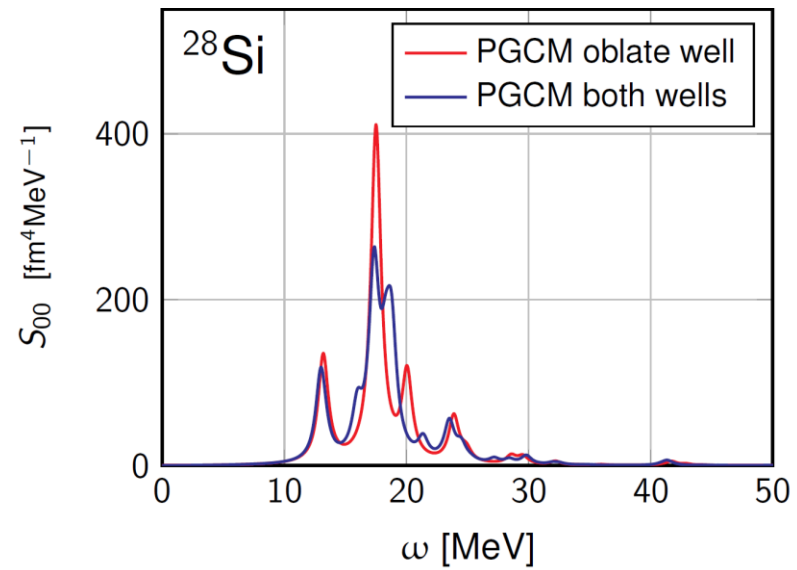
- Oblate minimum and prolate-shape isomer
- Qualitatively similar results QRPA/PGCM
 - QRPA response less fragmented
- Proper study of shape coexistence in PGCM
 - Shape coexistence but no shape mixing

Shape coexistence effects in ^{28}Si



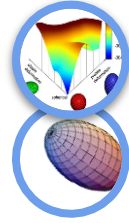
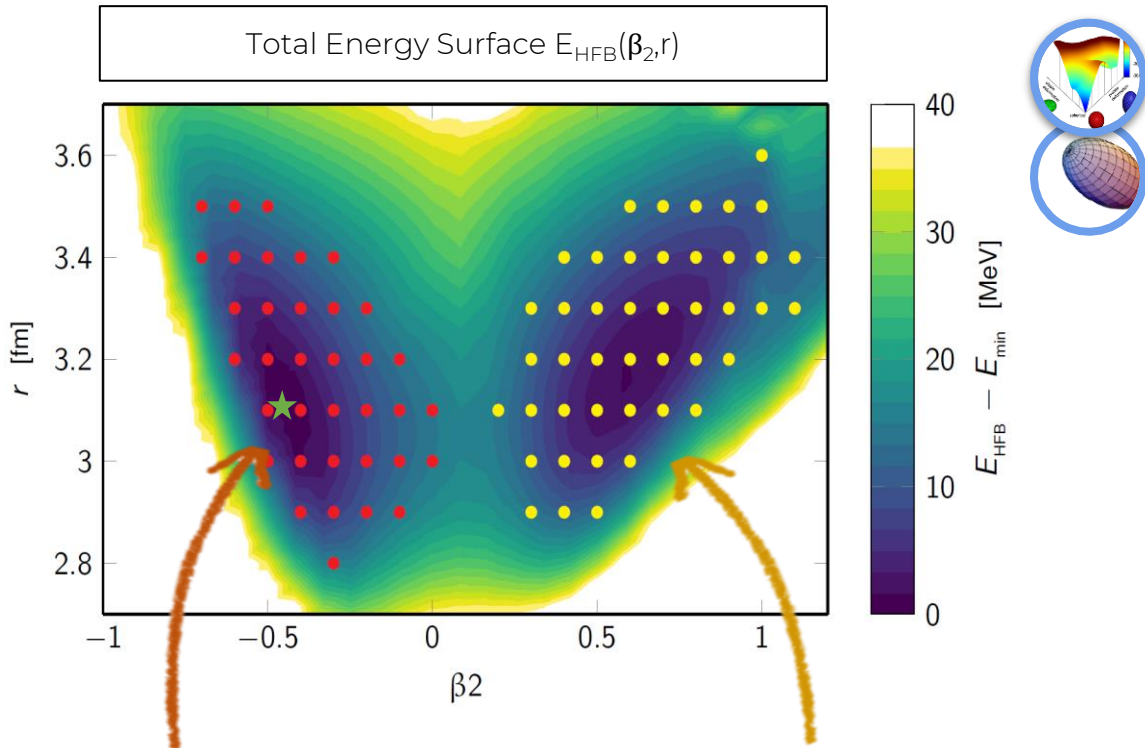
Shape coexistence [Jenkins et al., 2012]

Deformation



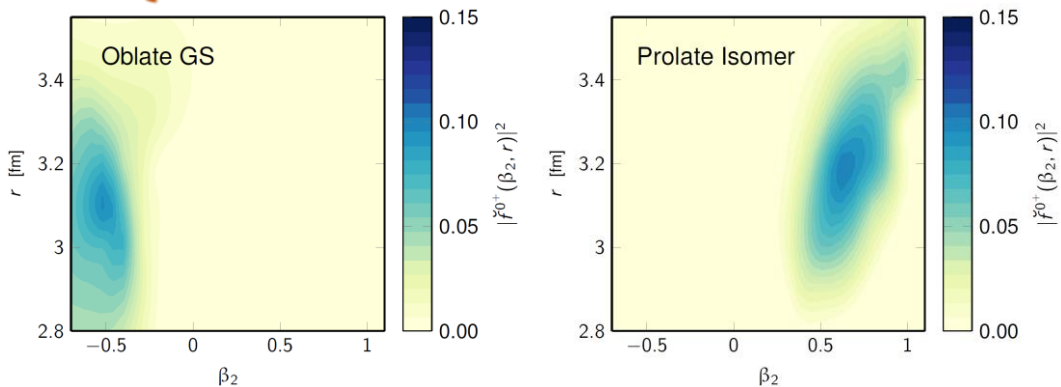
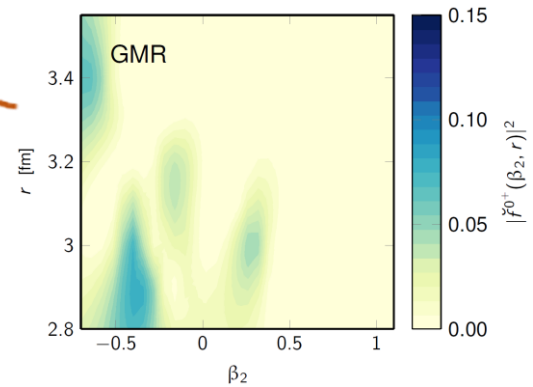
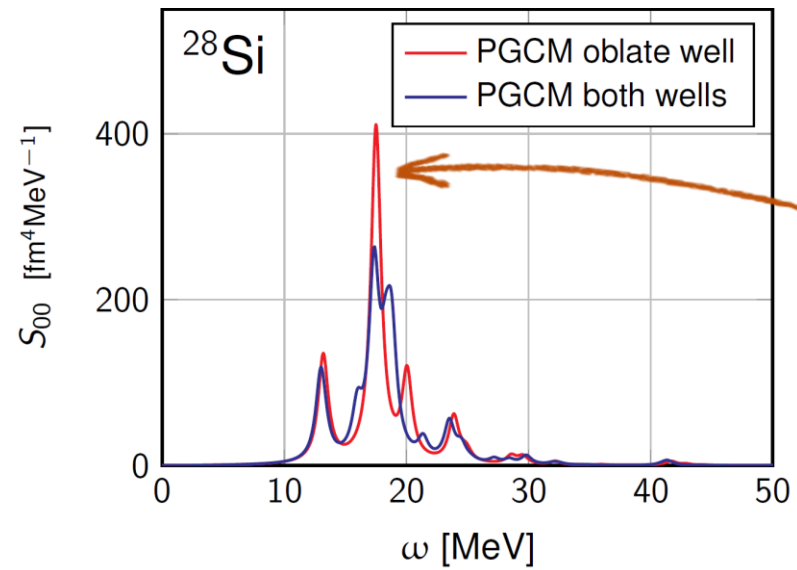
- Oblate minimum and prolate-shape isomer
- Qualitatively similar results QRPA/PGCM
 - QRPA response less fragmented
- Proper study of shape coexistence in PGCM
 - Shape coexistence but no shape mixing

Shape coexistence effects in ^{28}Si



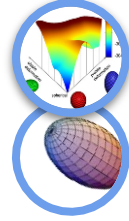
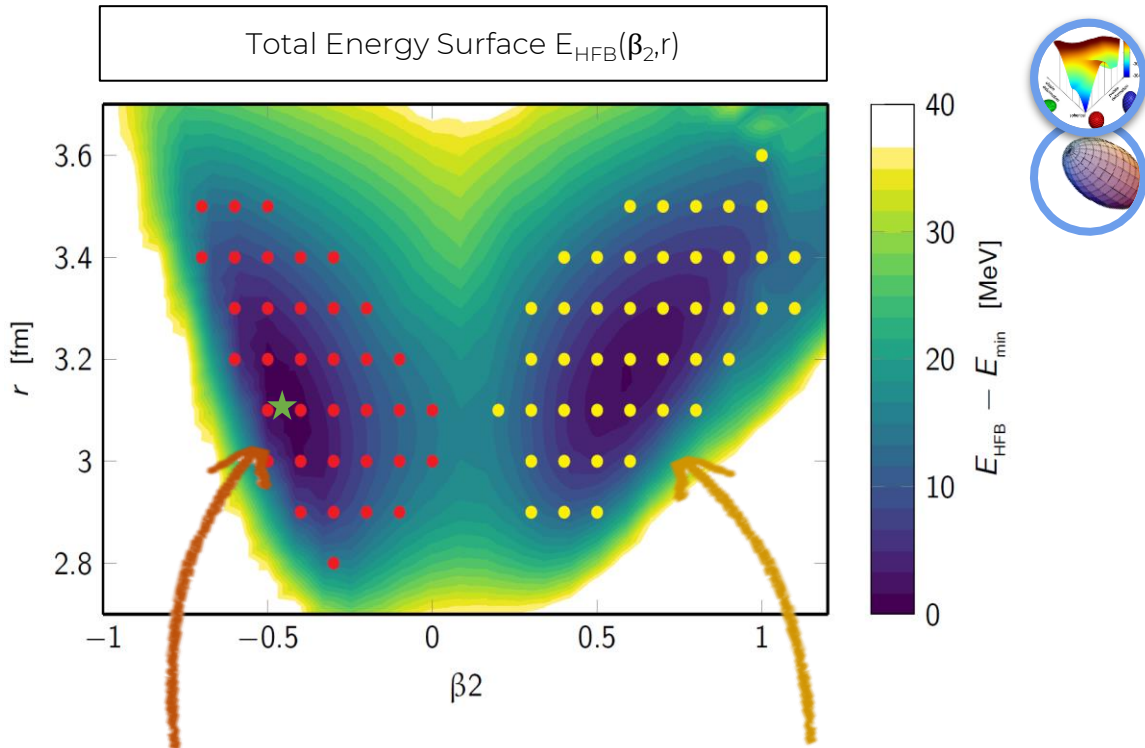
Shape coexistence [Jenkins et al., 2012]

Deformation



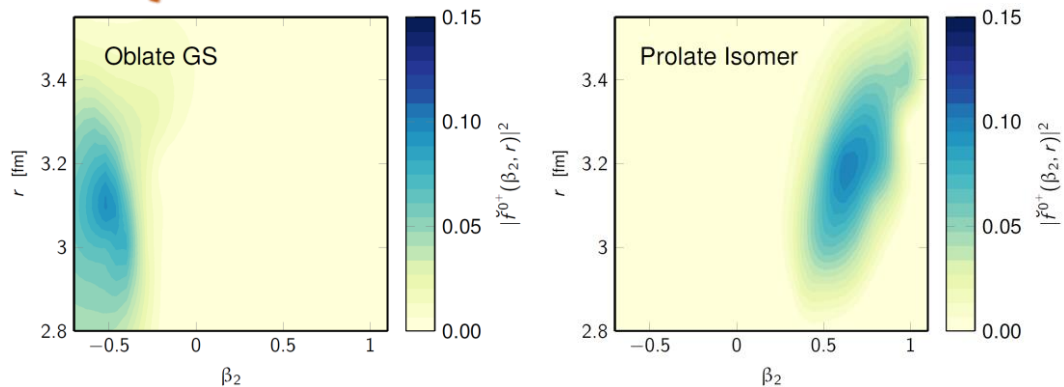
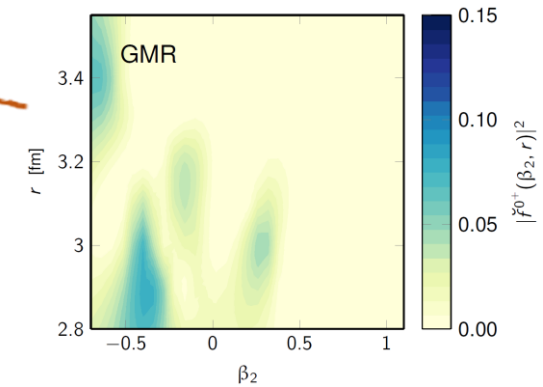
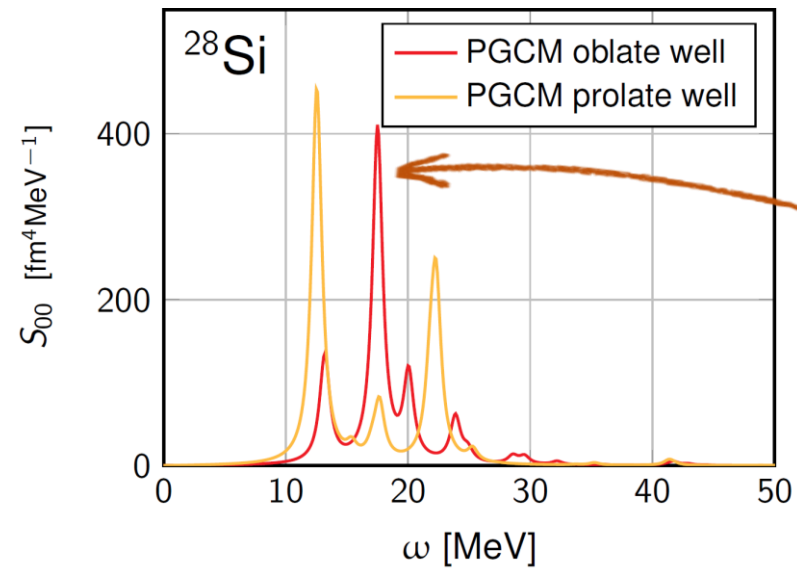
- Oblate minimum and prolate-shape isomer
- Qualitatively similar results QRPA/PGCM
 - QRPA response less fragmented
- Proper study of shape coexistence in PGCM
 - Shape coexistence but no shape mixing

