Nuclear structure, nuclear models and symmetries

P. Van Isacker, GANIL, France

Recent trends in nuclear structure theory.

Why do we (still) need nuclear models?

The role of symmetries

- Historical landmarks
- Examples of recent applications

CGS17, Grenoble, July 2023

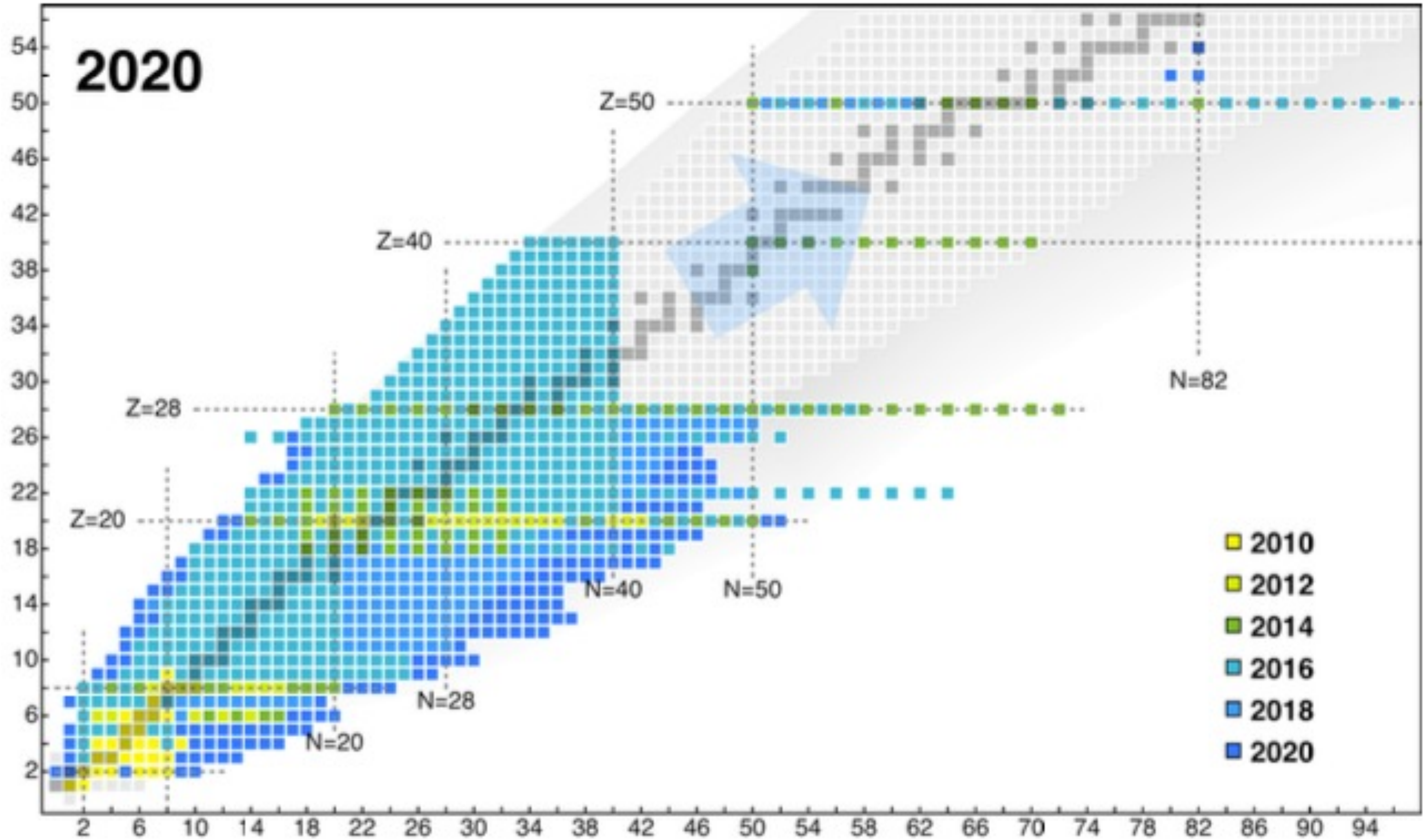
Ab initio nuclear structure

The *ab initio* approach aims to describe nuclei

- *from the interactions between the nucleons, ideally as derived from QCD (not possible as yet → EFT, effective field theory);*
- *in a comprehensive manner ("all" nuclei with their "complete" phenomenology);*
- *with an estimate of theoretical uncertainties.*

Question: Will new (unexpected) structural features emerge from this reductionist approach?

Ab initio progress



H. Hergert, Front. Phys. **8** (2020) 379

Chiral effective field theory (EFT)

χEFT is a low-energy realisation of QCD.

DOFs: nucleons+pions.

