Shape coexistence and mixing behind the isomers of ^{94}Pd

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Outline

- *complex* **EXCITED VAMPIR beyond-mean-field variational model**

- **shape coexistence phenomena in N~Z nuclei near N=50**

  - isomeric states in $^{94}Pd_{48}$

  - Gamow-Teller β decay of the 7^+ ***isomer in*** $^{94}Ag_{47}$ ***feeding*** $^{94}Pd_{48}$

  - superallowed Fermi β decay of the 0^+ ***ground state of*** ^{94}Ag ***feeding*** ^{94}Pd

The heaviest proton-rich nuclei manifest exotic structure and dynamics *generated by*

- ***shape coexistence and shape mixing***
- ***competing T=0 and T=1 proton-neutron and like-nucleon pairing correlations***
- ***isospin symmetry breaking interactions***

responsible for

drastic changes in structure with number of nucleons, spin, and excitation energy

Challenges for theory

- ***realistic effective Hamiltonians in adequate model spaces***
- ***beyond-mean-field methods***

aiming to

- ***unitary description of the evolution in structure at low, intermediate, and high spins***
- ***comprehensive understanding of the structure phenomena and β-decay properties***

complex VAMPIR model family

• **the model space is defined by a finite dimensional set of spherical single particle states**

• **the effective many-body Hamiltonian is represented as a sum of one- and two-body terms**

• **the basic building blocks are Hartree-Fock-Bogoliubov (HFB) vacua**

• **the HFB transformations are essentially *complex* and allow for proton-neutron, parity, and angular momentum mixing being restricted by time-reversal and axial symmetry**

(include natural- and unnatural-parity two-body correlations and T=1 and T=0 neutron-proton pairing correlations already at the mean-field level)

• **the broken symmetries (s=N, Z, I, p) are restored by projection before variation**

**** The models allow to use rather large model spaces and realistic effective interactions***

Beyond-mean-field variational procedure: complex Excited VAMPIR model

VAMPIR approach

$$E^s[F_1^s] = \frac{\langle F_1^s|\hat{H}\hat{\Theta}^s_{00}|F_1^s\rangle}{\langle F_1^s|\hat{\Theta}^s_{00}|F_1^s\rangle}$$

$$|\psi(F_1^s); sM\rangle = \frac{\hat{\Theta}^s_{M0}|F_1^s\rangle}{\sqrt{\langle F_1^s|\hat{\Theta}^s_{00}|F_1^s\rangle}}$$

$\Theta^s{}_{00}$ - symmetry projector

$|F^s{}_1>$ - HFB vacuum

Excited VAMPIR

$$|\psi(F_i^s); sM\rangle = \Sigma_{j=1}^{i}|\phi(F_j^s)\rangle\, \alpha_j^i \qquad \text{for} \quad i = 1, ..., n-1$$

$$|\phi(F_i^s); sM\rangle = \hat{\Theta}^s_{M0}|F_i^s\rangle$$

Allows to identify in a small energy interval spherical, oblate, prolate deformed orthogonal configurations of a given symmetry.

$$|\psi(F_n^s); sM\rangle = \Sigma_{j=1}^{n-1}|\phi(F_j^s)\rangle\, \alpha_j^n + |\phi(F_n^s)\rangle\, \alpha_n^n$$

$$(H - E^{(n)}N)f^n = 0$$

$$(f^{(n)})^+ N f^{(n)} = 1$$

Projected configurations significantly correlated could become strongly mixed by the final diagonalization.

$$|\Psi_\alpha^{(n)}; sM > = \sum_{i=1}^{n} |\psi_i; sM > f_{i\alpha}^{(n)}, \qquad \alpha = 1, ..., n$$

^{40}Ca - core

model space for protons and neutrons

$1p_{1/2}$ $1p_{3/2}$ $0f_{5/2}$ $0f_{7/2}$ $1d_{5/2}$ $0g_{9/2}$

(charge-symmetric basis + Coulomb contributions to the π-spe from the core)

renormalized G-matrix (Bonn CD potential)

- ***pairing properties enhanced by short range Gaussians for:***

T = 1 : pp (*-35 MeV*), np (*-20 MeV*), nn (*-35 MeV*)

T = 0: np (*-35 MeV*)