Shape coexistence effects in ^{28}Si



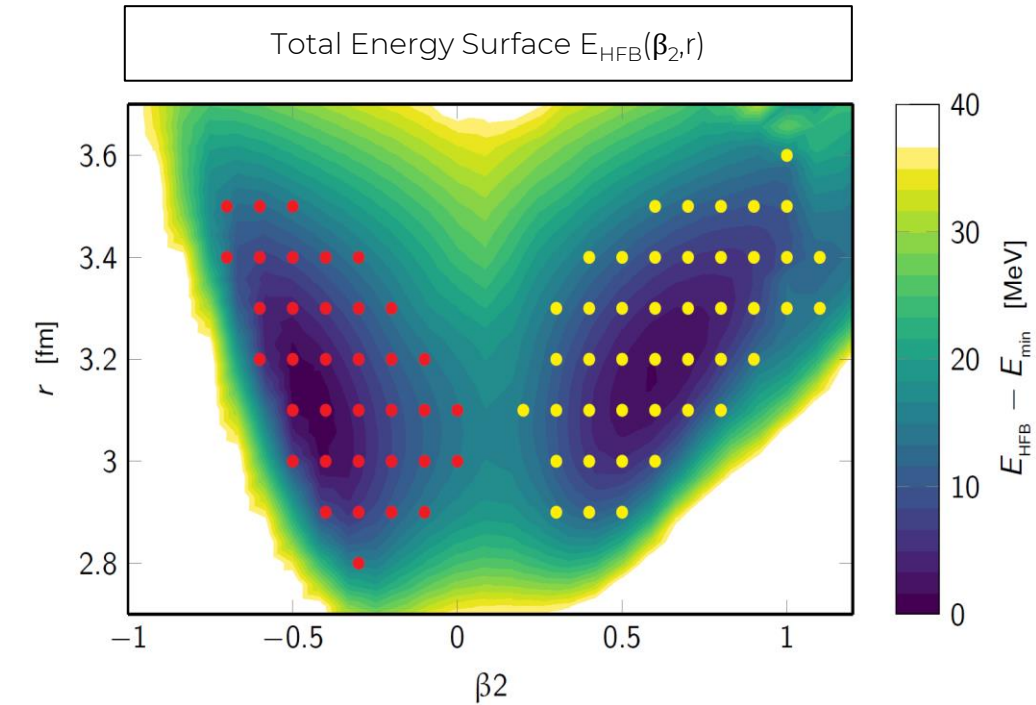
Shape coexistence [Jenkins et al., 2012]

Deformation

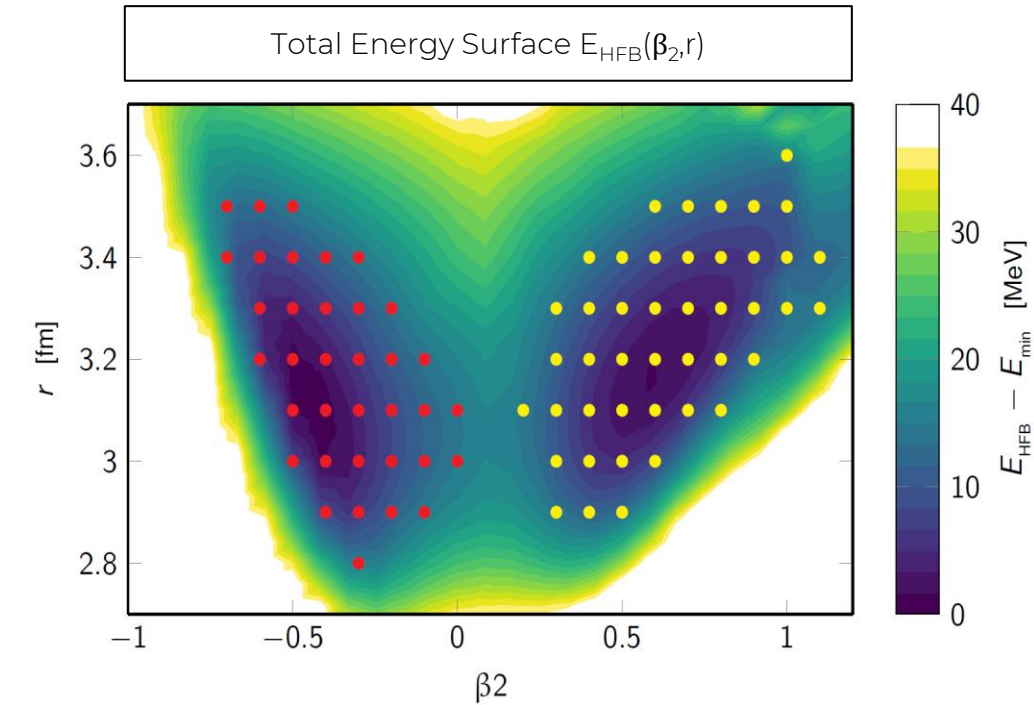


- Oblate minimum and prolate-shape isomer
- Qualitatively similar results QRPA/PGCM
 - QRPA response less fragmented
- Proper study of shape coexistence in PGCM
 - Shape coexistence but no shape mixing

Deformation effects in prolate ^{28}Si

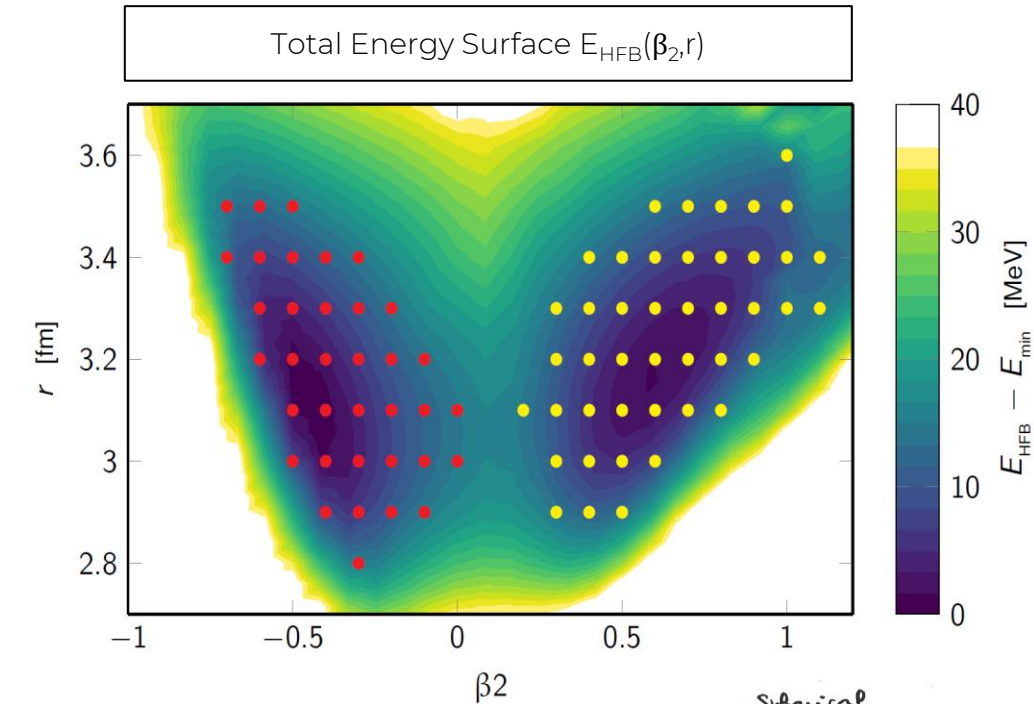


Deformation effects in prolate ^{28}Si



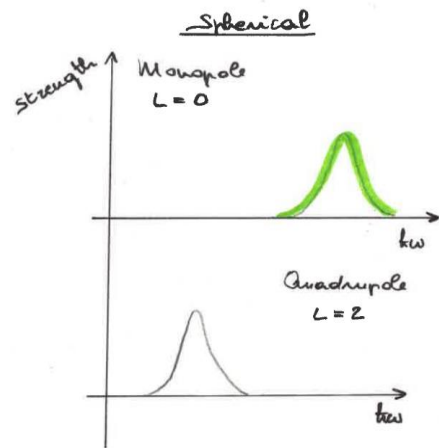
- Focus on the prolate-shape isomer

Deformation effects in prolate ^{28}Si

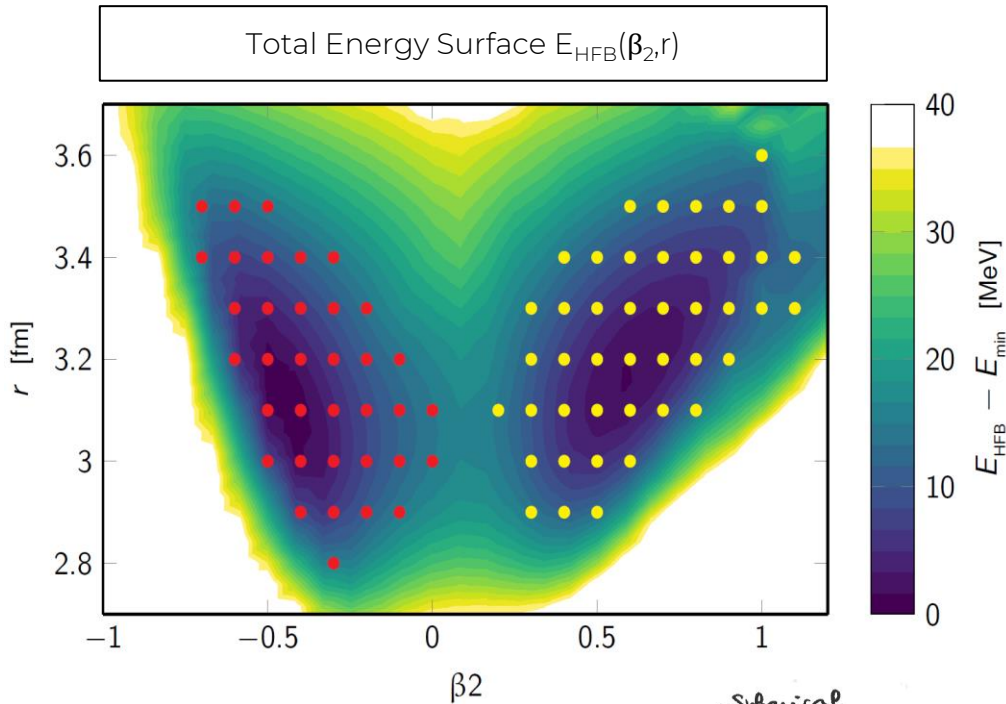


- Focus on the prolate-shape isomer

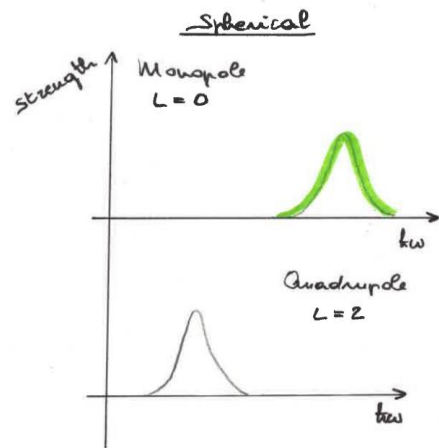
Spherical
(no deformation)



Deformation effects in prolate ^{28}Si



Spherical
(no deformation)

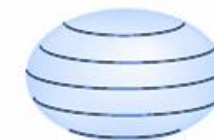


- Focus on the prolate-shape isomer
- Coupling to GQR generates **splitting**
 - ✗ High peak = shifted “spherical” breathing mode
 - ✗ Low peak = induced by coupling to GQR ($K=0$)

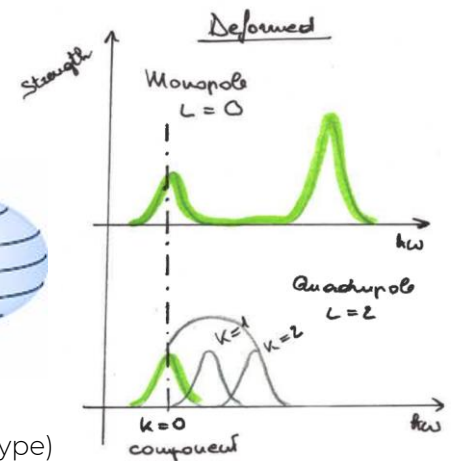
Spheroidal
(deformed)



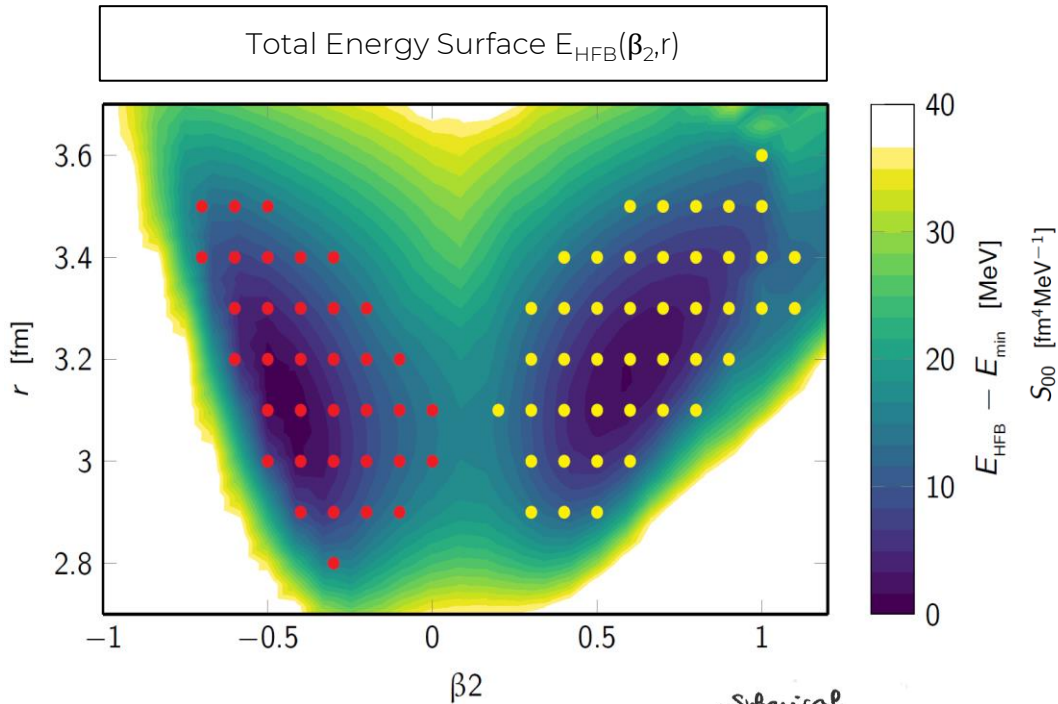
Prolate
(cigar type)



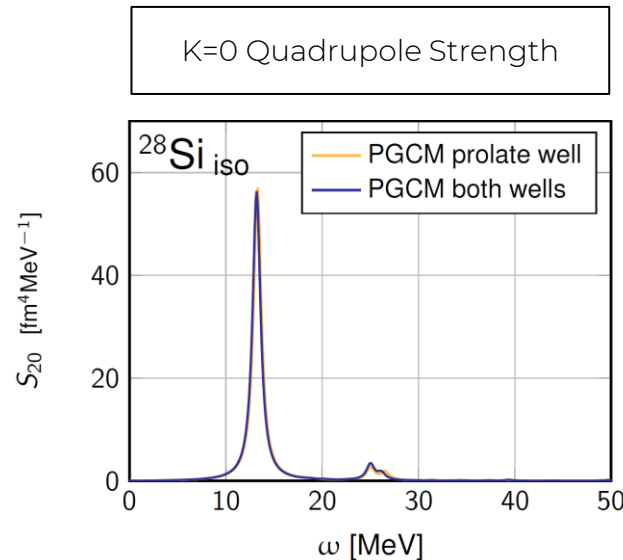
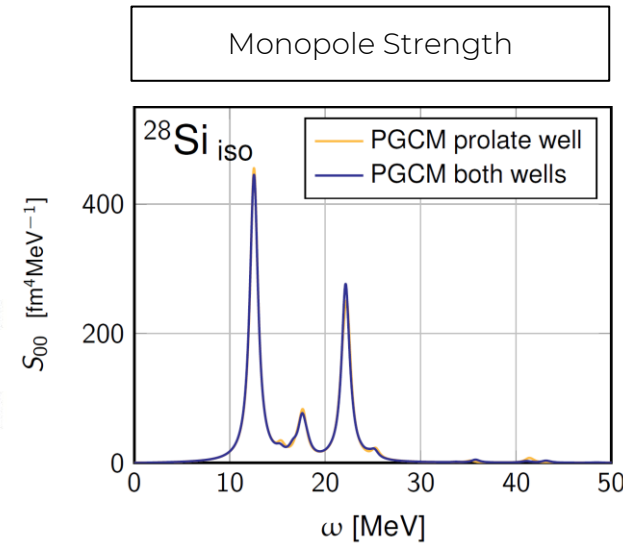
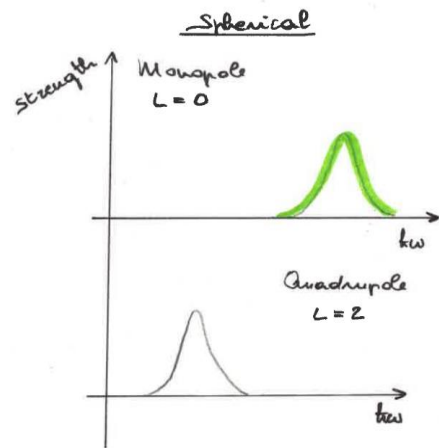
Oblate
(pancake type)