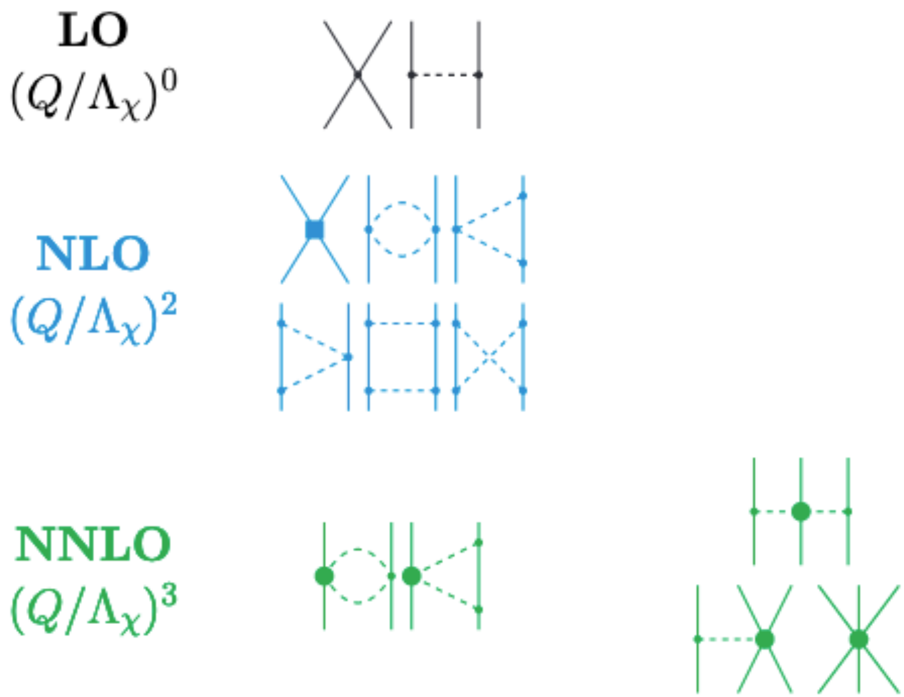
Interactions:

explicit pion-driven

others: contact $\rightarrow$

coupling constants

Power counting provides an organisational scheme.



S. Weinberg, Nucl. Phys. B **363** (1991) 3

E. Epelbaum *et al.*, Rev. Mod. Phys. **81** (2009) 1773

R. Machleidt, F. Sammarruca, Phys. Scripta **91** (2016) 083007

The nuclear many-body problem

With the Hamiltonian $H=T+H_2+H_3+\ldots$ from χEFT solve the A-body eigenvalue problem:

$$\hat{H}\left|\Psi_k^A\right\rangle = E_k^A\left|\Psi_k^A\right\rangle$$

Truncate infinite 1-body Hilbert (single-particle) space to one with dimension D.

Dimension of the A-body Hilbert space:

$$\begin{pmatrix} D \\ A \end{pmatrix} = \frac{D!}{A!(A-D)!}$$

Exact diagonalisation quickly becomes intractable $\rightarrow$ need for approximate methods.

Many-body approximations

Partitioning of H in H_0 (easy) and H_1 (rest).

The eigenvalue problem is easy for H_0:

$$\hat{H}_0 \left| \varphi_k^{(0)} \right\rangle = E_k^{(0)} \left| \varphi_k^{(0)} \right\rangle$$

An expansion series for the true eigenfunction:

$$\left| \Psi_k^A \right\rangle = \hat{\Omega}_k \left| \varphi_k^{(0)} \right\rangle$$

Two possible expansions of the wave operator:

$$\text{perturbative}: \quad \Omega_k = \sum_{r=0}^{\infty} \left(\frac{\hat{H}_1}{\hat{H}_0 - E_k^{(0)}} \right)^r \quad (\text{MBPT})$$

$$\text{non-perturbative}: \quad \Omega_k = \sum_{r=0}^{\infty} f_r\left(\hat{H}_1\right) \quad (\text{SRG,CC,GF})$$

A. Tichai *et al.*, Front. Phys. **8** (2020) 164

Symmetry breaking & restoration

Exact symmetries of the true Hamiltonian:

$$\left[\hat{H},\hat{A}\right]=\left[\hat{H},\hat{J}^2\right]=0$$

In closed-shell nuclei:

$$\left[\hat{H}_0,\hat{A}\right]=\left[\hat{H}_0,\hat{J}^2\right]=0$$

In open-shell nuclei:

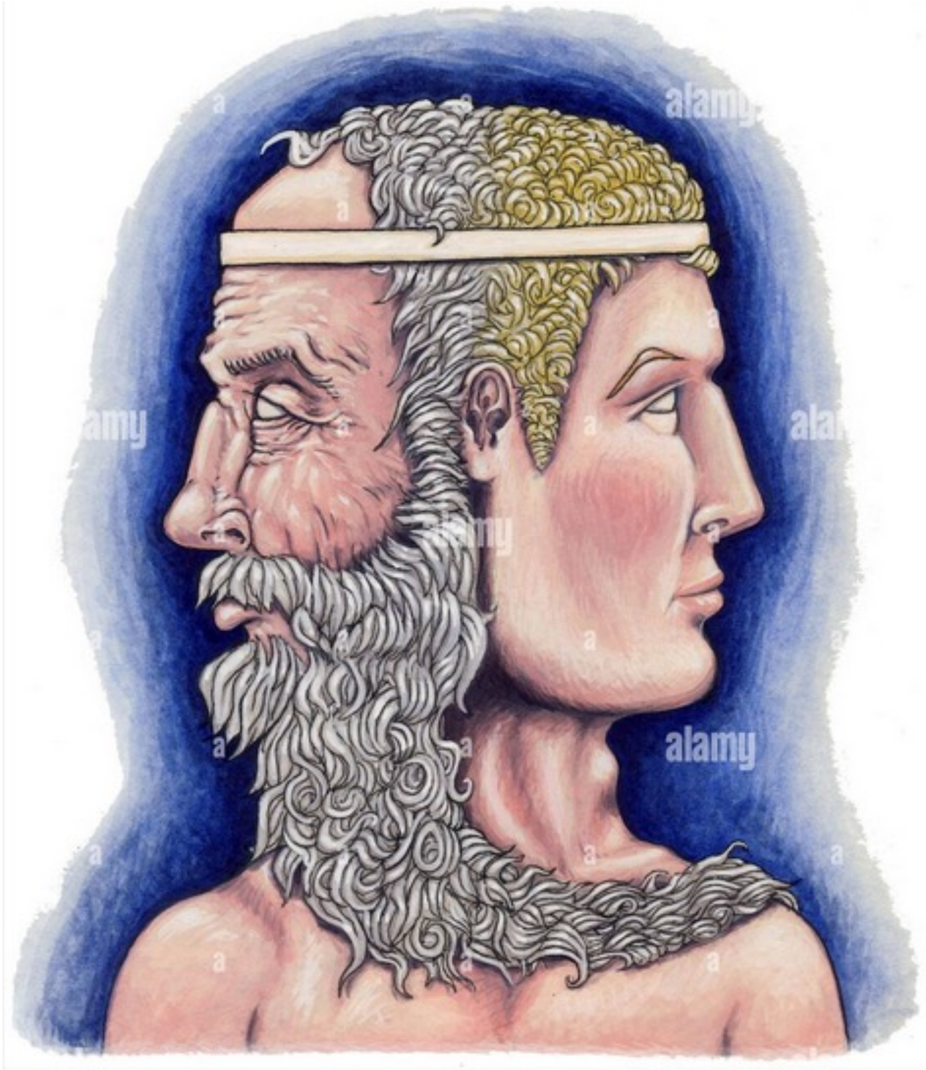
$$\left[\hat{H}_0,\hat{A}\right]\neq 0 \quad \text{and / or} \quad \left[\hat{H}_0,\hat{J}^2\right]\neq 0$$