- ***onset of deformation influenced by monopole shifts:***

$<0g_{9/2}\ 0f;\ T=0\ |G|\ 0g_{9/2}\ 0f;T=0>$ $(0f_{5/2},\ 0f_{7/2})$

$<1d_{5/2}\ 1p;\ T=0\ |G|\ 1d_{5/2}\ 1p;T=0>$ $(1p_{1/2},\ 1p_{3/2})$

- ***Coulomb interaction between valence protons added***

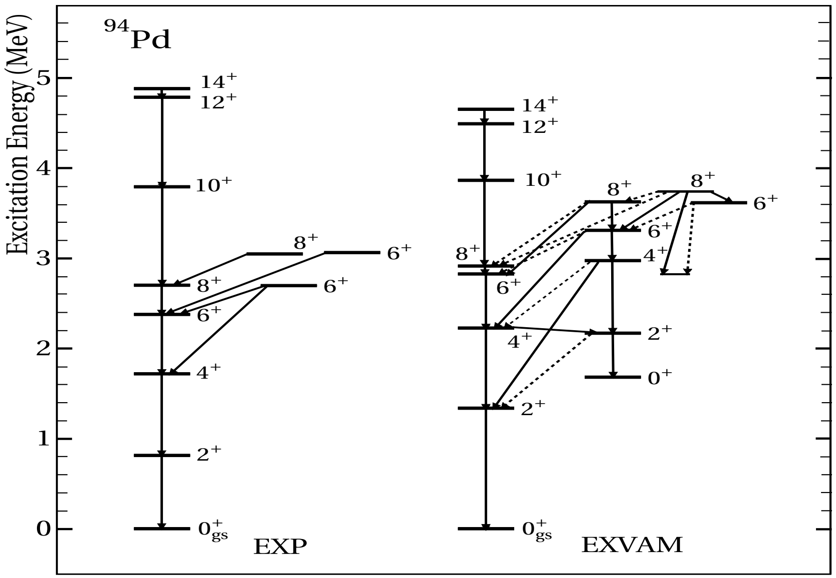
Shape coexistence phenomena in N~Z nuclei near N=50 : $^{94}Pd_{48}$ case

A. S. Mare and A. Petrovici, Phys. Rev. C 106, 054306 (2022)

Open questions:

- ***nature of the isomeric states at spin 8^+ and 14^+ (irregularities in the spectrum)***
- ***feeding by the Gamow-Teller β decay of the 7^+ isomer in $^{94}Ag_{47}$***
- ***feeding by the superallowed Fermi β decay of the 0^+ ground state of $^{94}Ag_{47}$***

Challenge: simultaneous description of all these phenomena within the same theoretical framework



Evolution of shape coexistence and mixing with increasing spin and excitation energy

Structure of the wave functions of positive parity states in ^{94}Pd

$I[\hbar]$	oblate-content	prolate-content
0_1^+	91(4)%	4%
0_2^+	86%	10%
2_1^+	23(3)%	73%
2_2^+	71%	24(3)%
4_1^+	2%	93(3)%
4_2^+	53(2)%	40(4)%
6_1^+	9(2)%	87%
6_2^+	67(16)(5)%	7(5)%
6_3^+	29(28)(20)(2)%	13(4)(3)%
8_1^+	77(7)%	9(5)%
8_2^+	66(15)(3)%	16%
8_3^+	23(3)%	72%
10_1^+	23(9)%	64%
12_1^+	59(22)%	14(2)%
14_1^+	95(2)%	

Spectroscopic quadrupole moments (efm^2) for positive parity states in ^{94}Pd

$I[\hbar]$		$I[\hbar]$	
2_1^+	-9.5	8_1^+	50.4
2_2^+	12.4	8_2^+	36.8
		8_3^+	-8.9
4_1^+	-37.5	10_1^+	15.3
4_2^+	1.5		
		12_1^+	8.3
6_1^+	-28.9		
6_2^+	36.3	14_1^+	45.6
6_3^+	19.1		

$0^+ \div 8^+$ $\beta_2 = 0.07 \div 0.14$ $\beta_2 = -0.07 \div -0.17$

$10^+ \div 14^+$ $\beta_2 = 0.03 \div 0.10$ $\beta_2 = -0.03 \div -0.14$

$B(E2;I \rightarrow I\text{-}2)$ values (e^2fm^4) for the lowest bands in ^{94}Pd