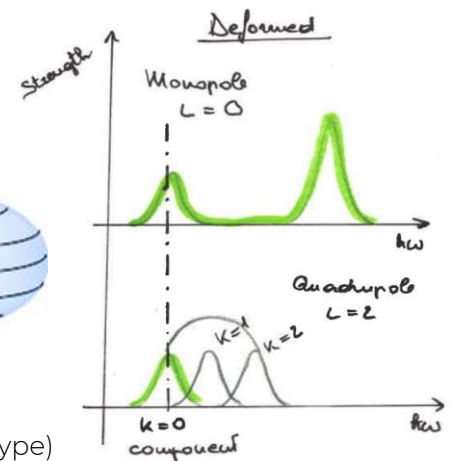
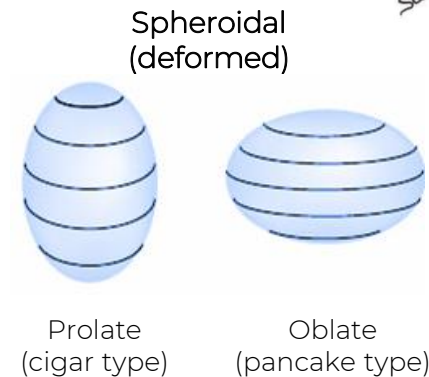
Deformation effects in prolate ^{28}Si



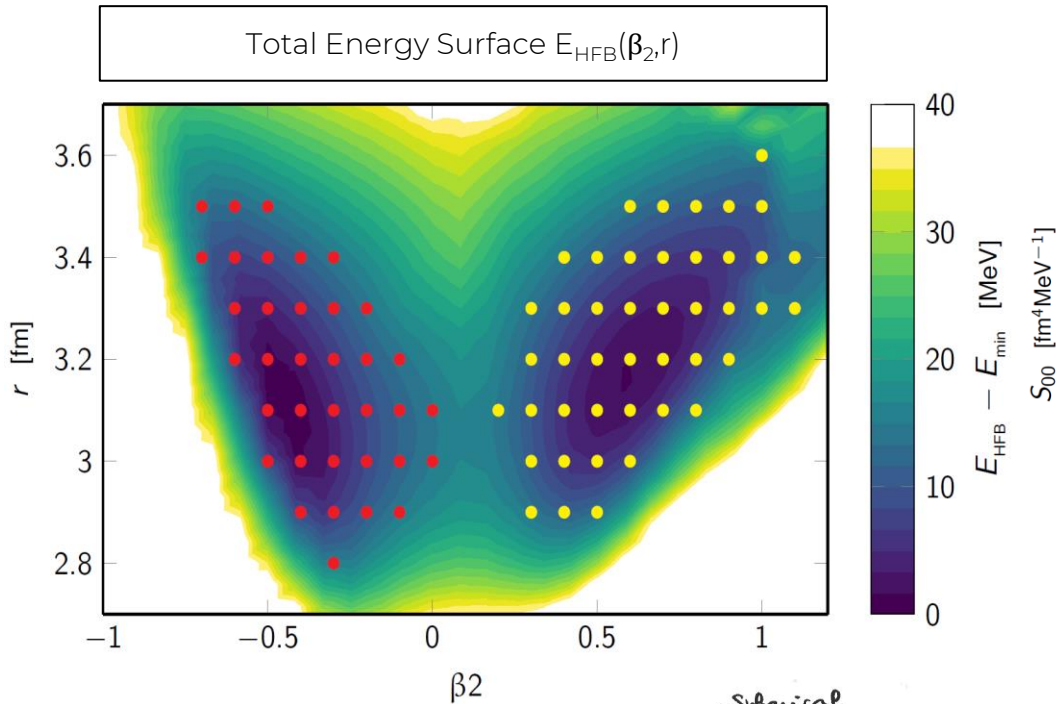
Spherical
(no deformation)



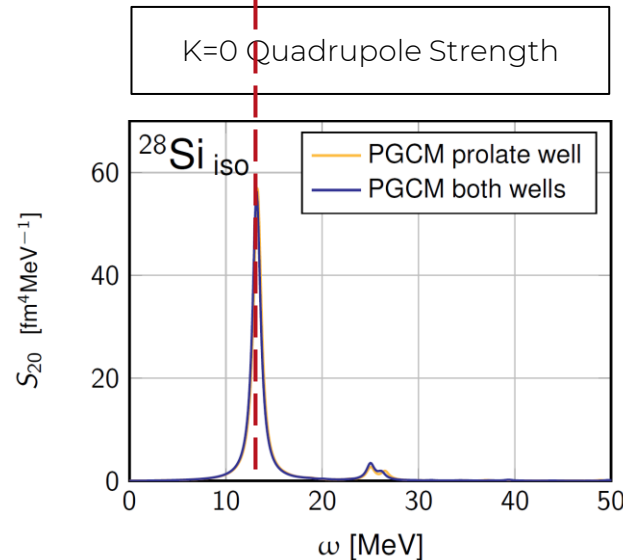
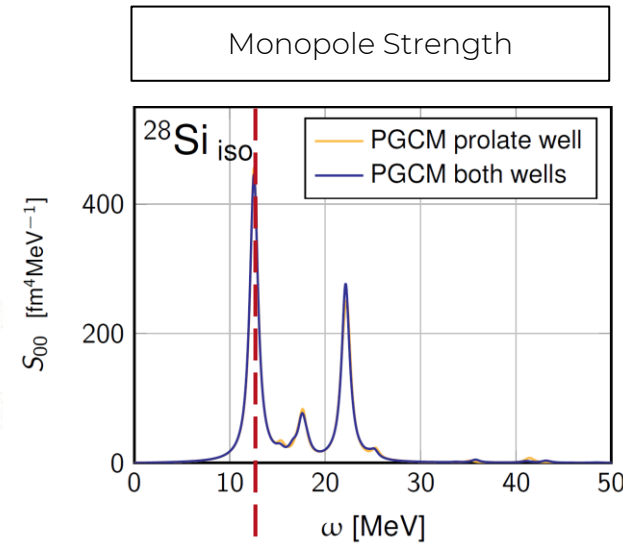
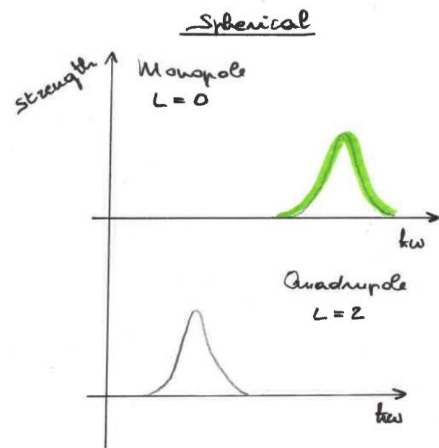
- Focus on the prolate-shape isomer
- Coupling to GQR generates **splitting**
 - ✗ High peak = shifted “spherical” breathing mode
 - ✗ Low peak = induced by coupling to GQR ($K=0$)
- Two-peak GMR on the prolate shape isomer



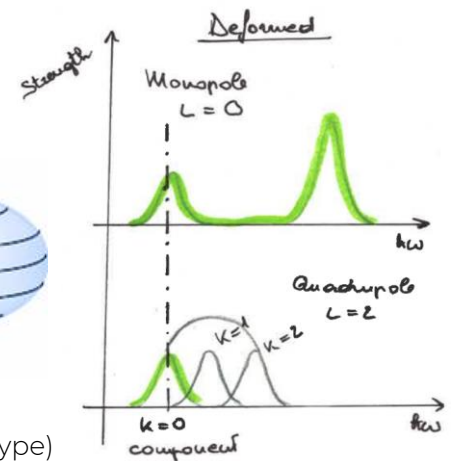
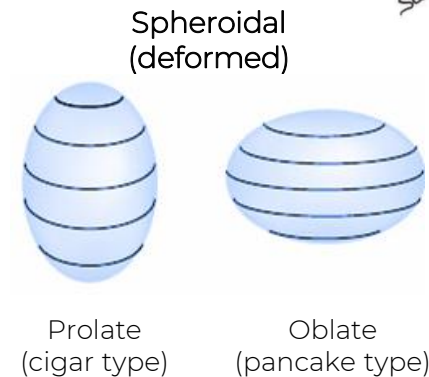
Deformation effects in prolate ^{28}Si



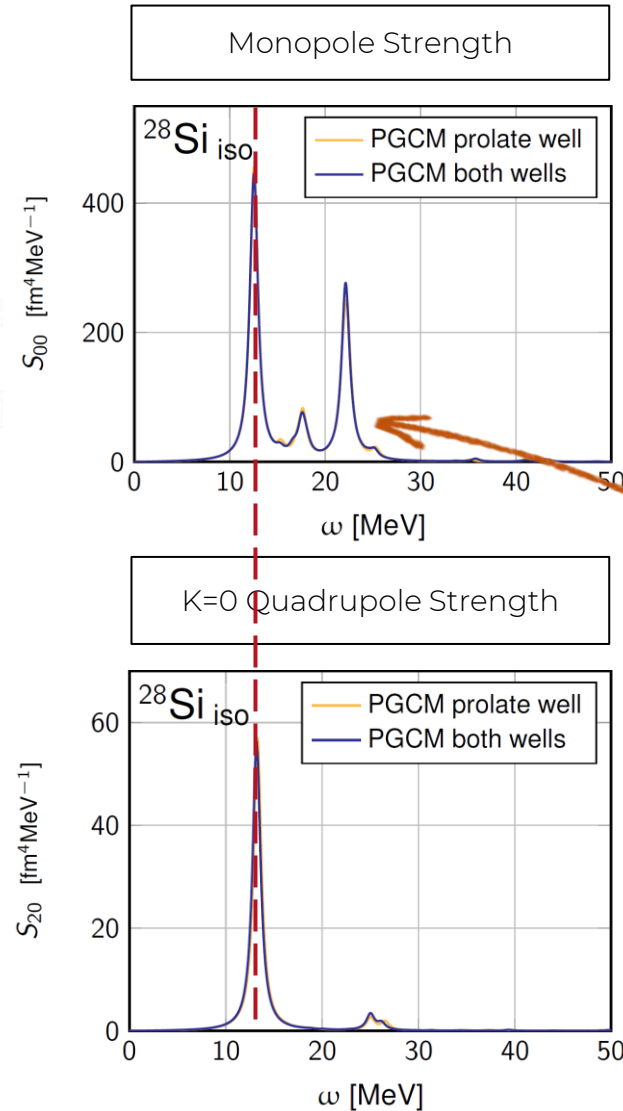
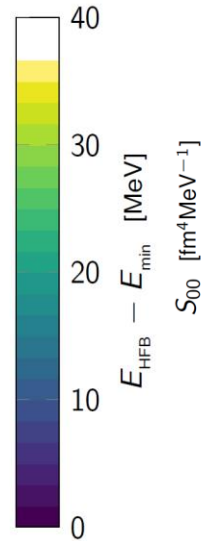
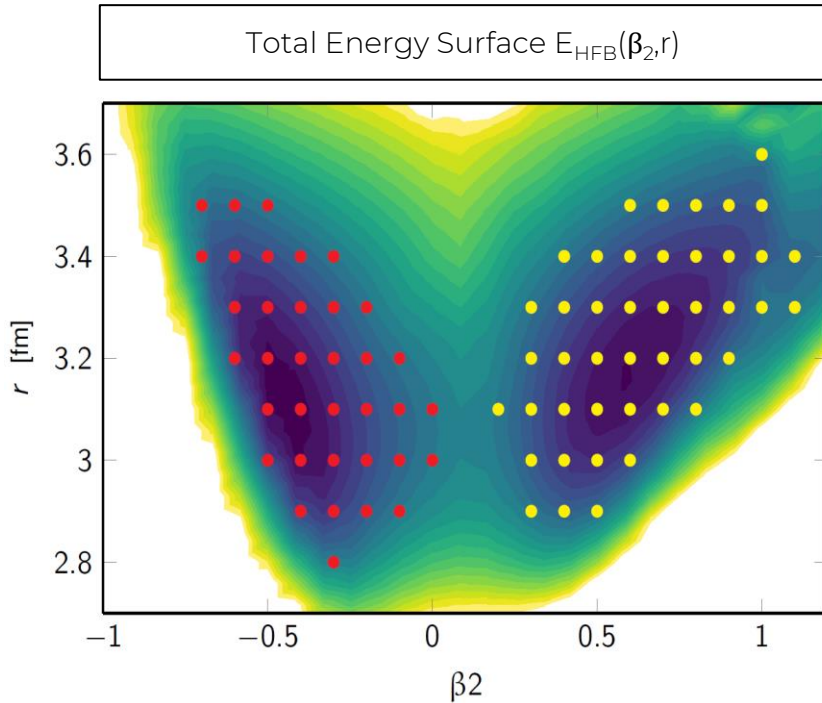
Spherical
(no deformation)



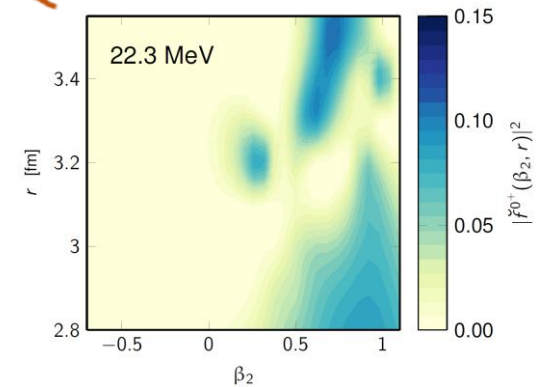
- Focus on the prolate-shape isomer
- Coupling to GQR generates **splitting**
 - ✗ High peak = shifted “spherical” breathing mode
 - ✗ Low peak = induced by coupling to GQR ($K=0$)
- Two-peak GMR on the prolate shape isomer



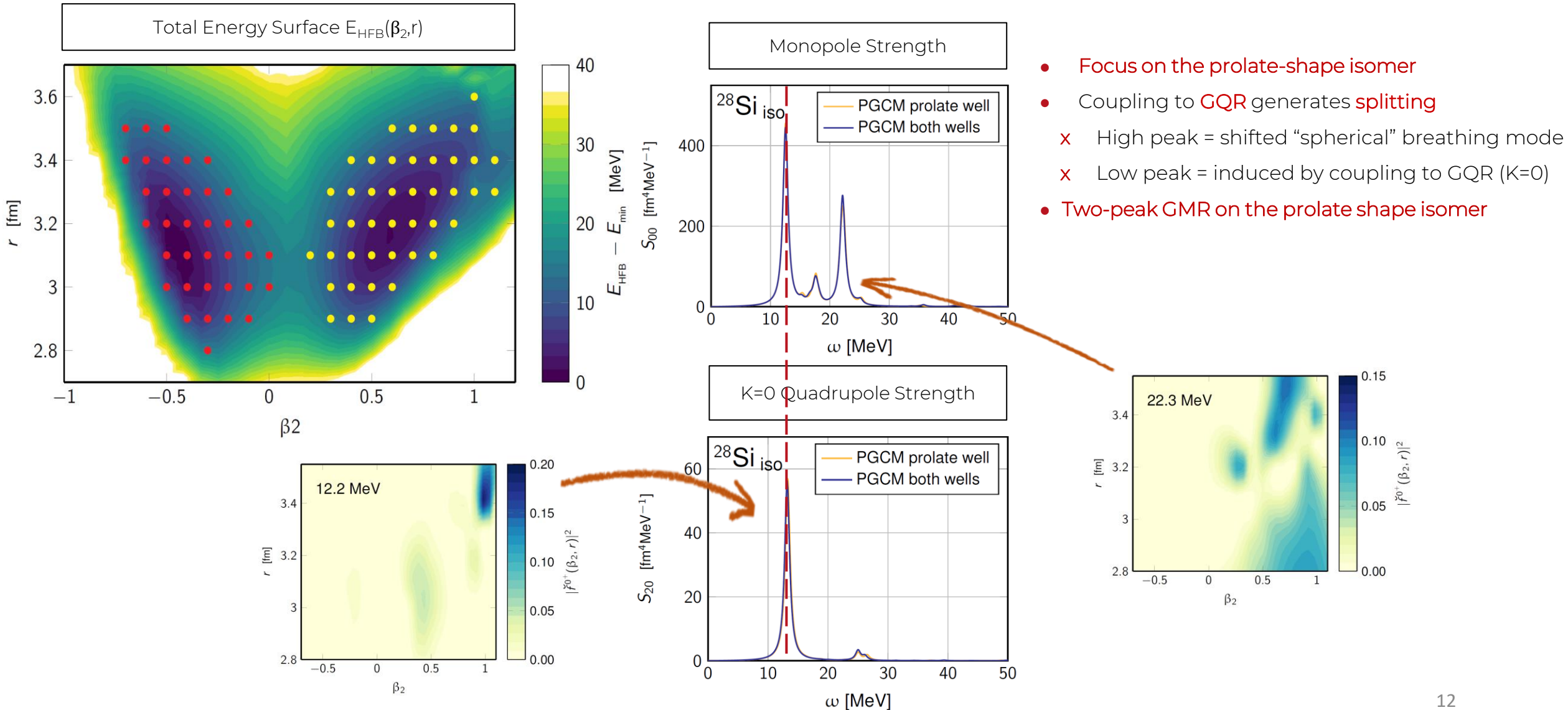
Deformation effects in prolate ^{28}Si



- Focus on the prolate-shape isomer
- Coupling to GQR generates **splitting**
 - ✗ High peak = shifted “spherical” breathing mode
 - ✗ Low peak = induced by coupling to GQR (K=0)
- Two-peak GMR on the prolate shape isomer