$\rightarrow$ zoo of many-body methods [closed/open, U(1) and/or SU(2), perturbative (MBPT), non-perturbative (SRG, CC, CF,...)].

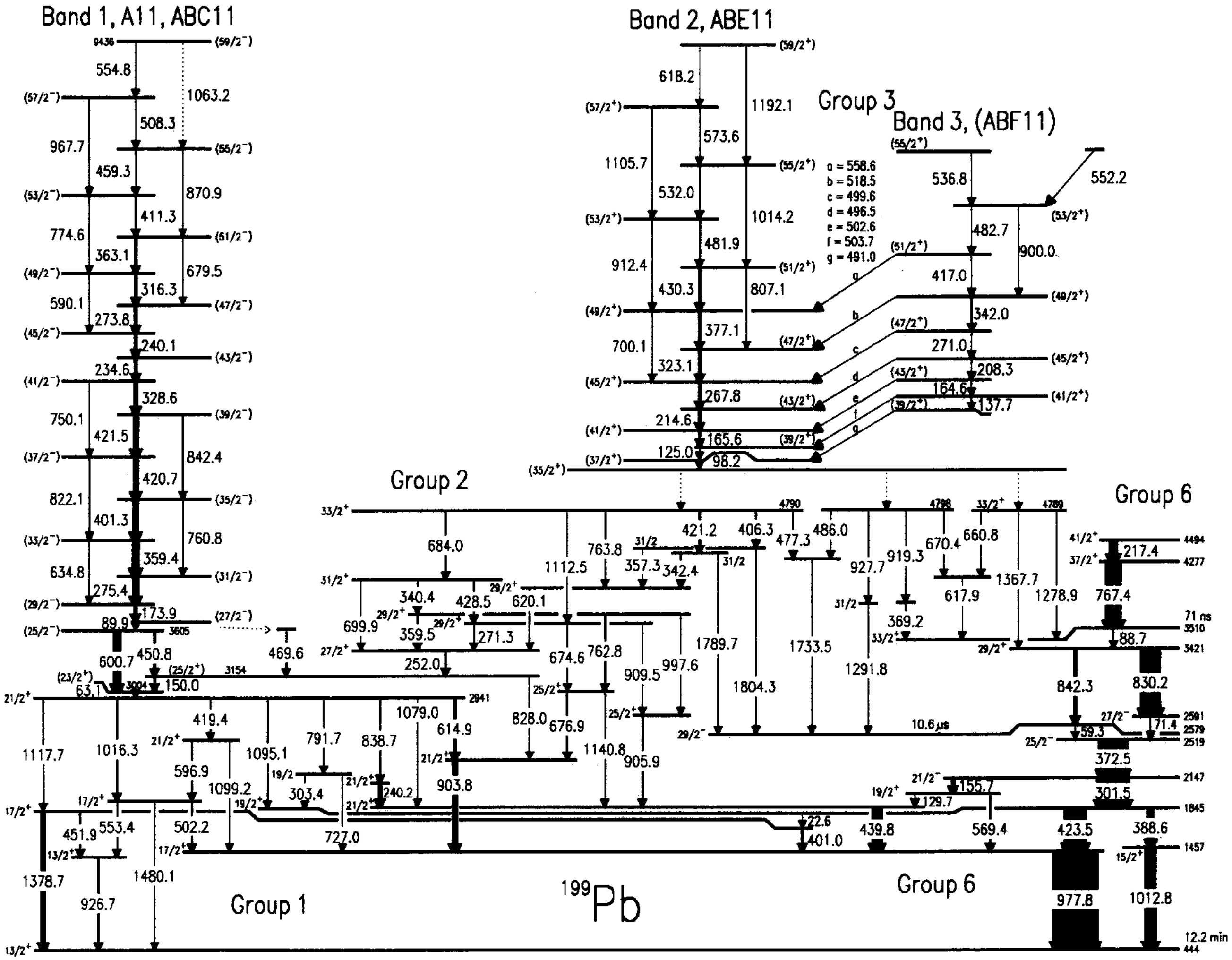
A. Tichai *et al.*, Front. Phys. **8** (2020) 164