$I[\hbar]$	yrast band	first excited band
2^+	367	426
4^+	383(81)	213(104)[134]
6^+	336[105]	240(99)
8^+	165[60][111]	118(31)[51]
10^+	278[34]	
12^+	160[32]	
14^+	56	

$T^{EXP}_{1/2}(8^+ \rightarrow 6^+) = 1.2(3)$ ns

$B^{EXP}(E2;\ 8^+ \rightarrow 6^+) = 130(30)\ e^2fm^4$

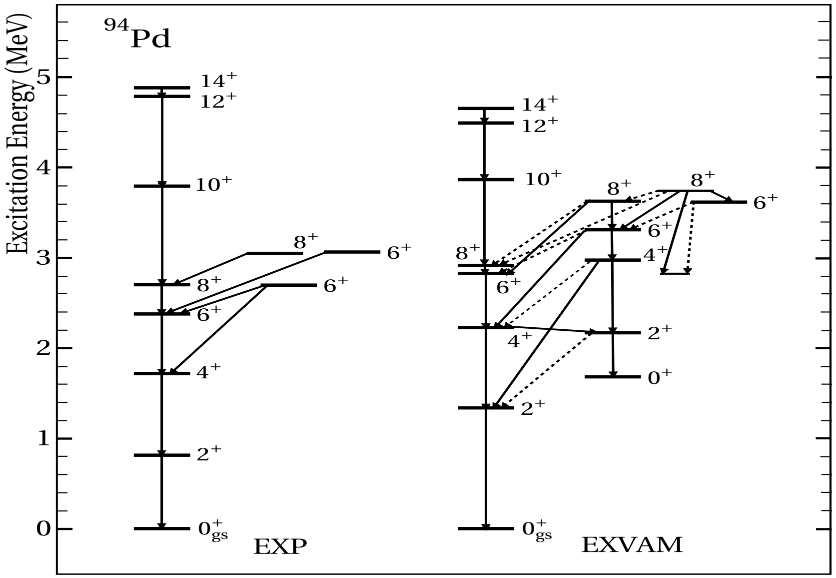
$B^{EXVAM}(E2;\ 8^+ \rightarrow 6^+) = 165\ e^2fm^4$

$T^{EXP}_{1/2}(14^+ \rightarrow 12^+) = 499(9)$ ns

$B^{EXP}(E2;\ 14^+ \rightarrow 12^+) = 53(1)\ e^2fm^4$

$B^{EXVAM}(E2;\ 14^+ \rightarrow 12^+) = 56\ e^2fm^4$

Decay pattern of the lowest three 6^+ and 8^+ states in ^{94}Pd



B(E2) ($e^2 fm^4$) and B(M1) (μ^2_B) values for the lowest 6^+ and 8^+ states in ^{94}Pd

$I[\hbar]$	B(E2)	B(M1)
$6_2^+ \rightarrow 6_1^+$	103	0.62
$6_3^+ \rightarrow 6_1^+$	99	0.26
$6_3^+ \rightarrow 6_2^+$	37	0.75
$8_1^+ \rightarrow 6_1^+$	165	
$8_2^+ \rightarrow 6_1^+$	31	
$8_2^+ \rightarrow 6_2^+$	118	
$8_3^+ \rightarrow 6_1^+$	159	
$8_3^+ \rightarrow 6_2^+$	44	
$8_3^+ \rightarrow 6_3^+$	67	
$8_2^+ \rightarrow 8_1^+$	61	3.07
$8_3^+ \rightarrow 8_1^+$	128	0.41
$8_3^+ \rightarrow 8_2^+$	82	3.97