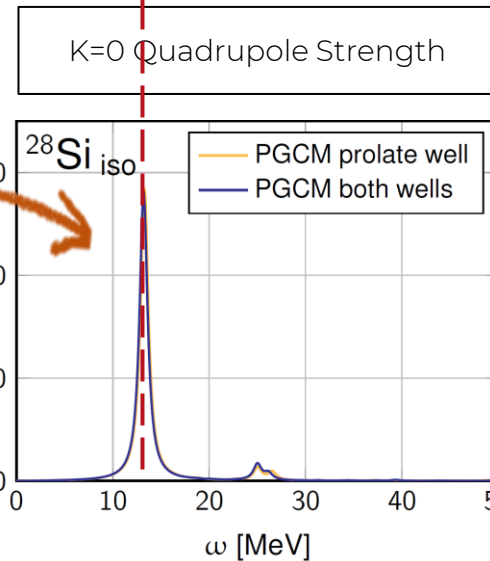
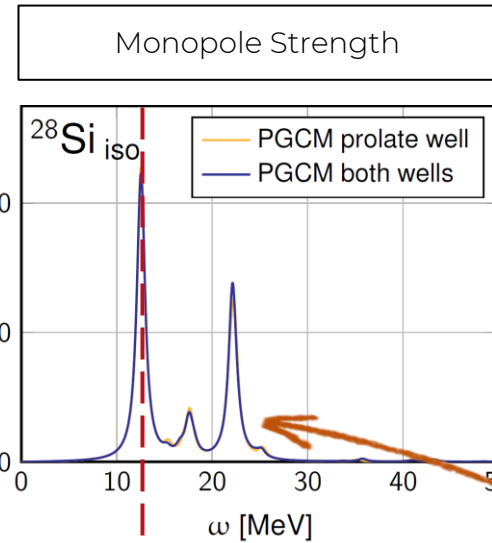
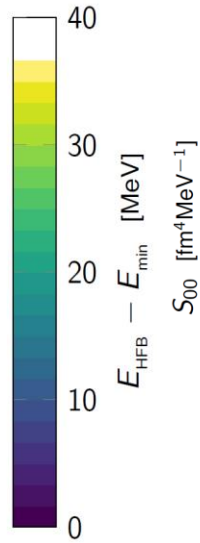
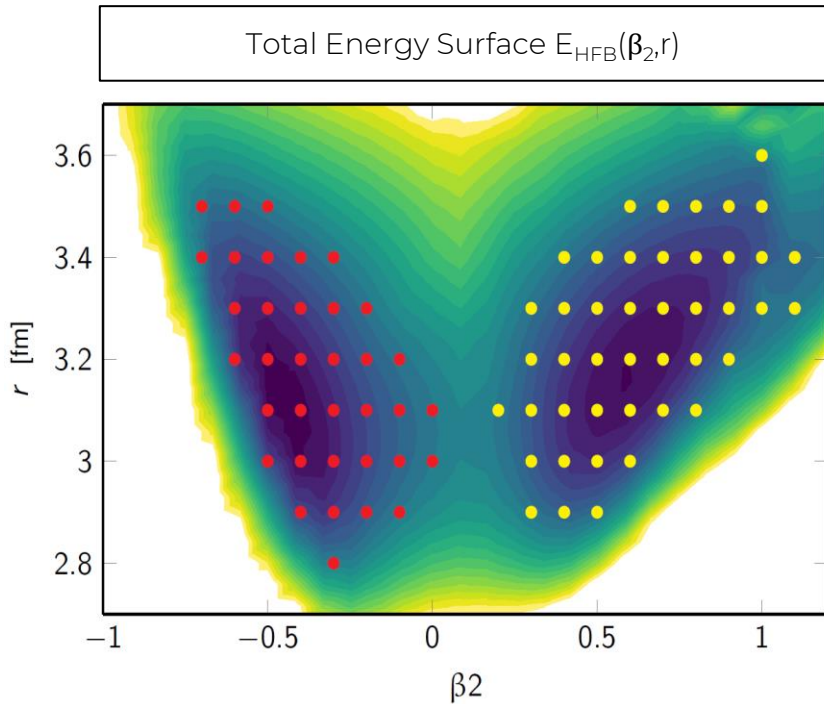


Deformation effects in prolate ^{28}Si



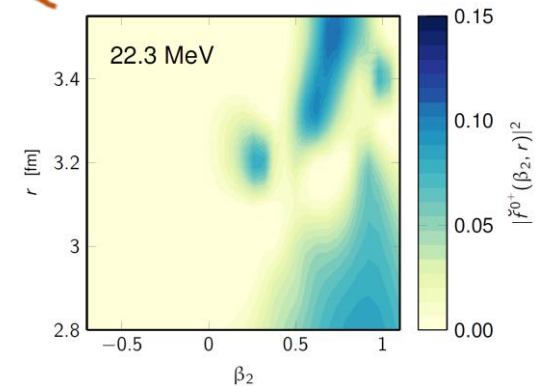
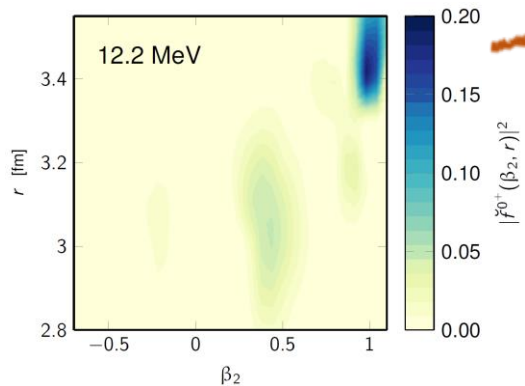
- Focus on the prolate-shape isomer
- Coupling to GQR generates splitting
 - ✗ High peak = shifted “spherical” breathing mode
 - ✗ Low peak = induced by coupling to GQR (K=0)
- Two-peak GMR on the prolate shape isomer

Deformation effects in prolate ^{28}Si

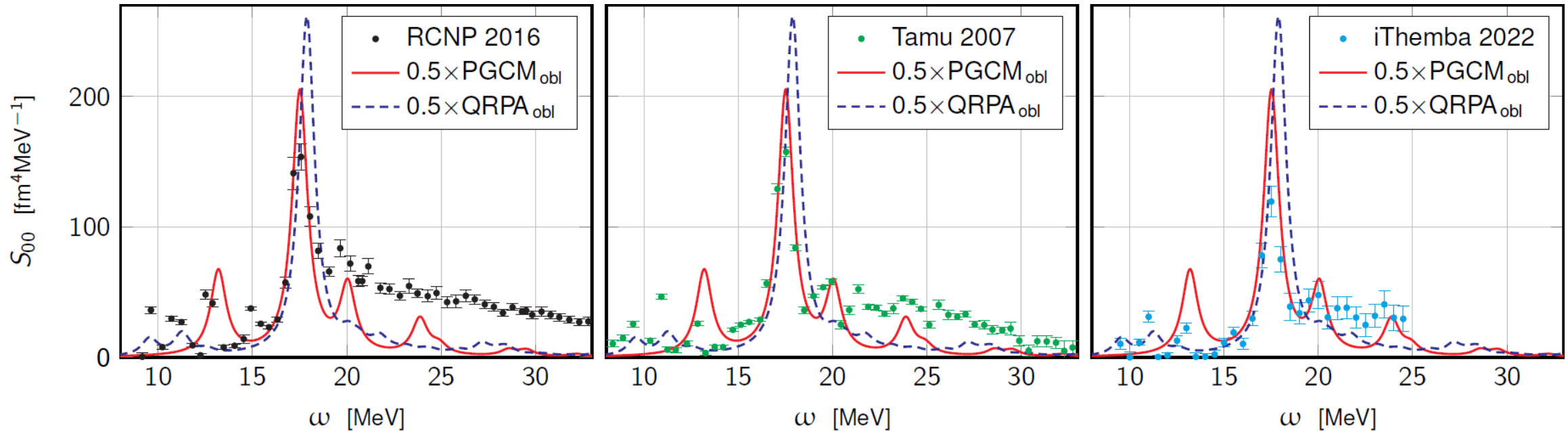


- Focus on the prolate-shape isomer
- Coupling to GQR generates **splitting**
 - ✗ High peak = shifted “spherical” breathing mode
 - ✗ Low peak = induced by coupling to GQR (K=0)
- Two-peak GMR on the prolate shape isomer

Experimentally accessible ?



Comparison to experiment ^{28}Si



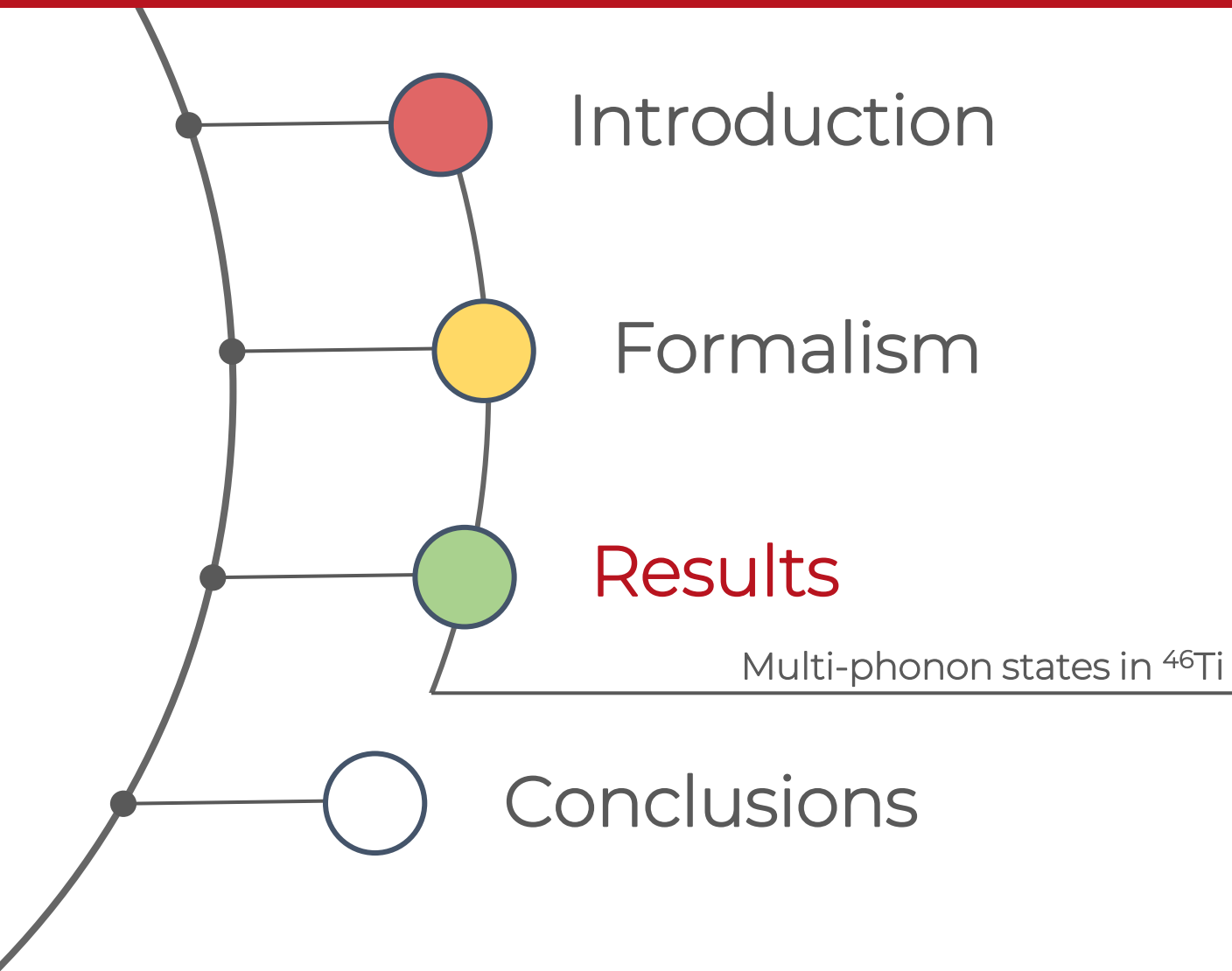
PGCM better reproduces the experimental data

- Better description of the main resonance
- Fragmentation effects are better captured than in QRPA

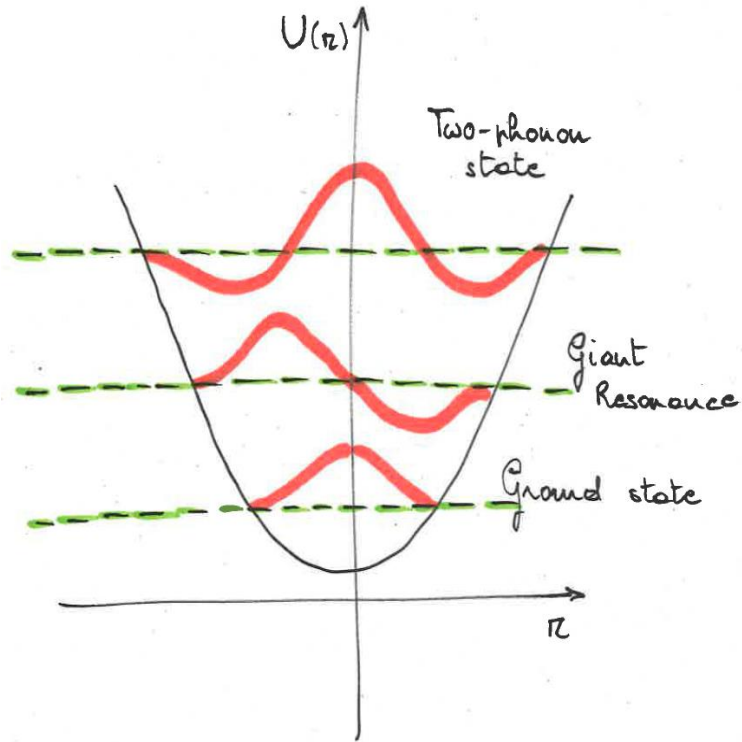
Experimental data are useful and promising to **test different many-body methods**