The need for nuclear models

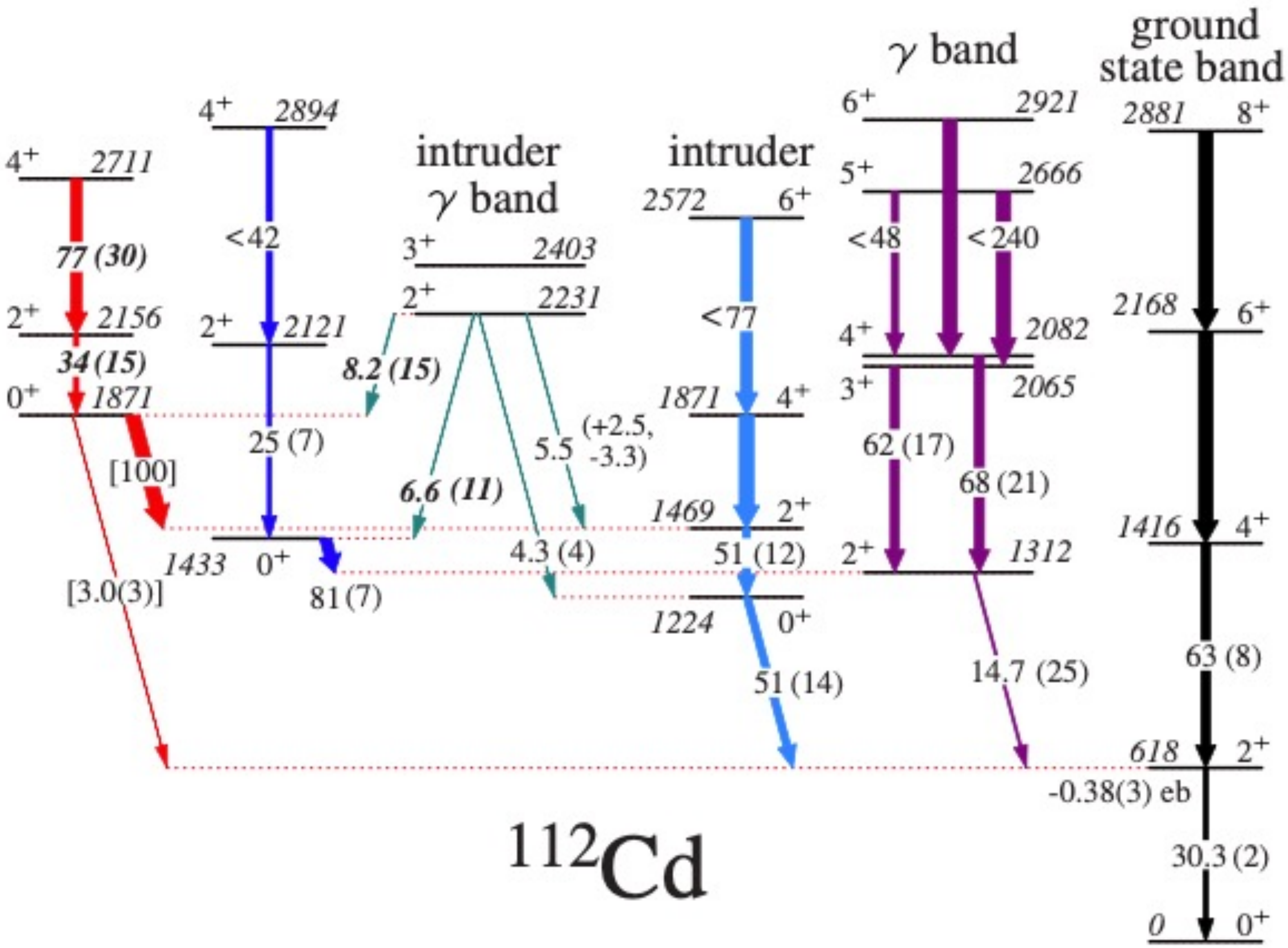
- *Ab initio* nuclear theory is invariably complicated.
- Experiments reveal a Janus-faced nucleus, the properties of which are often complex but sometimes simple.