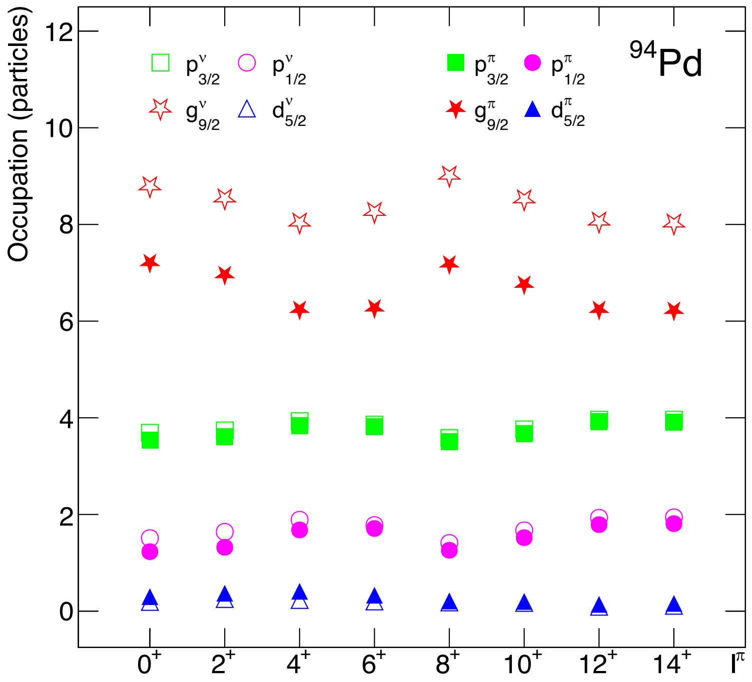
E0 transitions for the lowest 0^+ and 2^+ states: fingerprints of shape coexistence and mixing

ρ^2 values for 0^+ and 2^+ states in ^{94}Pd

Transition	EXVAM
$\rho^2(E0; 0_2^+ \to 0_1^+)$	0.019
$\rho^2(E0; 0_3^+ \to 0_1^+)$	0.007
$\rho^2(E0; 0_3^+ \to 0_2^+)$	0.005
$\rho^2(E0; 2_2^+ \to 2_1^+)$	0.022
$\rho^2(E0; 2_3^+ \to 2_1^+)$	0.011
$\rho^2(E0; 2_3^+ \to 2_2^+)$	0.008

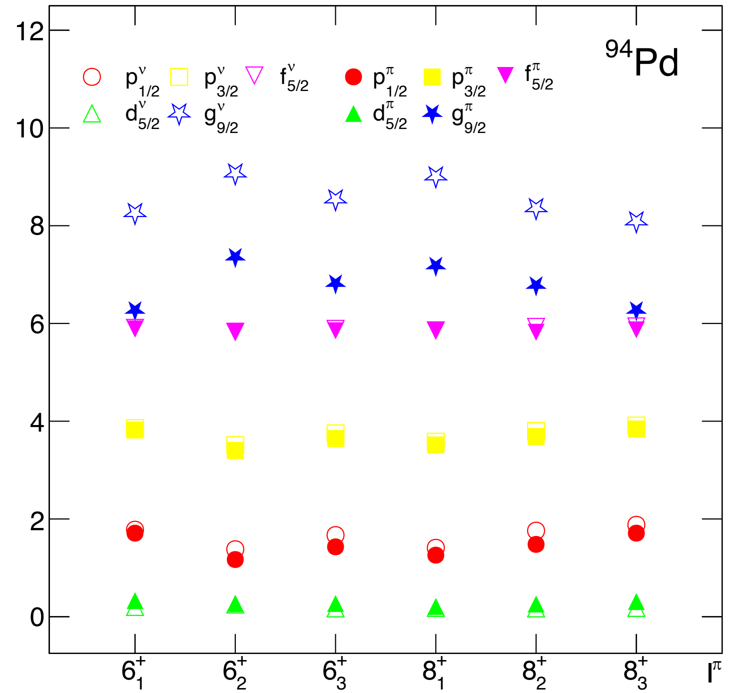
Evolution of the occupations of valence spherical orbitals for positive parity states in ^{94}Pd