Data are not unambiguous, i.e. **better resolution** would be beneficial

Outline

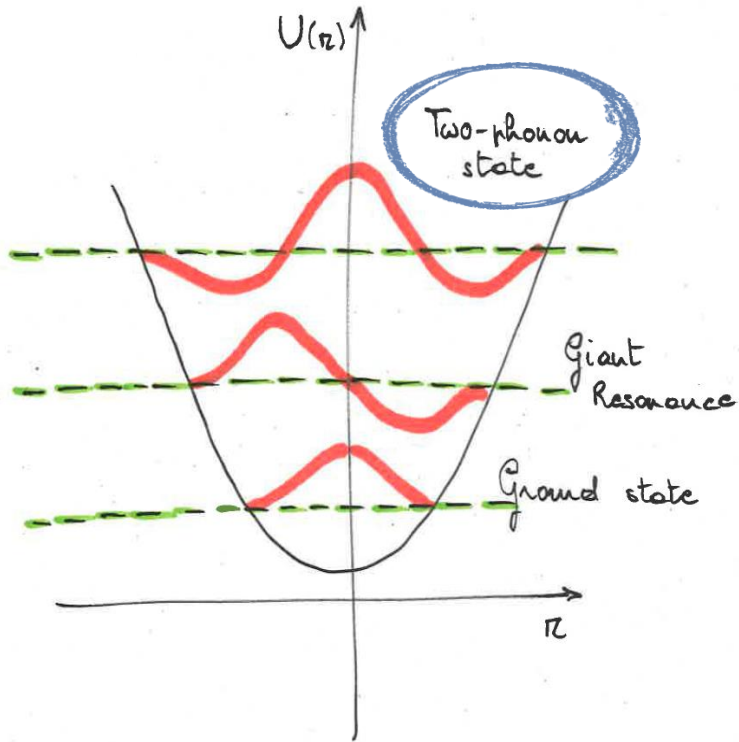


Multi-phonon states in ^{46}Ti



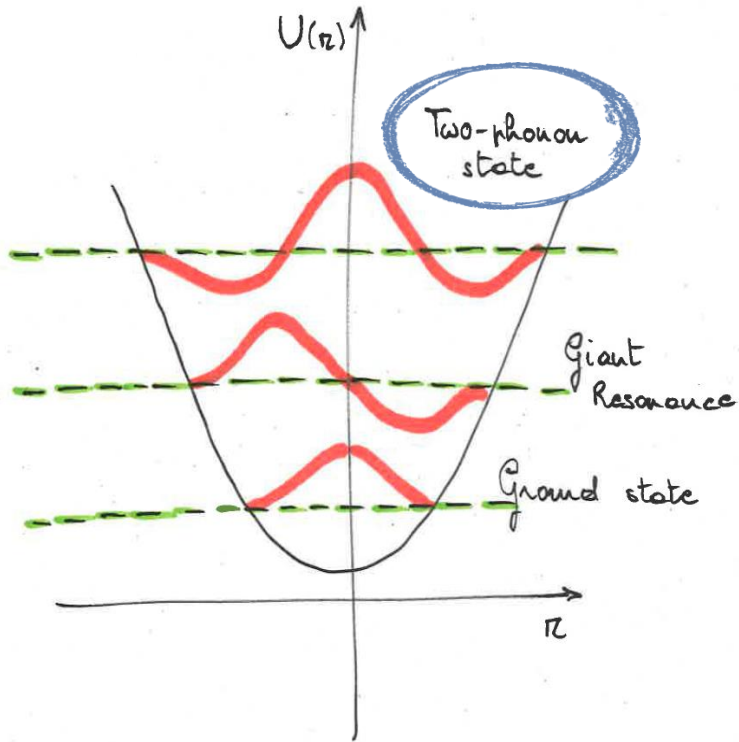
- GRs are the **first phonon** of a collective excitation

Multi-phonon states in ^{46}Ti



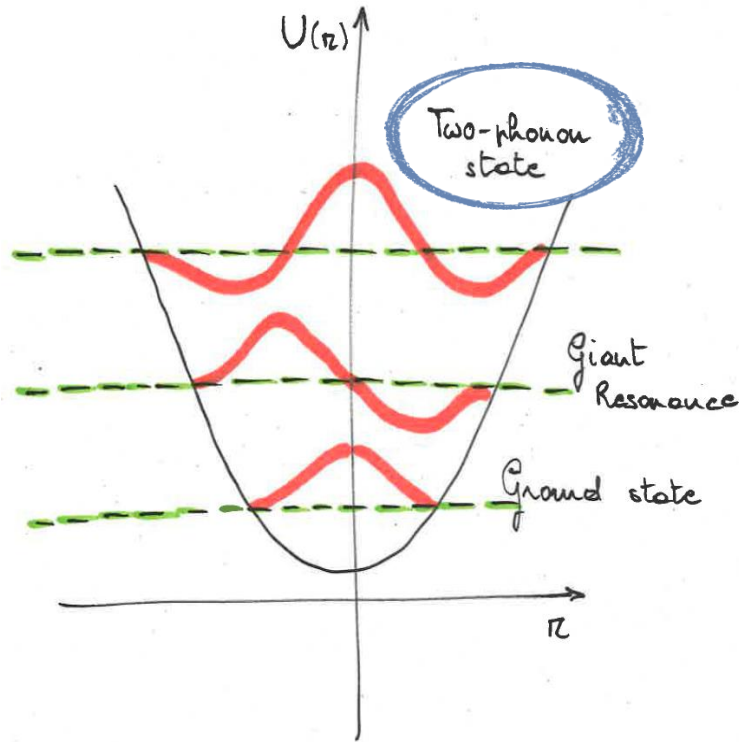
- GRs are the **first phonon** of a collective excitation
- Higher phonons also exist ! **Multi-phonon states**

Multi-phonon states in ^{46}Ti



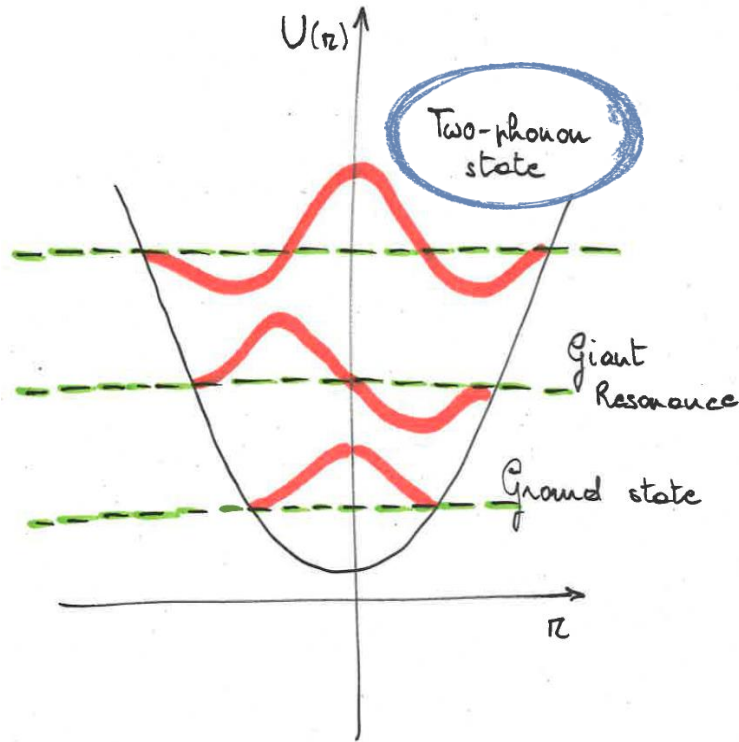
- GRs are the **first phonon** of a collective excitation
- **Higher phonons also exist!** **Multi-phonon states**
- **Not** accessible to QRPA

Multi-phonon states in ^{46}Ti



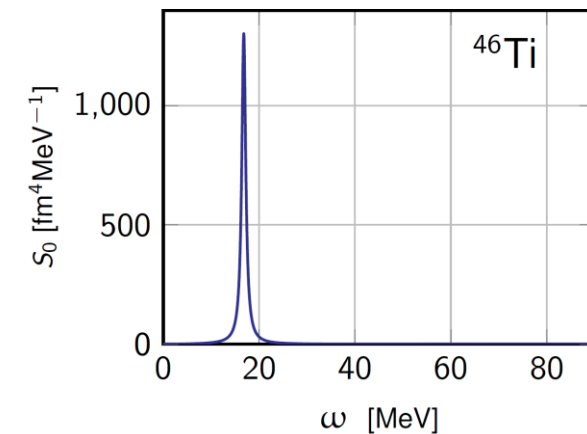
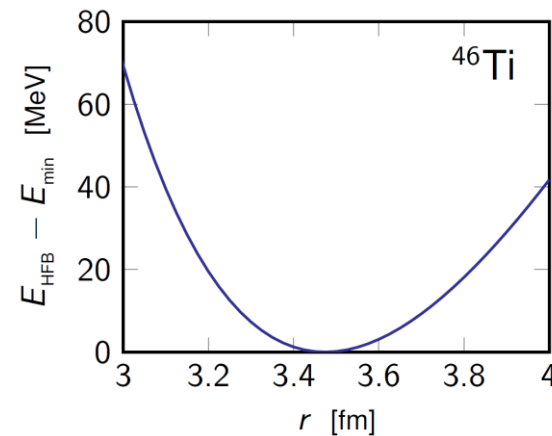
- GRs are the **first phonon** of a collective excitation
- **Higher phonons also exist!** **Multi-phonon states**
- **Not** accessible to QRPA
- Equally spaced in the harmonic limit

Multi-phonon states in ^{46}Ti

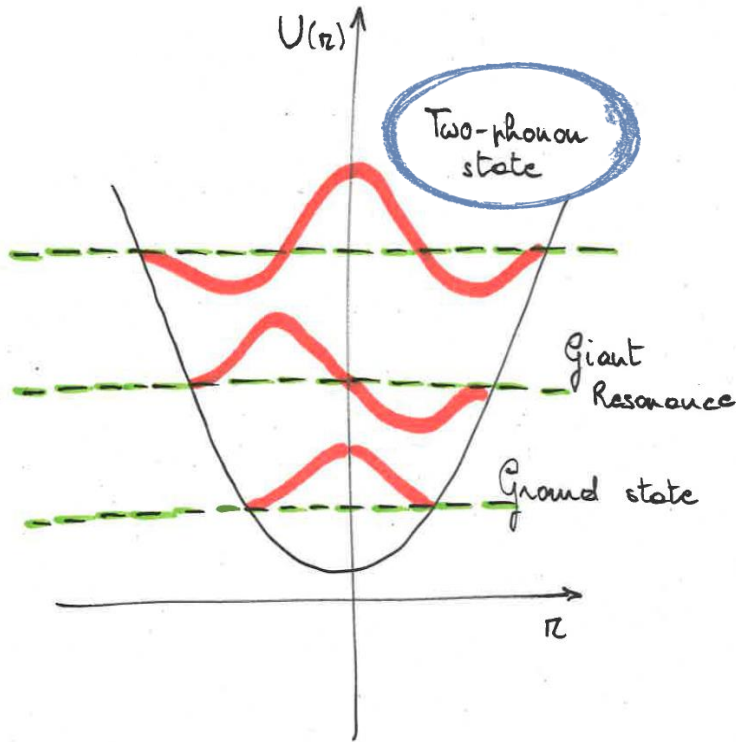


- GRs are the **first phonon** of a collective excitation
- **Higher phonons also exist!** **Multi-phonon states**
- **Not** accessible to QRPA
- Equally spaced in the harmonic limit

One-dimensional PGCM calculation

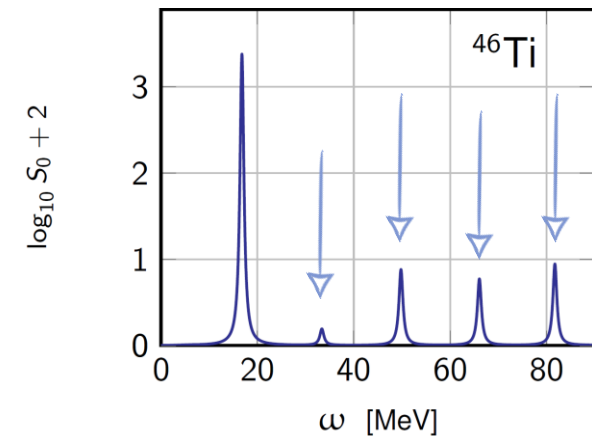
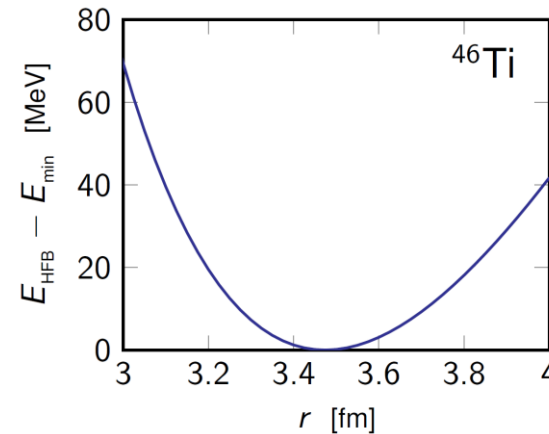


Multi-phonon states in ^{46}Ti



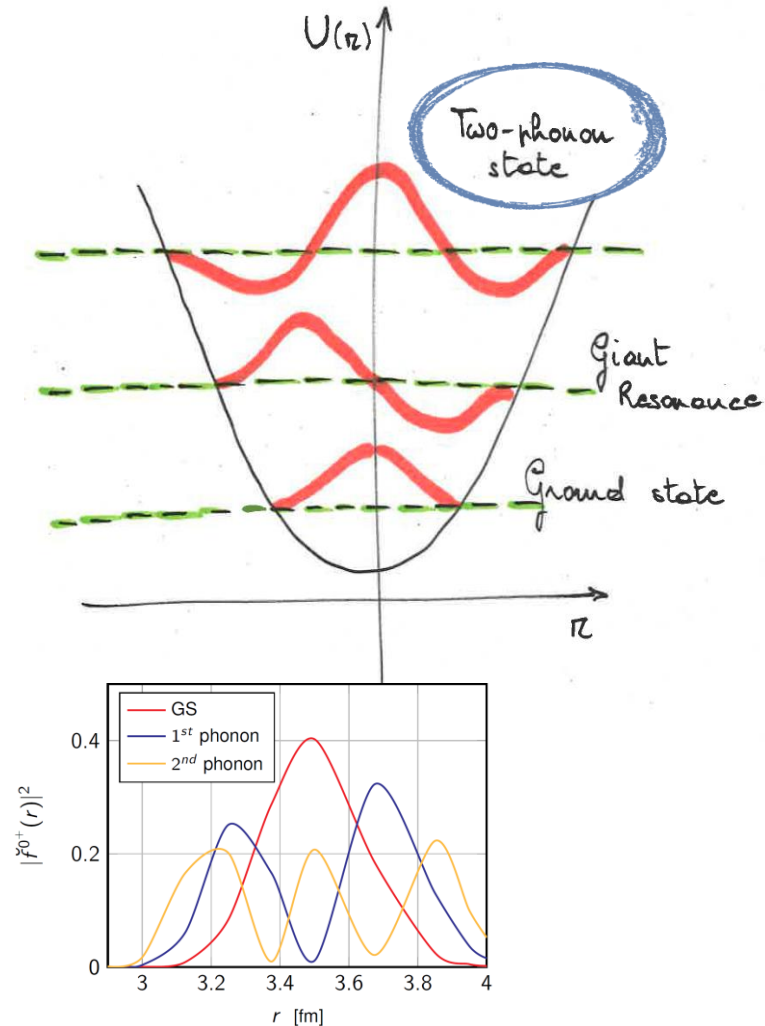
- GRs are the **first phonon** of a collective excitation
- **Higher phonons also exist !** **Multi-phonon states**
- **Not** accessible to QRPA
- Equally spaced in the harmonic limit