A complex nucleus: ^{199}Pb

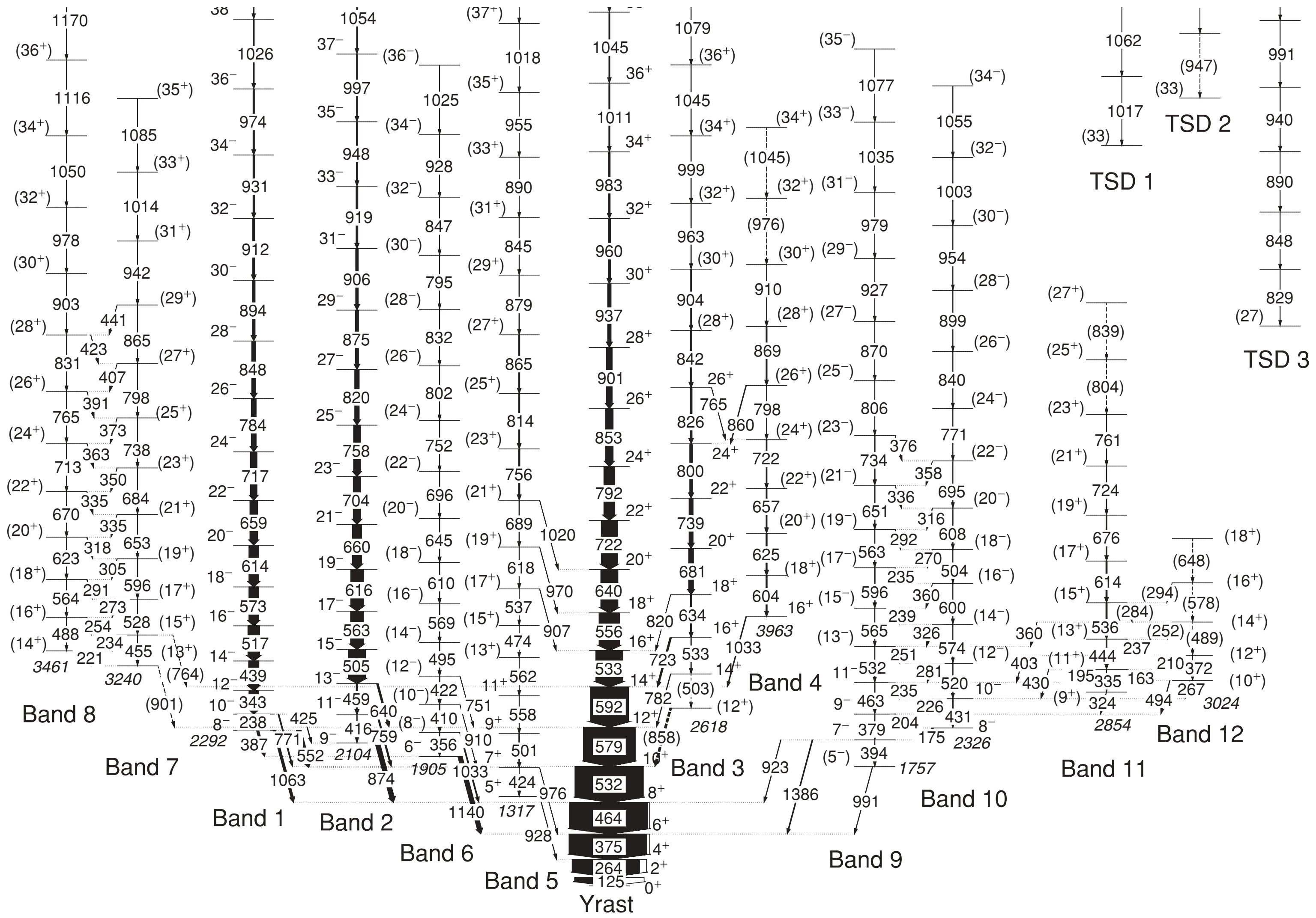


A complex nucleus: ^{112}Cd



P.E. Garrett *et al.*, Phys. Rev. Lett. **123** (2019) 142502

A complex nucleus: ^{160}Er



Random matrices, GOE…

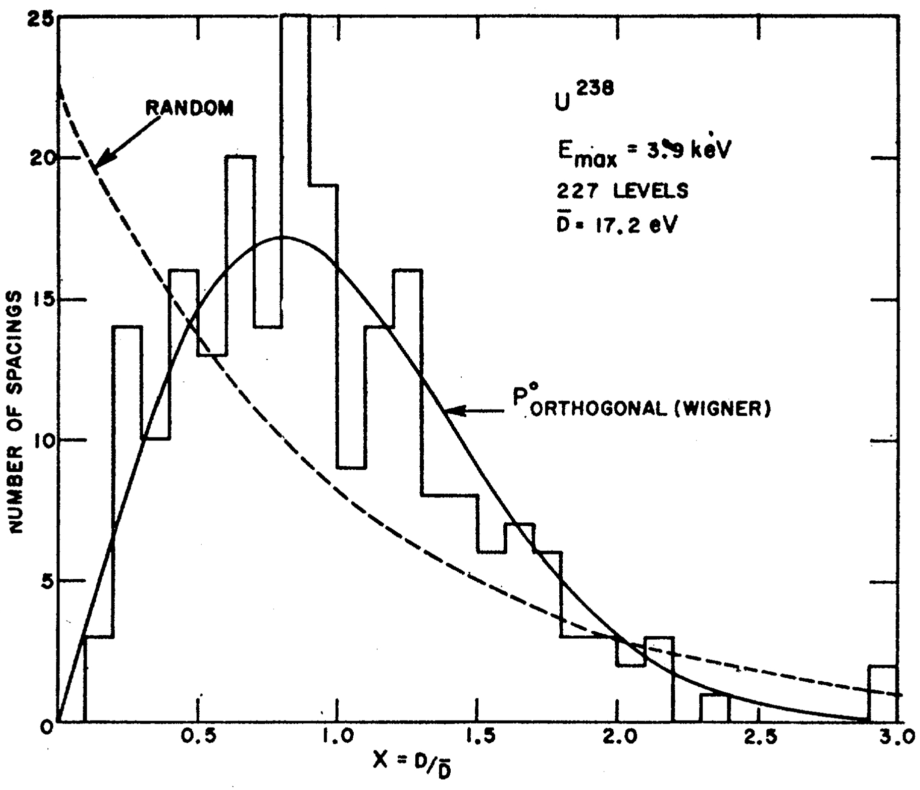


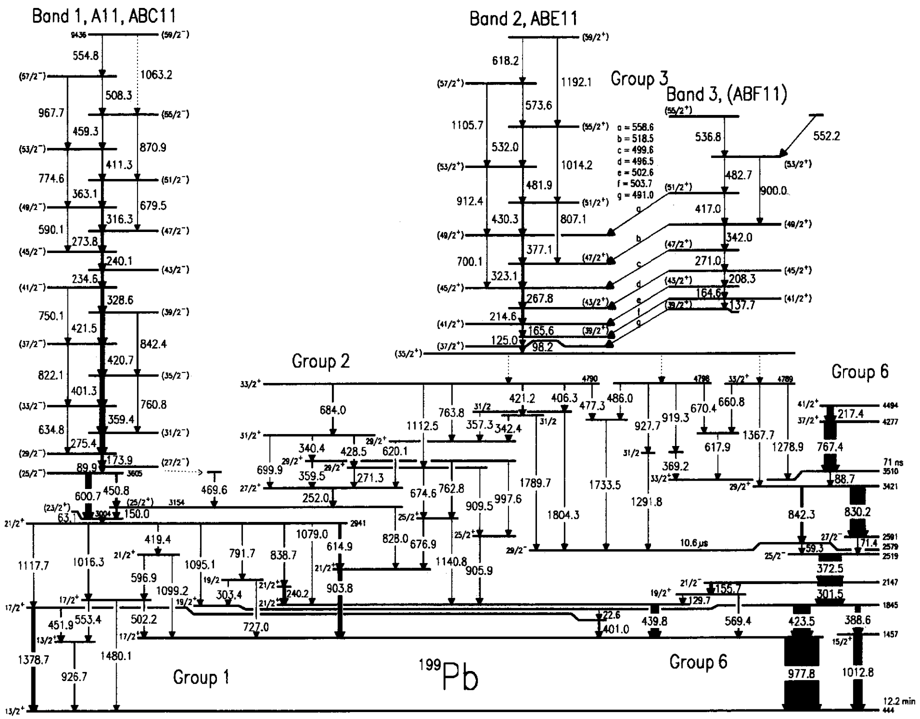
TABLE II. Neutron resonance level parameters of U^{238}.