Occupation of valence spherical orbitals for the yrast band



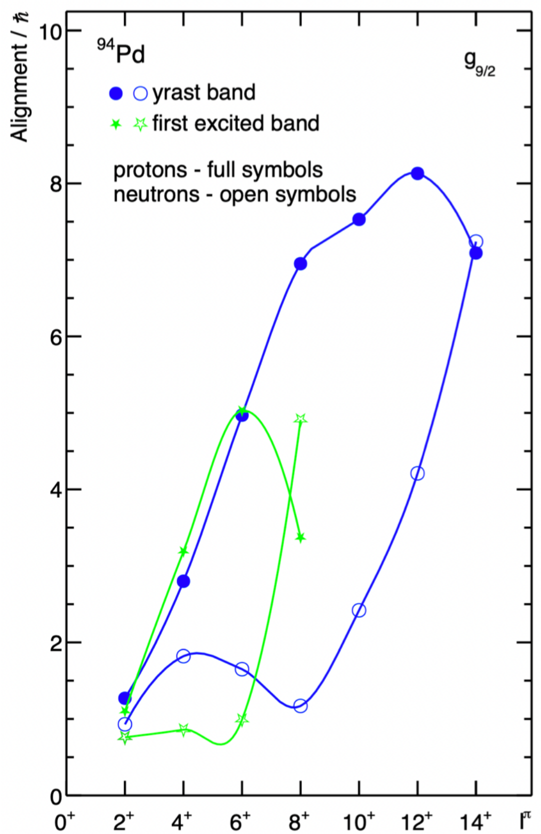
Particular changes in the $0g_{9/2}$, $1p_{1/2}$, and $1p_{3/2}$ occupations at the 8^+ isomer, but not for the 14^+ isomer

Occupation of valence spherical orbitals for 6^+ and 8^+ states

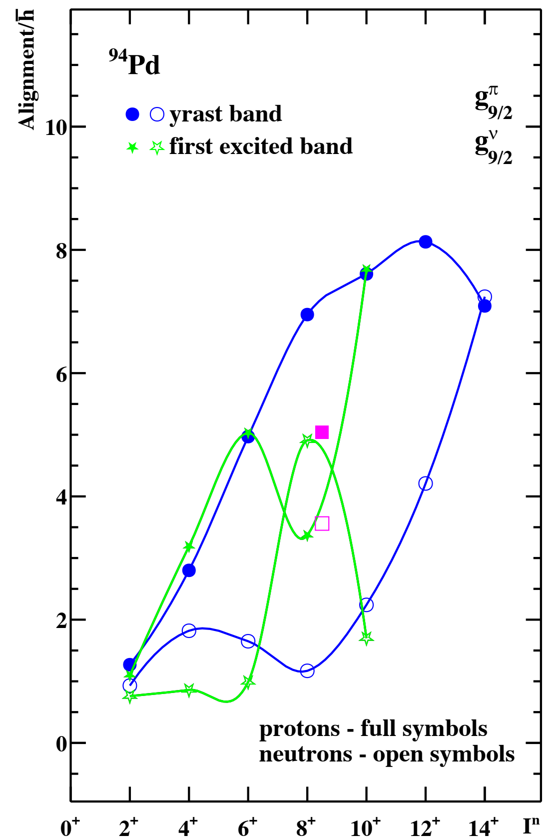


Significant changes in the $0g_{9/2}$, $1p_{1/2}$, and $1p_{3/2}$ occupations for the lowest three 6^+ as well as 8^+ states

Evolution of the alignment in the lowest bands in ^{94}Pd



The alignment plot for the lowest two bands



Alignment plot - extended

The evolution of shape coexistence and mixing *corroborated with changes in* ***$0g_{9/2}$ proton and neutron alignment*** *are responsible for the* ***isomeric nature of the yrast 8^+ and 14^+ states***

$g(8^+_1) = 1.22$ $g(8^+_2) = 0.37$ $g(8^+_3) = 0.70$

$g(10^+_1) = 1.03$

$g(12^+_1) = 0.85$

$g(14^+_1) = 0.53$

Weak interaction rates: self-consistent treatment

Fermi transition probabilities

$$B_{if}(F) = \frac{1}{2J_i + 1}\frac{g_V^2}{4\pi}|M_F|^2$$

$$\begin{aligned} M_F &\equiv (\xi_f J_f||\hat{1}||\xi_i J_i) \\ &= \delta_{J_i J_f}\sum_{ab} M_F(ab)(\xi_f J_f||[c_a^\dagger \tilde{c}_b]_0||\xi_i J_i) \end{aligned}$$