One-dimensional PGCM calculation



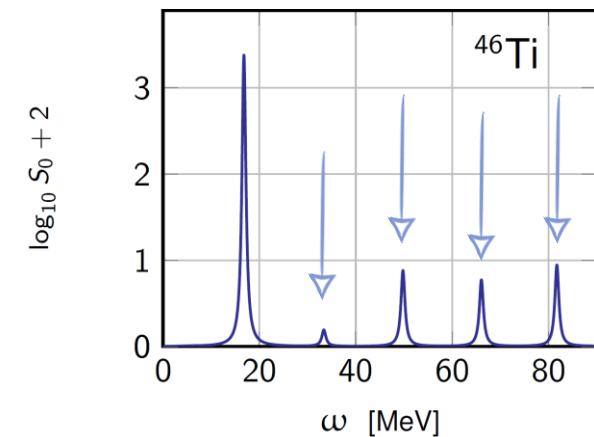
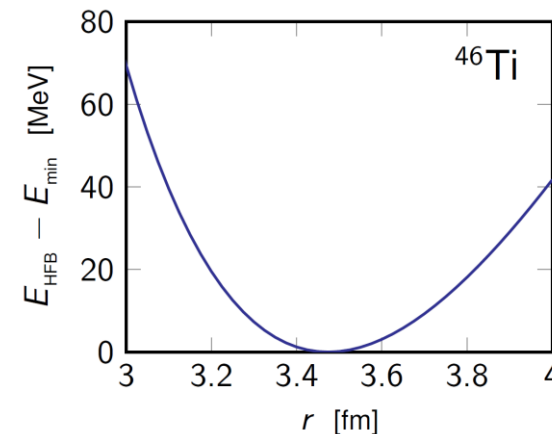
- PGCM predicts high-lying states

Multi-phonon states in ^{46}Ti



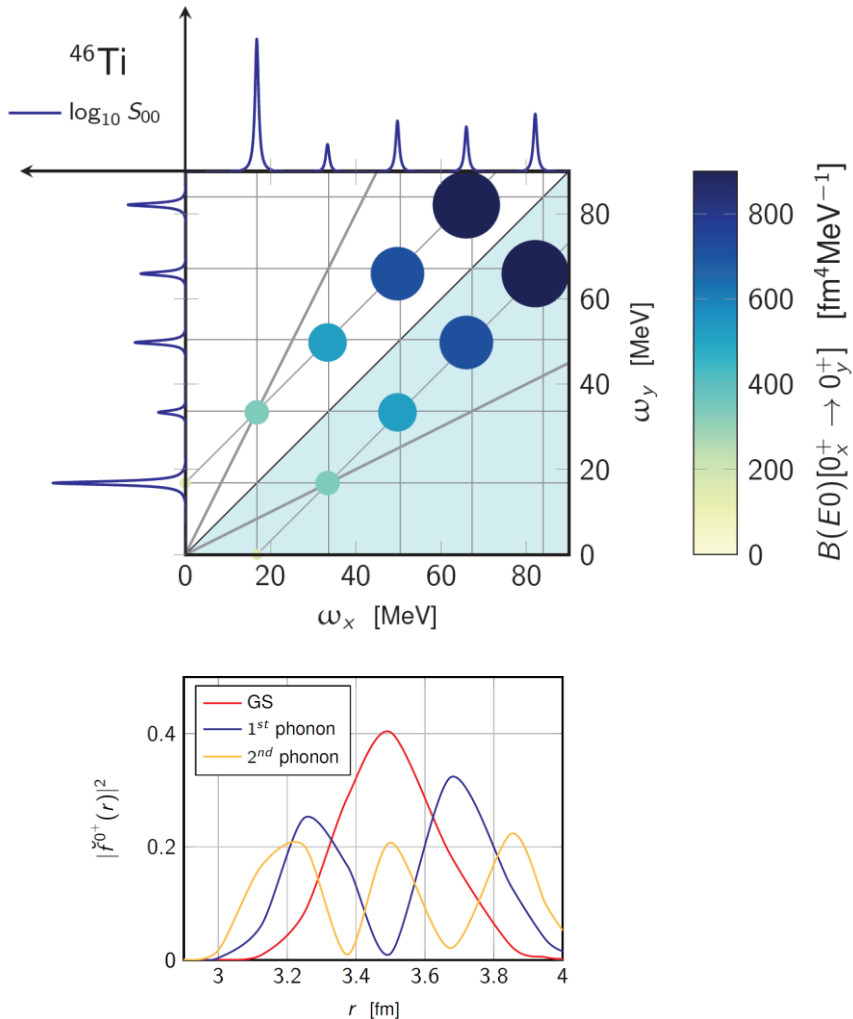
- GRs are the **first phonon** of a collective excitation
- **Higher phonons also exist !** **Multi-phonon states**
- **Not** accessible to QRPA
- Equally spaced in the harmonic limit

One-dimensional PGCM calculation



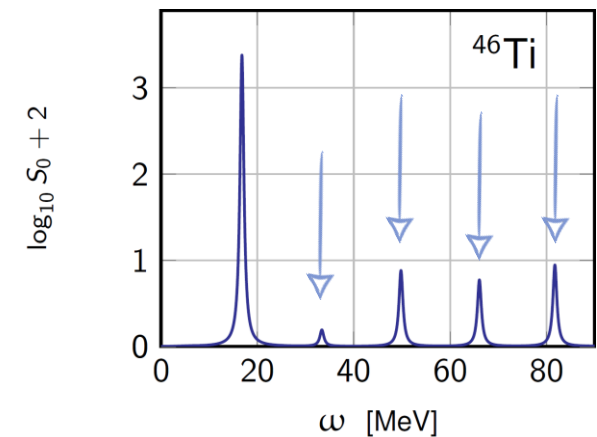
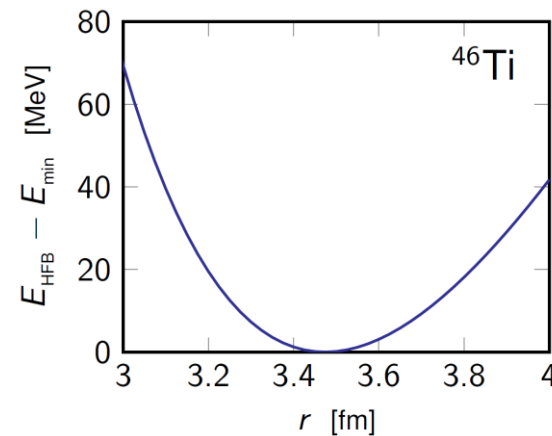
- PGCM predicts high-lying states
- Close to the harmonic oscillator eigen-solutions

Multi-phonon states in ^{46}Ti



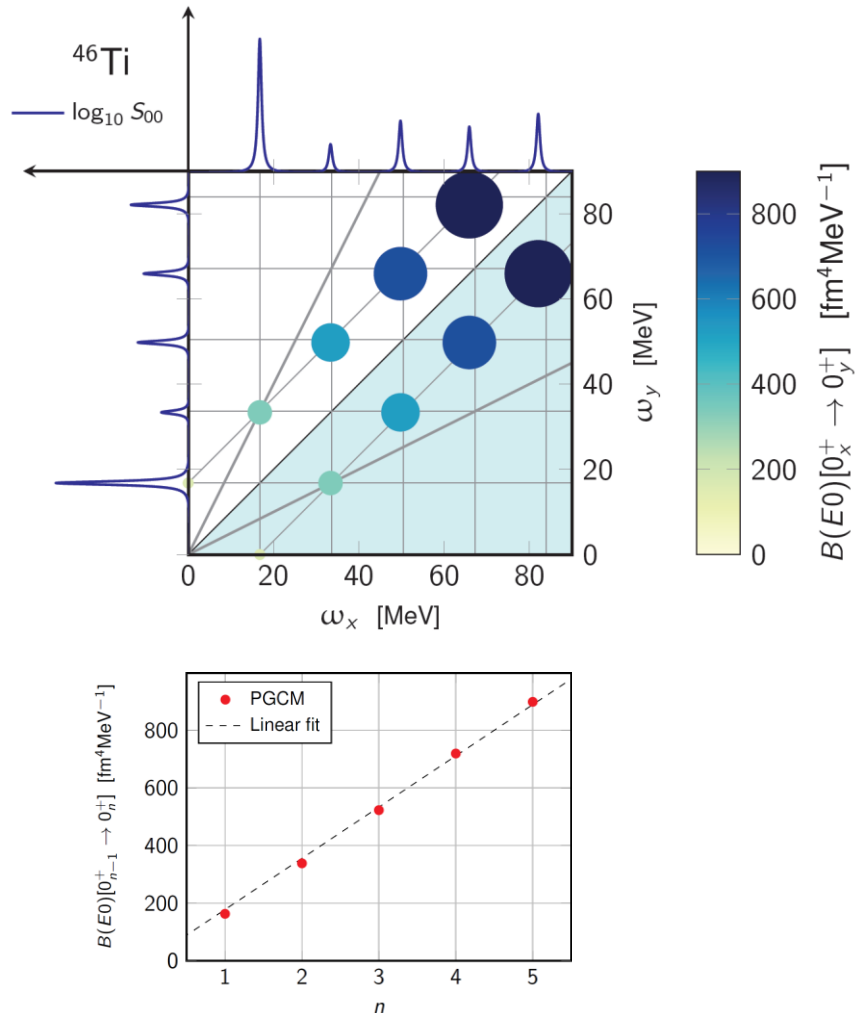
- GRs are the **first phonon** of a collective excitation
- Higher phonons also exist !** **Multi-phonon states**
- Not accessible to QRPA**
- Equally spaced in the harmonic limit

One-dimensional PGCM calculation



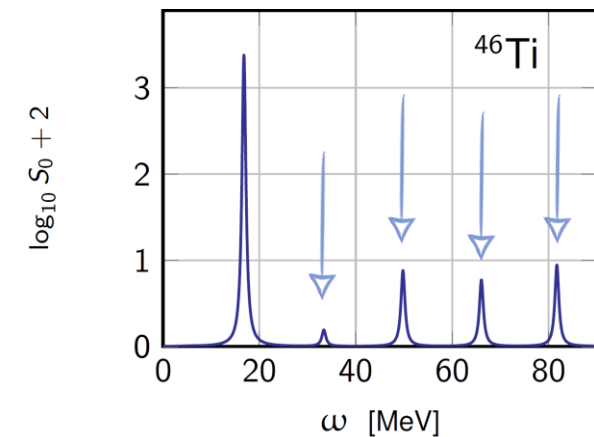
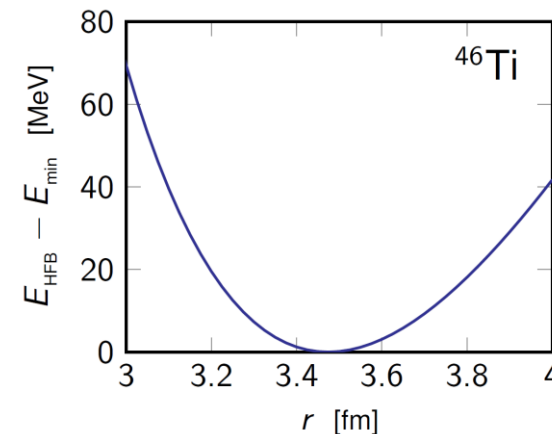
- PGCM predicts high-lying states**
- Close to the harmonic oscillator eigen-solutions
- Transitions maximised between neighbouring phonons

Multi-phonon states in ^{46}Ti



- GRs are the **first phonon** of a collective excitation
- Higher phonons also exist !** Multi-phonon states
- Not accessible to QRPA**
- Equally spaced in the harmonic limit

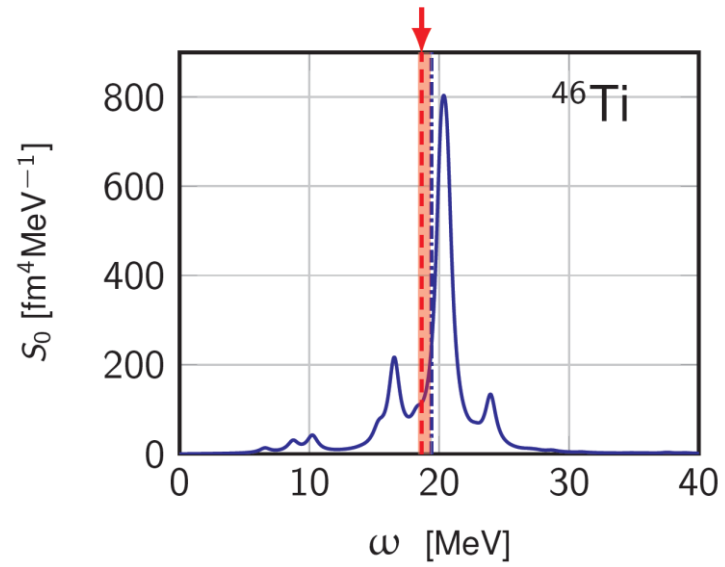
One-dimensional PGCM calculation



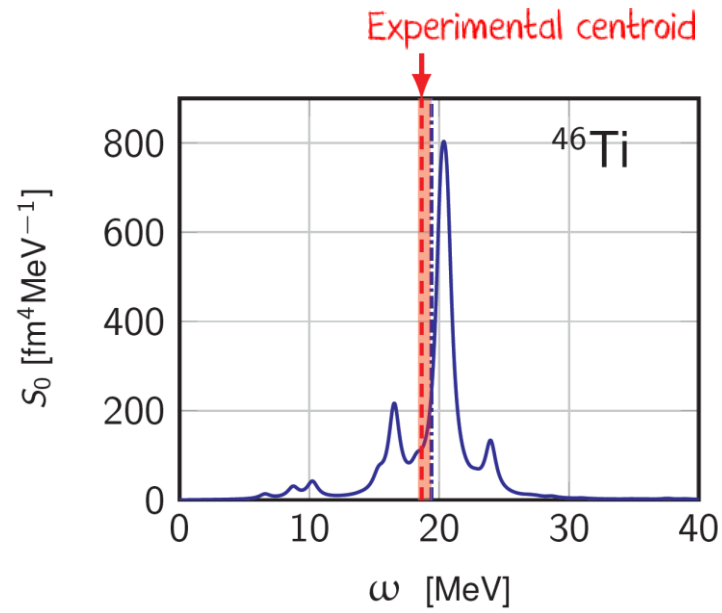
- PGCM predicts high-lying states**
- Close to the harmonic oscillator eigen-solutions
- Transitions maximised between neighbouring phonons
- x** Linear trend in the transition strength

Realistic calculations

- Realistic PGCM in the (r, β_2) plane

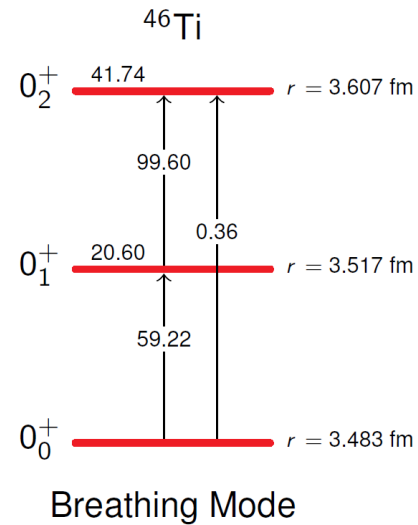
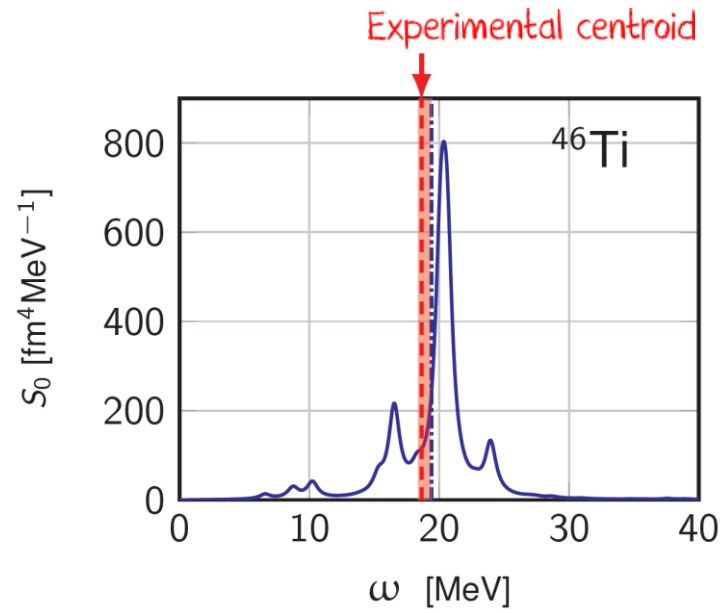