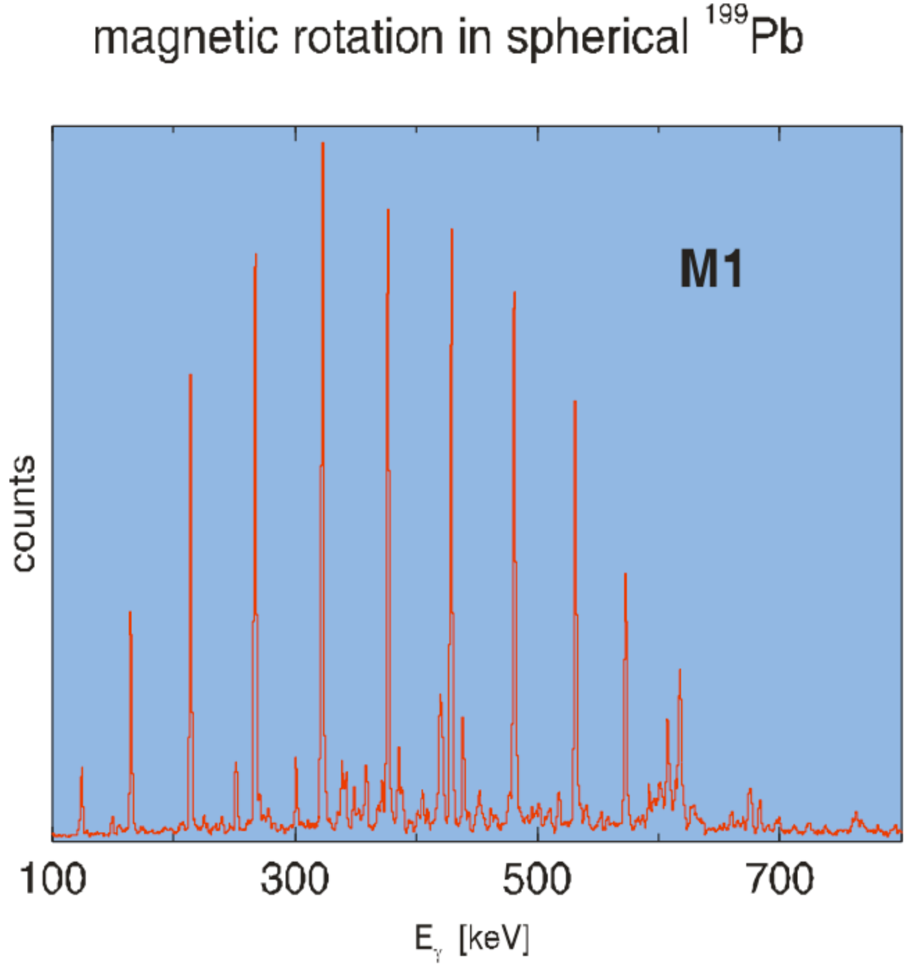
E_0 (eV)	ΔE_0	Γ_n^0 (meV)	$\Delta\Gamma_n^0$	E_0 (eV)	ΔE_0	Γ_n^0 (meV)	$\Delta\Gamma_n^0$	E_0 (eV)	ΔE_0	Γ_n^0 (meV)	$\Delta\Gamma_n^0$
6.68		0.59	0.004	1177.62	0.55	1.85	0.15	2620.6	1.85	0.80	0.40
*10.2[a]		0.0004	0.0002	1194.96	0.55	2.65	0.30	2631.6	1.85	0.02	0.02
21.00		1.90	0.065	1210.93	0.60	0.26	0.05	2672.8	1.90	3.40	1.00
36.70		5.14	0.15	1245.12	0.60	6.50	0.50	2695.6	1.90	0.45	0.10
66.30		3.09	0.12	1267.01	0.60	0.75	0.05	2716.8	1.95	1.36	0.30
80.77		0.23	0.02	1273.20	0.60	0.80	0.05	*2730.0	1.95	0.05	0.05
*90.00	0.15	0.008	0.001	1298.44	0.65	0.08	0.03	2750.1	2.00	0.75	0.25
102.78	0.20	6.50	0.20	1317.21	0.65	0.11	0.02	2761.9	2.00	0.30	0.05
116.93	0.25	3.33	0.14	1335.72	0.65	0.03	0.02	2787.9	2.00	0.20	0.08
145.80	0.25	0.07	0.027	1393.00	0.70	3.70	0.50	*2798.0	2.00	0.05	0.05
165.54	0.30	0.27	0.03	1405.11	0.70	2.05	0.20	3806.2	2.05	0.13	0.05
190.34	0.30	10.90	0.20	*1410.00	0.75	0.03	0.03	2828.6	2.05	0.17	0.05
208.65	0.35	3.90	0.40	*1417.00	0.75	0.03	0.02	*2845.2	2.10	0.05	0.05
237.40	0.10	1.80	0.10	1419.64	0.75	0.25	0.10	2866.1	2.10	1.48	0.10
*242.88	0.10	0.01	0.002	1427.73	0.75	0.80	0.10	2882.9	2.10	9.80	1.00
*263.94	0.10	0.014	0.002	1444.10	0.75	0.57	0.01	2897.8	2.15	0.50	0.25
273.74	0.10	1.52	0.10	1473.80	0.80	2.05	0.20	*2908.5	2.15	0.05	0.05
291.11	0.15	0.90	0.10	1523.10	0.80	5.50	0.50	2923.6	2.15	0.08	0.04
311.12	0.15	0.056	0.004	1532.00	0.80	0.05	0.02	2932.3	2.15	0.46	0.20
347.92	0.20	4.40	0.40	*1546.00	0.85	0.02	0.02	2956.3	2.20	0.28	0.10
376.92	0.20	0.058	0.004	1550.00	0.85	0.03	0.02	2967.4	2.20	0.15	0.05
397.56	0.20	0.30	0.05	1565.00	0.85	0.05	0.01	*2974.0	2.20	0.05	0.05
410.23	0.25	0.95	0.05	1598.16	0.85	8.00	0.50	2987.4	2.25	0.10	0.05
434.19	0.25	0.40	0.07	1622.89	0.90	2.10	0.30	3003.1	2.25	1.70	0.50
*454.17	0.25	0.02	0.005	1638.19	0.90	1.00	0.12	3015.0	2.25	0.13	0.05
463.31	0.30	0.24	0.02	*1645.40	0.90	0.02	0.02	3029.0	2.30	2.50	0.50
478.70	0.30	0.14	0.03	1662.08	0.95	4.00	0.50	3041.0	2.30	0.05	0.02
*488.89	0.30	0.02	0.005	1688.33	0.95	1.90	0.30	3060.2	2.30	0.50	0.10
518.59	0.30	1.90	0.10	*1700.71	0.95	0.02	0.02	3081.1	2.35	0.08	0.03
535.49	0.35	1.60	0.10	1709.40	0.95	1.35	0.15	3109.4	2.40	1.80	0.50
*556.05	0.35	0.02	0.01	1723.00	1.00	0.33	0.04	3133.2	2.40	0.10	0.05
580.20	0.40	1.12	0.03	1744.00	1.00	0.04	0.01	3149.0	2.40	1.10	0.20
595.15	0.20	3.35	0.20	1755.80	1.00	1.50	0.50	3169.0	2.45	0.18	0.02
619.94	0.20	1.14	0.04	1782.30	1.05	11.00	1.00	3179.4	2.45	1.10	0.40
*623.53	0.20	0.017	0.007	1797.70	1.05	0.05	0.02	3189.0	2.45	0.77	0.30
628.67	0.20	0.16	0.02	1808.26	1.05	0.40	0.10	3206.0	2.50	1.00	0.30
661.18	0.25	4.50	0.25	1845.60	1.10	0.31	0.05	3226.0	2.50	0.40	0.10