$$M_F(ab) = (a||\hat{1}||b)$$

Gamow-Teller transition probabilities

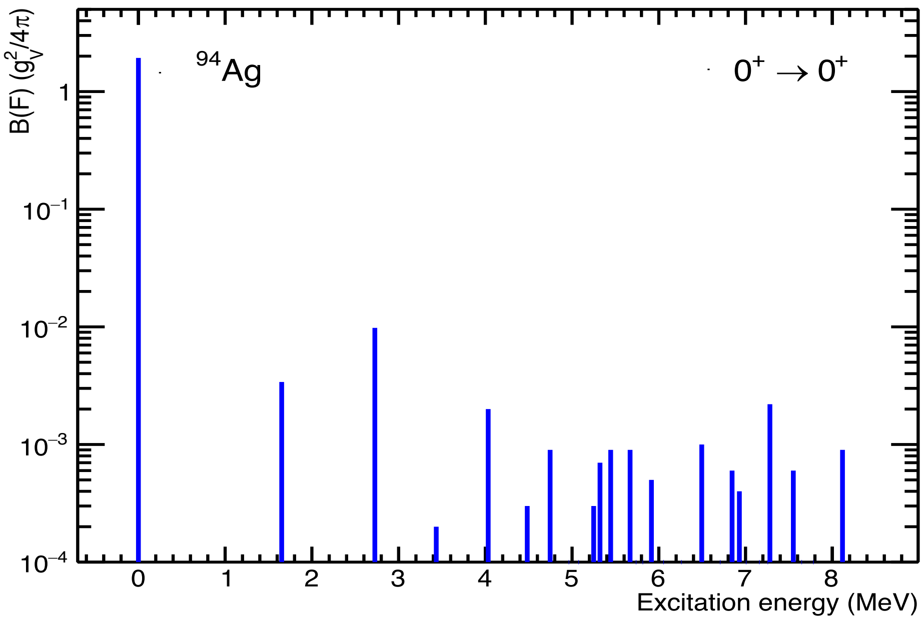
$$B_{if}(GT) = \frac{1}{2J_i + 1}\frac{g_A^2}{4\pi}|M_{GT}|^2$$

$$\begin{aligned} M_{GT} &\equiv (\xi_f J_f||\hat{\sigma}||\xi_i J_i) \\ &= \sum_{ab} M_{GT}(ab)(\xi_f J_f||[c_a^\dagger \tilde{c}_b]_1||\xi_i J_i) \end{aligned}$$

$$M_{GT}(ab) = 1/\sqrt{3}(a||\hat{\sigma}||b)$$

Independent chains of variational calculations for the parent and daughter states

Superallowed Fermi β decay of the ^{94}Ag to ^{94}Pd



^{94}Ag : 0^+_{gs} - 95% oblate content

Fermi strength distribution for the decay of the 0^+_{gs} in ^{94}Ag to 0^+ states in ^{94}Pd

1.32% depletion of the ground to ground decay

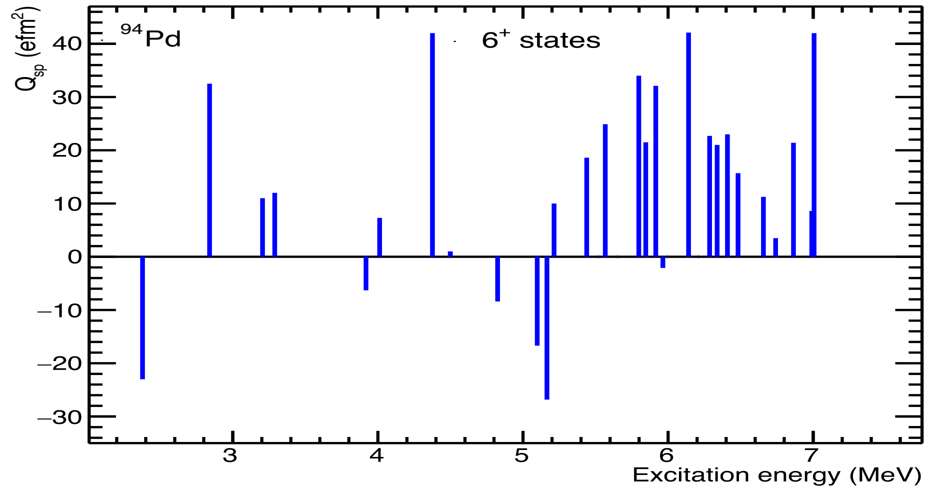
Interplay between shape mixing and isospin symmetry violation effects on superallowed Fermi β decay