Realistic calculations



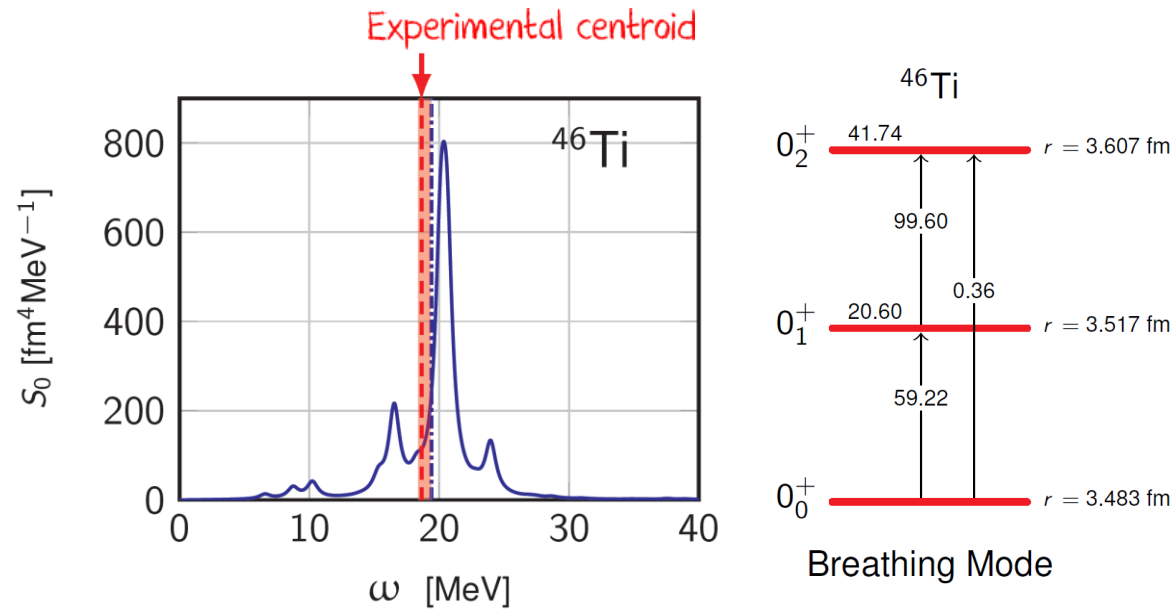
- Realistic PGCM in the (r, β_2) plane
- Good agreement with experiment

Realistic calculations

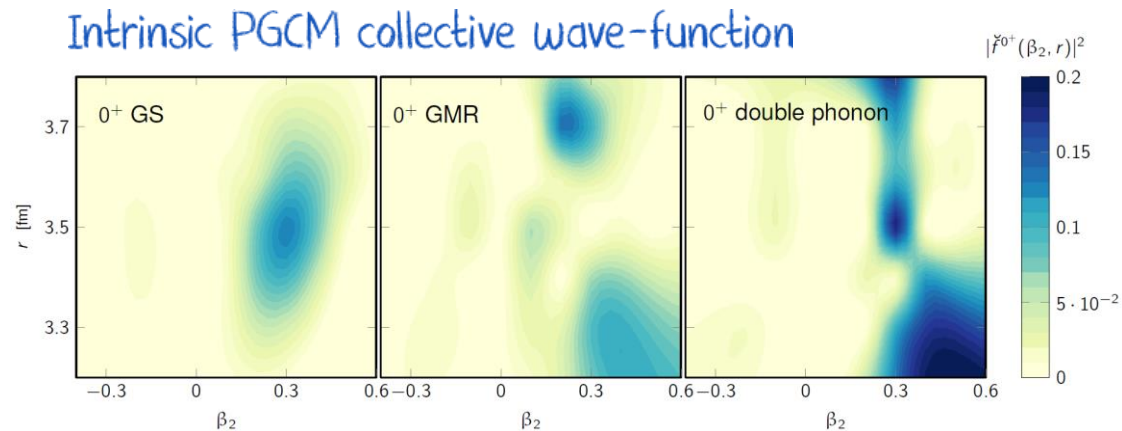


- Realistic PGCM in the (r, β_2) plane
- Good agreement with experiment
- Multi-phonon states observed

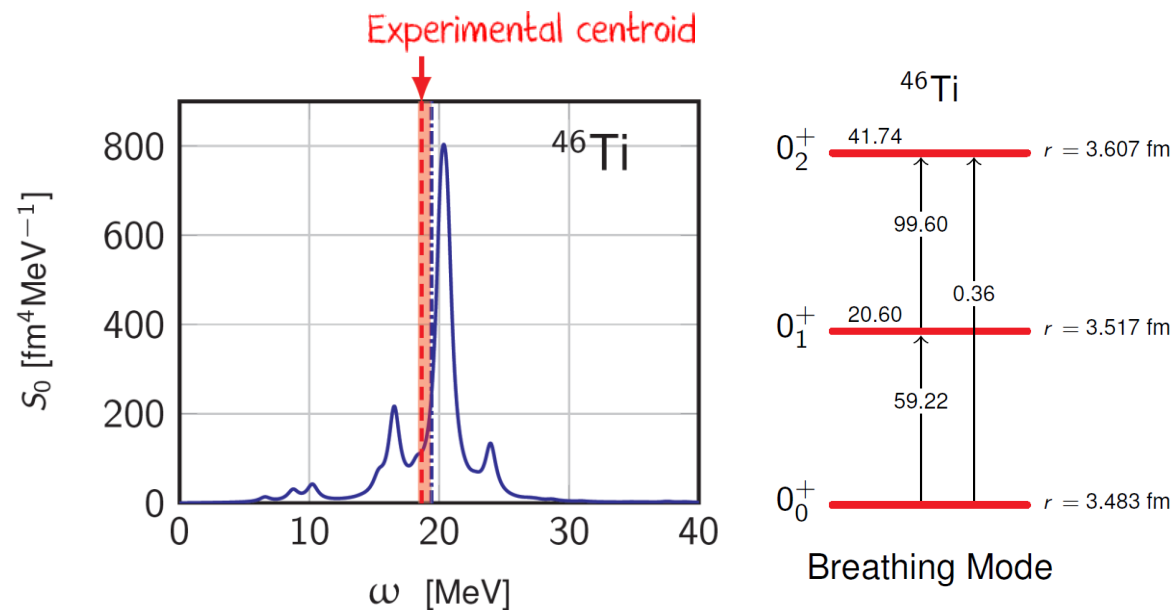
Realistic calculations



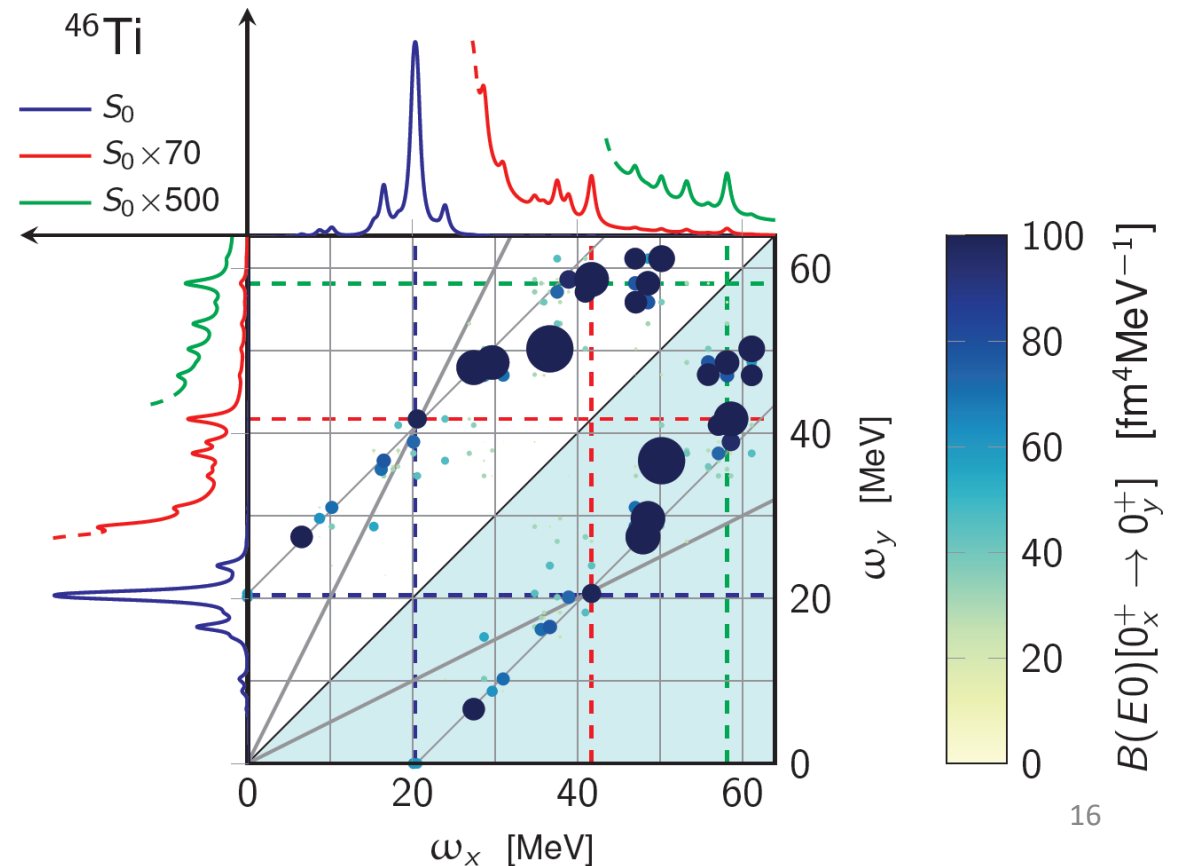
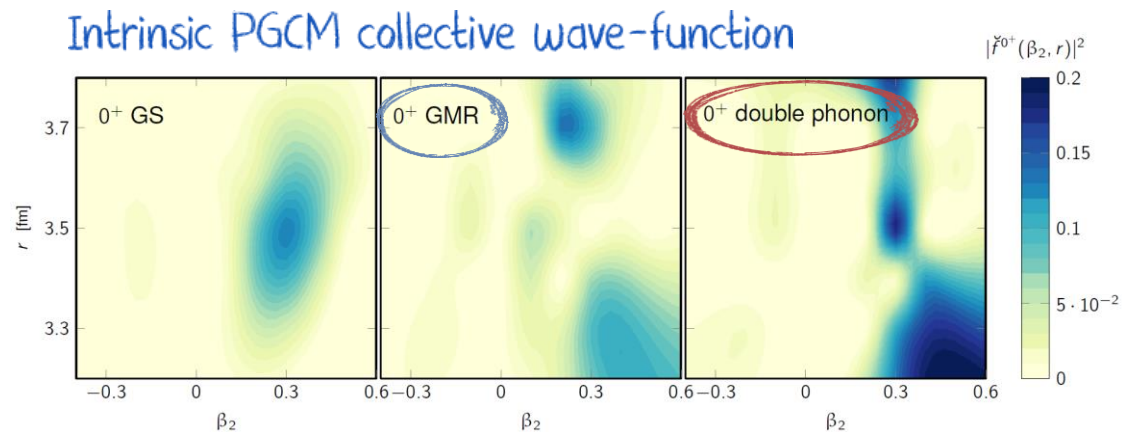
- Realistic PGCM in the (r, β_2) plane
- Good agreement with experiment
- Multi-phonon states observed



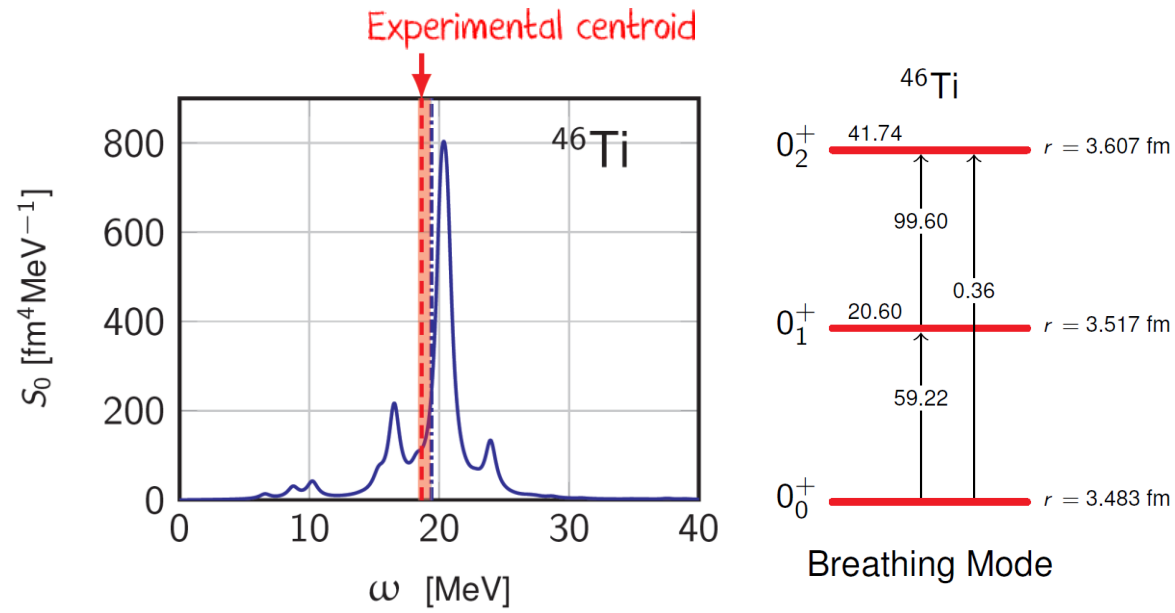
Realistic calculations



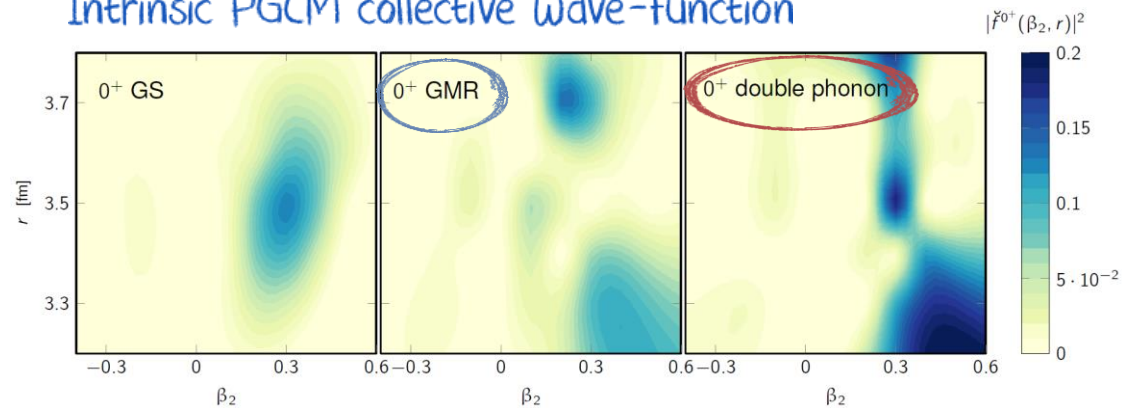
- Realistic PGCM in the (r, β_2) plane
- Good agreement with experiment
- Multi-phonon states observed