*677.00	0.25	0.02	0.01	1902.27	1.15	0.48	0.10	3249.2	2.55	0.20	0.05
693.23	0.25	1.30	0.05	1917.10	1.15	0.50	0.05	3280.0	2.55	1.80	0.20
708.46	0.25	0.70	0.10	1968.66	1.20	13.00	1.00	3295.0	2.60	0.15	0.05
721.80	0.25	0.05	0.01	1974.65	1.20	10.50	1.00	3310.9	2.60	1.65	0.20
*730.10	0.25	0.03	0.01	2023.58	1.25	4.50	0.50	3321.3	2.60	1.42	0.20
732.26	0.30	0.05	0.005	2031.06	1.25	1.10	0.10	3334.0	2.65	1.00	0.15
*742.95	0.30	0.02	0.005	2088.63	1.30	0.30	0.05	3355.7	2.65	1.30	0.20
765.05	0.30	0.24	0.04	2096.49	1.30	0.22	0.05	3371.0	2.65	0.05	0.02
779.14	0.30	0.06	0.005	2124.35	1.35	0.10	0.05	3387.8	2.70	0.14	0.04
790.88	0.30	0.18	0.02	2145.95	1.35	0.75	0.10	3409.0	2.70	1.80	0.50
821.58	0.35	2.05	0.10	2152.77	1.35	3.80	0.40	*3419.0	2.75	0.05	0.05
*846.62	0.35	0.02	0.005	2172.00	1.40	0.05	0.03	3436.9	2.75	3.25	0.50
851.02	0.35	1.90	0.10	2185.99	1.40	7.80	0.80	3459.1	2.80	6.50	1.00
856.15	0.35	2.75	0.15	*2194.00	1.40	0.05	0.05	*3470.0	2.80	0.02	0.02
866.52	0.35	0.14	0.02	2201.42	1.40	2.40	0.40	3484.3	2.80	2.00	1.00
*891.29	0.35	0.03	0.01	2229.96	1.45	0.10	0.03	3492.0	2.80	0.19	0.10
905.11	0.35	1.50	0.05	2235.73	1.45	0.10	0.05	3512.0	2.85	0.05	0.02
909.90	0.38	0.03	0.01	*2241.53	1.45	0.03	0.03	3526.0	2.85	0.18	0.10
925.18	0.40	0.28	0.02	2259.06	1.45	1.38	0.15	3561.5	2.90	2.40	0.80
*932.50	0.40	0.01	0.01	2266.43	1.50	3.05	0.20	3574.0	2.90	4.00	1.00
936.87	0.40	4.80	0.50	2281.27	1.50	2.30	0.10	3593.0	2.95	0.26	0.05
958.43	0.40	5.10	0.50	2288.70	1.50	0.05	0.02	*3600.0	2.95	0.05	0.05
991.78	0.45	11.00	0.50	*2302.0	1.50	0.02	0.02	3611.0	2.95	0.05	0.02
1000.30	0.45	0.04	0.04	2315.9	1.50	0.30	0.10	3625.0	3.00	0.05	0.02
1011.25	0.45	0.06	0.02	2337.4	1.55	0.10	0.05	3630.0	3.00	3.60	0.50
1023.00	0.45	0.20	0.04	2352.0	1.55	1.30	0.50	*3647.0	3.00	0.05	0.05
1029.08	0.45	0.10	0.03	2356.0	1.55	1.30	0.50	*3674.0	3.05	0.05	0.05
*1033.16	0.45	0.02	0.02	2392.5	1.60	0.23	0.10	3693.0	3.05	4.00	1.00
1053.93	0.45	2.30	0.50	2410.2	1.60	0.09	0.03	3717.7	3.10	1.00	0.25
1068.10	0.45	0.02	0.02	2426.5	1.65	1.65	0.30	3733.3	3.10	2.50	1.00
*1070.50	0.50	0.01	0.01	2446.2	1.65	2.25	0.25	3764.7	3.15	0.56	0.10
*1081.10	0.50	0.02	0.01	2454.0	1.65	0.05	0.03	3783.7	3.20	4.50	1.00
*1094.80	0.50	0.02	0.01	2489.8	1.70	1.10	0.10	*3799.7	3.20	0.05	0.05
1098.35	0.50	0.45	0.10	2520.7	1.75	0.20	0.10	3832.0	3.25	0.10	0.05
*1102.34	0.50	0.02	0.01	2548.7	1.75	6.80	0.80	3858.1	3.30	5.50	1.00
1108.88	0.50	0.90	0.05	2559.3	1.75	4.30	0.50	3871.3	3.30	4.00	1.50
1131.45	0.50	0.06	0.02	2580.7	1.80	4.80	0.50	3895.0	3.30	0.08	0.05
1140.38	0.55	6.50	0.05	2598.7	1.80	11.00	2.00	3904.4	3.35	3.60	0.06
1167.46	0.55	2.35	0.15	*2604.0	1.80	0.05	0.05				