Gamow-Teller β decay of the 7^+ isomer in ^{94}Ag to ^{94}Pd

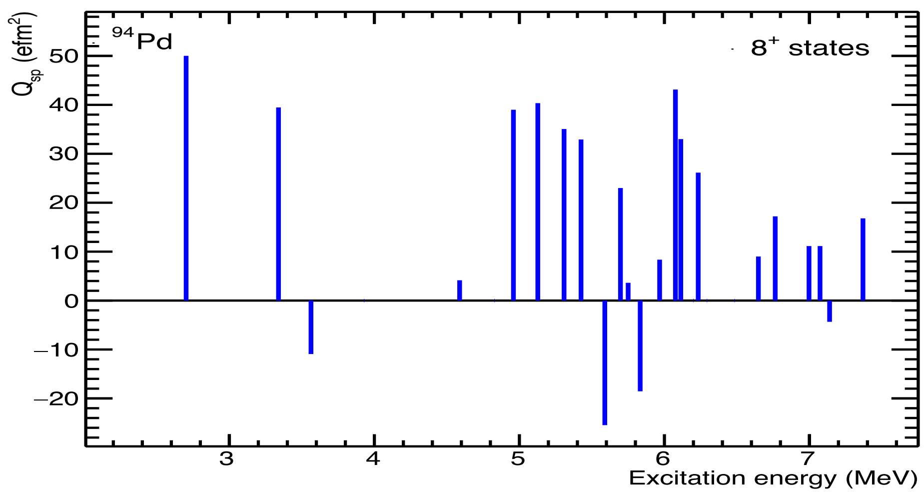
^{94}Ag : 7^+ isomer - 91% oblate content

$Q_{sp}(7^+) = 75.8$ efm^2

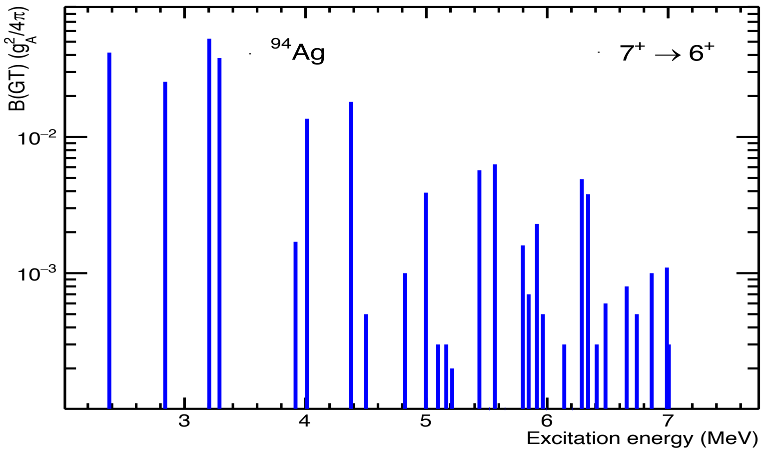
$g(7^+) = 0.54$



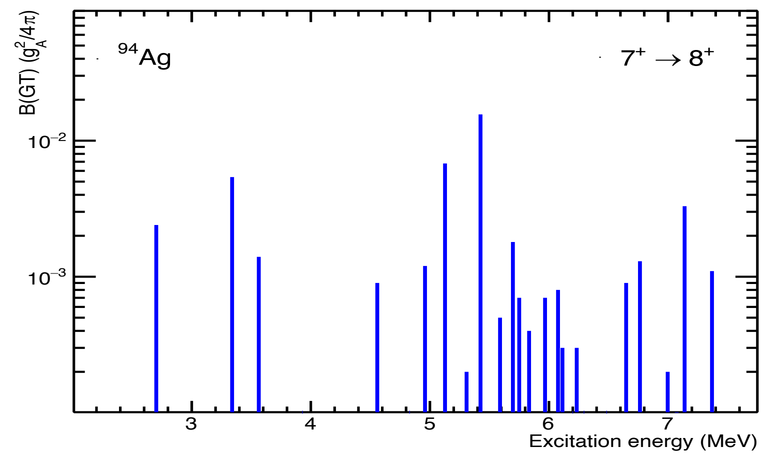
Spectroscopic quadrupole moments of 6^+ Gamow-Teller daughter states in ^{94}Pd