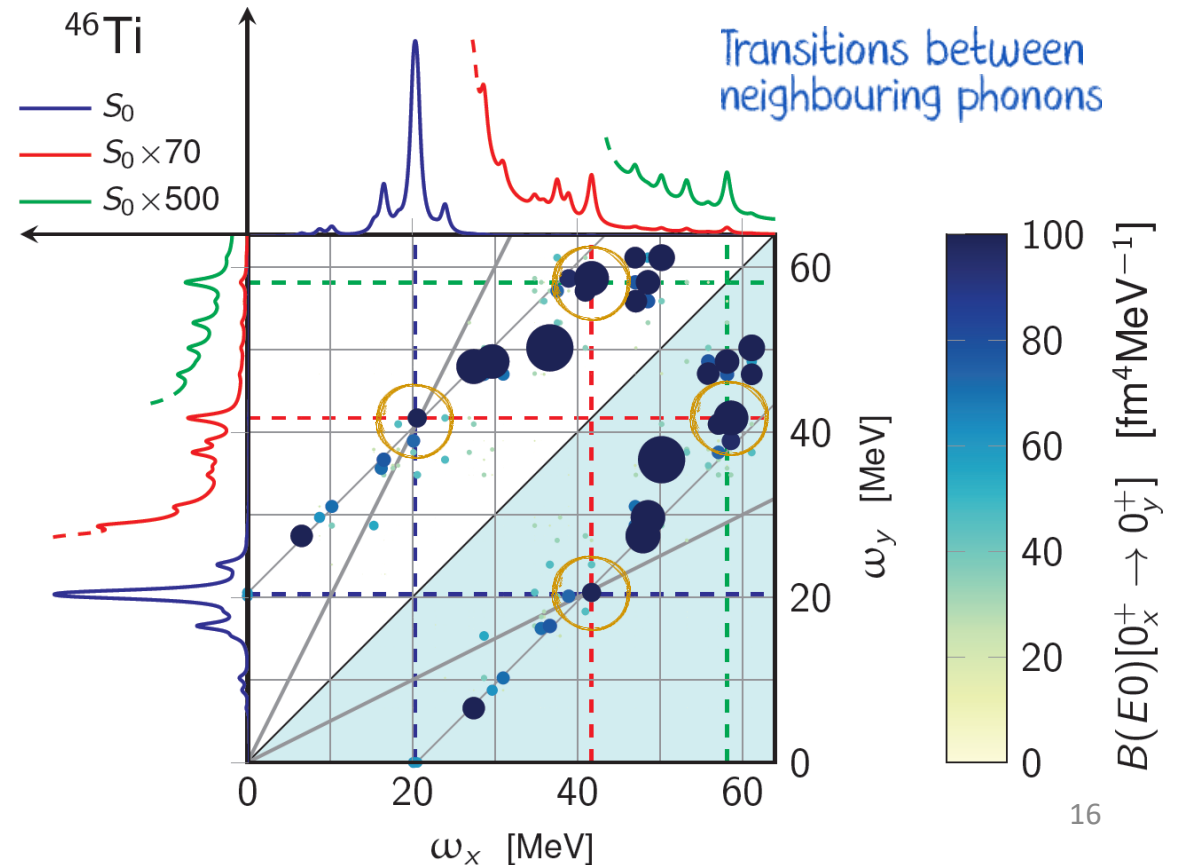
Realistic calculations



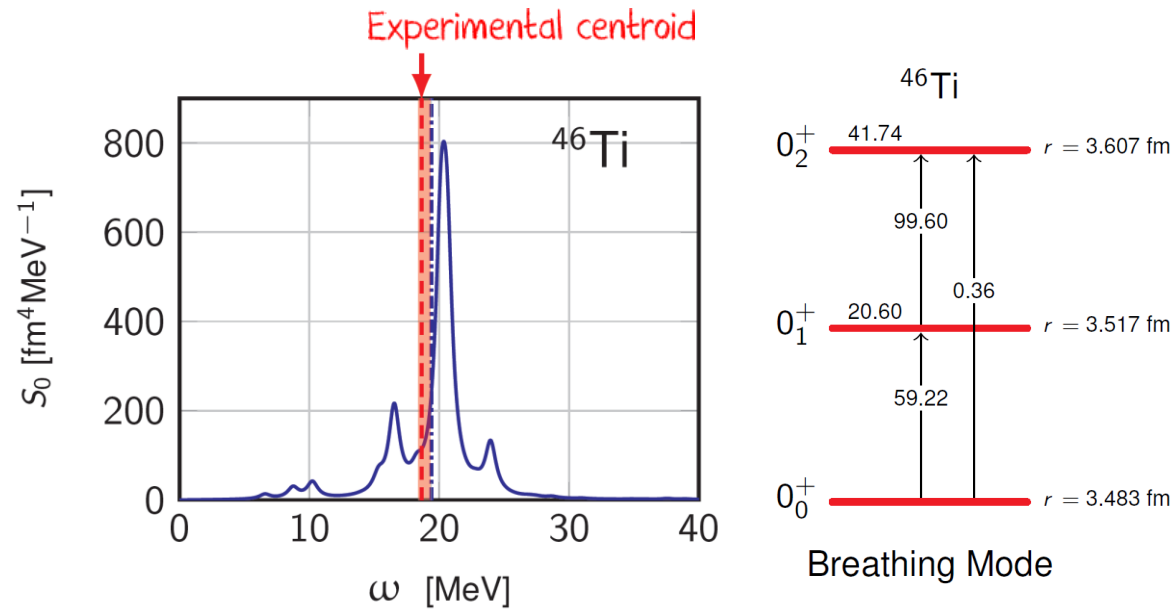
Intrinsic PGCM collective wave-function



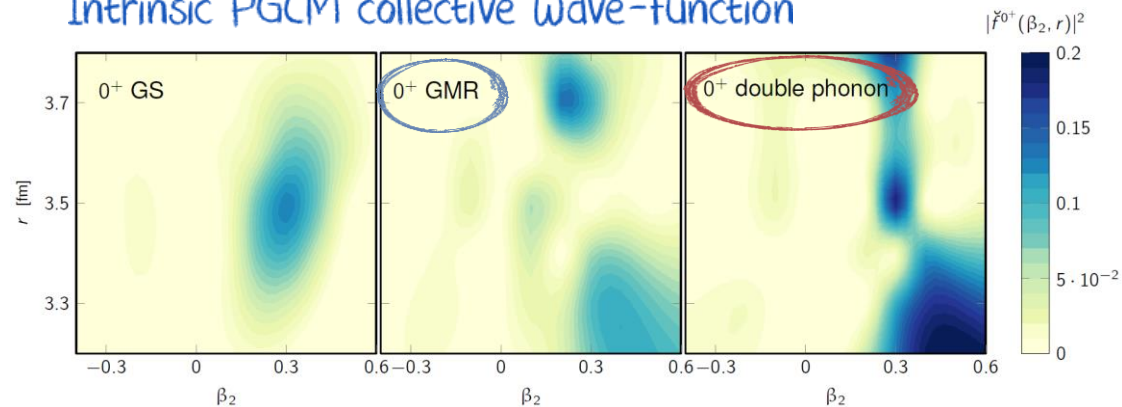
- Realistic PGCM in the (r, β_2) plane
- Good agreement with experiment
- Multi-phonon states observed



Realistic calculations

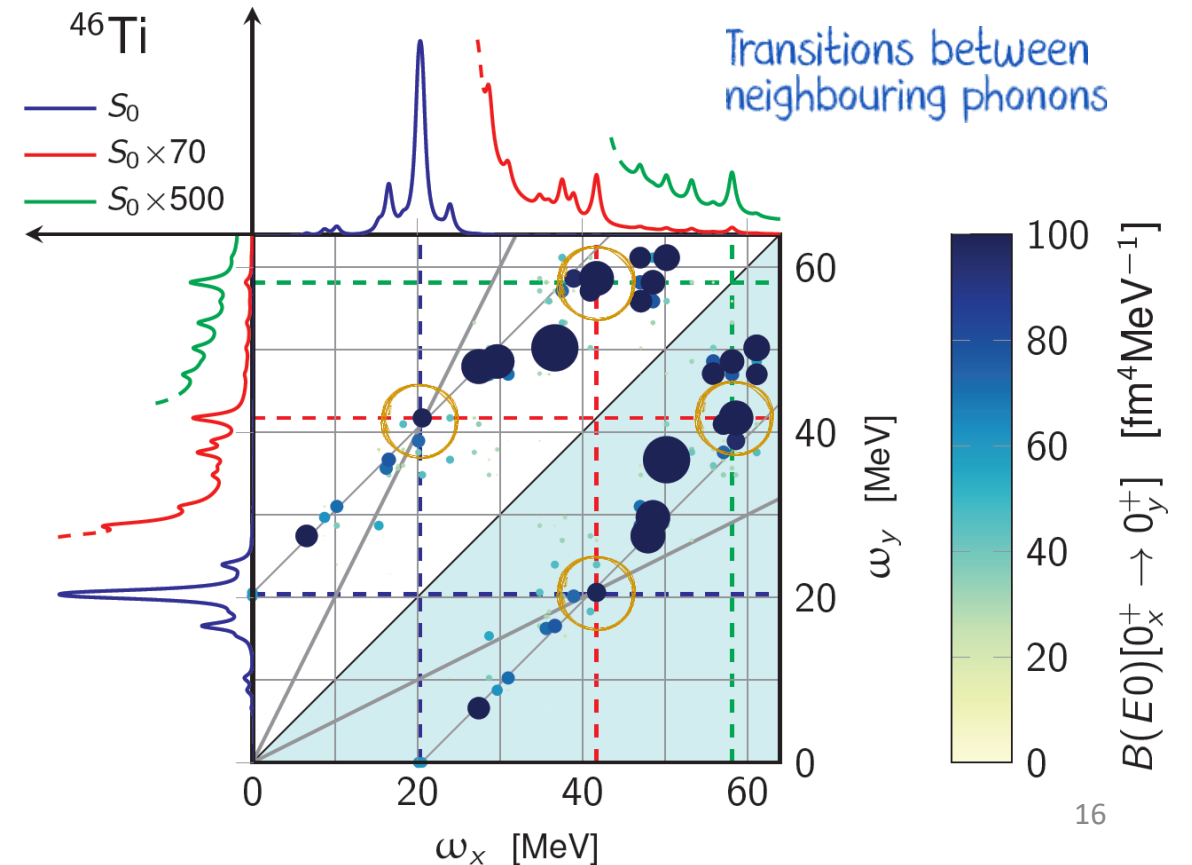


Intrinsic PGCM collective wave-function

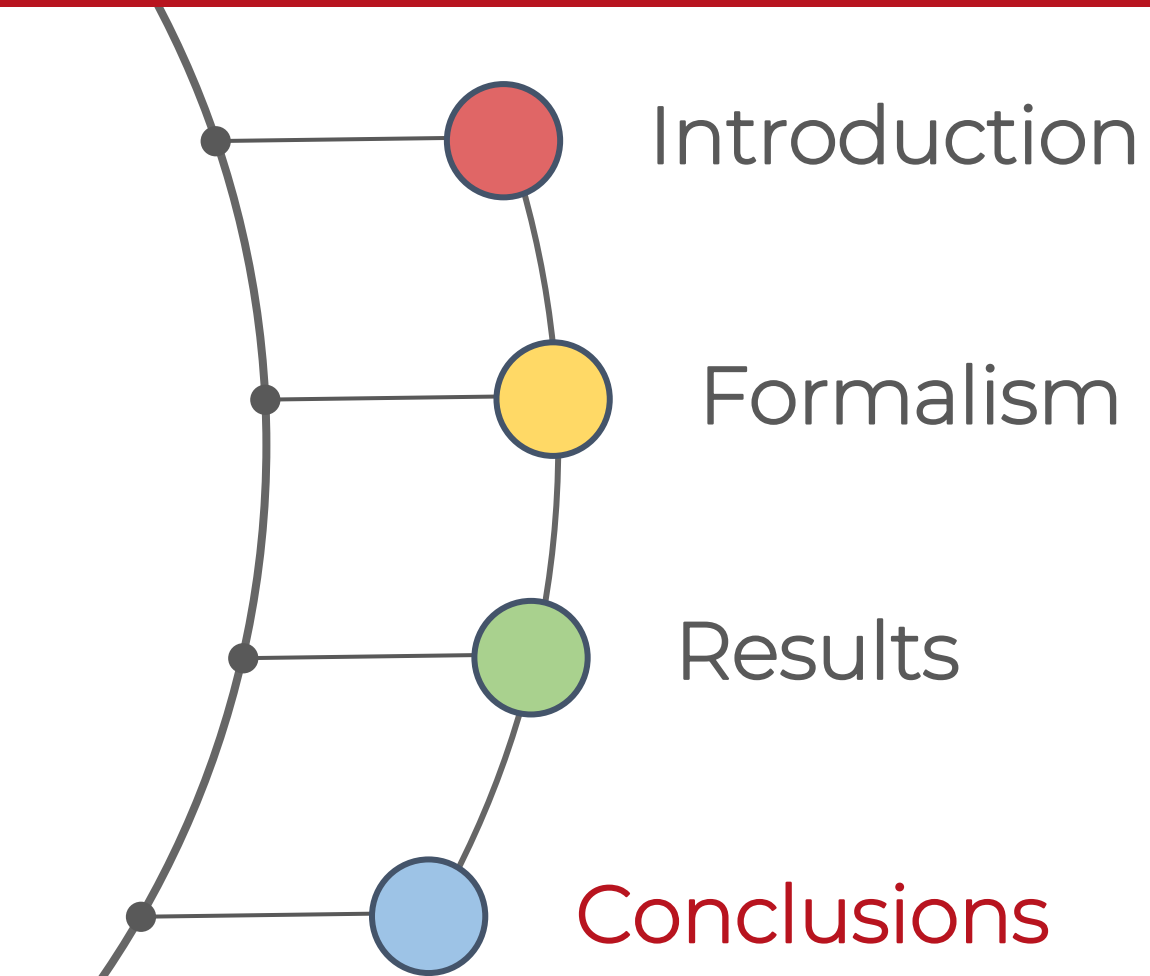


- Realistic PGCM in the (r, β_2) plane
- Good agreement with experiment
- Multi-phonon states observed

Experimentally accessible ?



Outline



Conclusions and Perspectives

Conclusions and Perspectives

- PGCM is a valuable and reliable tool for *ab-initio* spectroscopy

Conclusions and Perspectives

 PGCM is a valuable and reliable tool for *ab-initio* spectroscopy

Different levels of symmetry breaking and restoration can reveal
new physical and/or theoretical insights



Conclusions and Perspectives

 PGCM is a valuable and reliable tool for *ab-initio* spectroscopy


Different levels of symmetry breaking and restoration can reveal
new physical and/or theoretical insights



 Access to observables and phenomena scarcely investigated previously in *ab-initio*

Conclusions and Perspectives

 PGCM is a valuable and reliable tool for *ab-initio* spectroscopy

Different levels of symmetry breaking and restoration can reveal
new physical and/or theoretical insights 

 Access to observables and phenomena scarcely investigated previously in *ab-initio*

What's next ?

Conclusions and Perspectives

● PGCM is a valuable and reliable tool for *ab-initio* spectroscopy

Different levels of symmetry breaking and restoration can reveal
new physical and/or theoretical insights



● Access to observables and phenomena scarcely investigated previously in *ab-initio*

What's next ?

- Many upgrade concerning PGCM
 - Explore further generator coordinates
 - Deeper convergence study
 - Uncertainty quantification
 - Nuclear incompressibility

Conclusions and Perspectives

- PGCM is a valuable and reliable tool for *ab-initio* spectroscopy

Different levels of symmetry breaking and restoration can reveal new physical and/or theoretical insights

- Access to observables and phenomena scarcely investigated previously in *ab-initio*

What's next ?

- Many upgrade concerning PGCM
 - Explore further generator coordinates
 - Deeper convergence study
 - Uncertainty quantification
 - Nuclear incompressibility

Eigenvector Continuation

Conclusions and Perspectives

- PGCM is a valuable and reliable tool for *ab-initio* spectroscopy

Different levels of symmetry breaking and restoration can reveal new physical and/or theoretical insights

- Access to observables and phenomena scarcely investigated previously in *ab-initio*

What's next ?

- Many upgrade concerning PGCM
 - Explore further generator coordinates
 - Deeper convergence study
 - Uncertainty quantification
 - Nuclear incompressibility

Eigenvector Continuation

- Develop full symmetry-conserving QRPA

[Federschmidt and Ring, NucPhysA, 1985]

Thanks for the attention



CEA DRF

Thomas Duguet
Vittorio Somà
Benjamin Bally
Alberto Scalesi



CEA DES

Mikael Frosini



CEA DAM

Jean-Paul Ebran
Yann Beaujeault-Taudière
Antoine Roux



Gianluca Colò
Danilo Gambacurta



Technische Universität Darmstadt

Alexander Tichai



KU Leuven

Pepijn Demol