Spectroscopic quadrupole moments of 8^+ Gamow-Teller daughter states in ^{94}Pd



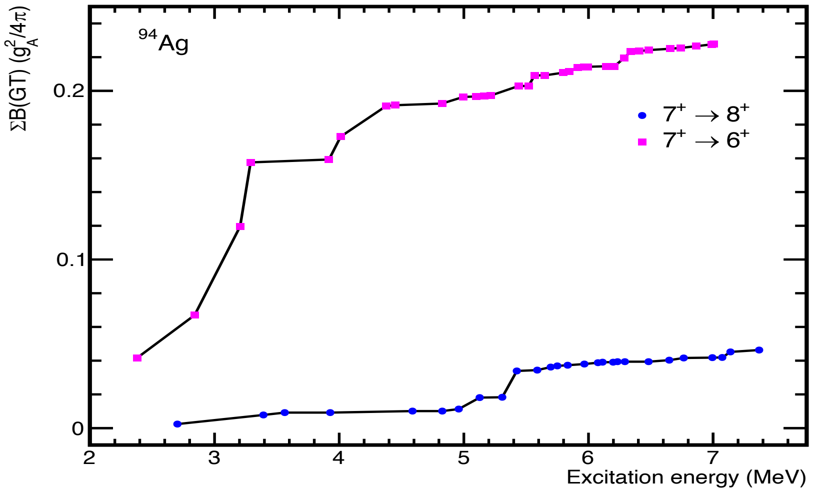
Gamow-Teller strength distribution for the decay of the 7^+ isomer in ^{94}Ag to 6^+ states in ^{94}Pd



Gamow-Teller strength distribution for the decay of the 7^+ isomer in ^{94}Ag to 8^+ states in ^{94}Pd

Contributions: - dominant: $g^{\nu}_{9/2}\, g^{\pi}_{9/2}$, small: $d^{\nu}_{5/2}\, d^{\pi}_{5/2}$ and $p^{\nu}_{1/2}\, p^{\pi}_{3/2}$ matrix elements (6^+states)

- dominant: $g^{\nu}_{9/2}\, g^{\pi}_{9/2}$, small: $p^{\nu}_{1/2}\, p^{\pi}_{3/2}$, $p^{\nu}_{3/2}\, p^{\pi}_{1/2}$, $p^{\nu}_{3/2}\, p^{\pi}_{3/2}$ matrix elements (8^+ states)



Gamow-Teller accumulated strength for the decay of the 7^+ isomer in ^{94}Ag to 6^+ and 8^+ states in ^{94}Pd

$$P_p = \frac{\Sigma_{S_p}^{Q_{EC}} f(Z, E_f) B(GT, E_f)}{\Sigma_{0}^{Q_{EC}} f(Z, E_f) B(GT, E_f)}$$

$Q_{EC} = 14.4\ MeV$ $\quad S_P = 4.378\ MeV\ (^{94}Pd)$

$P_p^{exp} = 20\%$ $\quad P_p^{EXVAM} = 27\%$

$$\frac{1}{T_{1/2}} = \frac{1}{K} \sum_{E_f} f(Z, E_f) B_{if}(GT)$$

$T^{exp}_{1/2}$ (7^+ isomer in ^{94}Ag) = 0.55(6) s

$T^{EXVAM}_{1/2}$ (7^+ isomer in ^{94}Ag) = 0.28 s

Summary

Comprehensive understanding of shape coexistence phenomena in proton-rich nuclei close to N=50

within the complex* Excited Vampir *beyond-mean-field variational model

- ***the evolution of shape coexistence and mixing in the structure of positive parity states in $^{94}Pd_{48}$***

- ***the nature of the isomeric states at spin 8^+ and 14^+***

- ***the complex decay pattern of the lowest three 6^+ and 8^+ states***

- ***the feeding of the intermediate spin states in ^{94}Pd by the Gamow-Teller β decay of the 7^+ isomer in ^{94}Ag***

- ***the feeding of the ground state as well as the nonanalog states in ^{94}Pd by the superallowed Fermi β decay of the 0^+ ground state in ^{94}Ag***