* Levels which are either p-wave or uncertain. E_0 and ΔE_0 in eV. Γ_n^0 and $\Delta\Gamma_n^0$ in meV. Γ_n^0 value up to $E_n = 208$ eV are previously published.

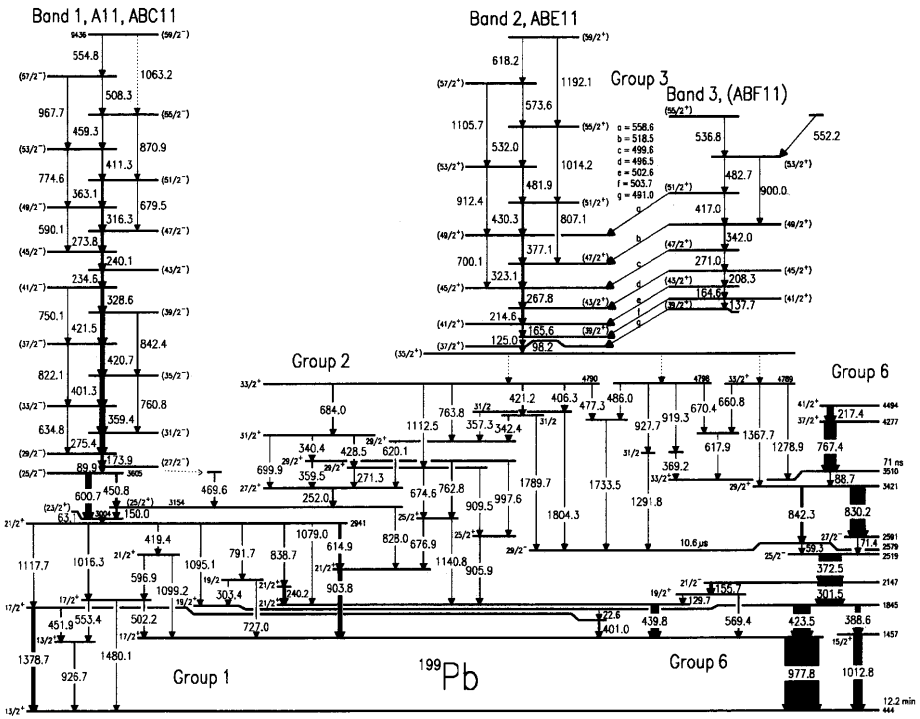
E.P. Wigner, Ann. Phys. **53** (1951) 26

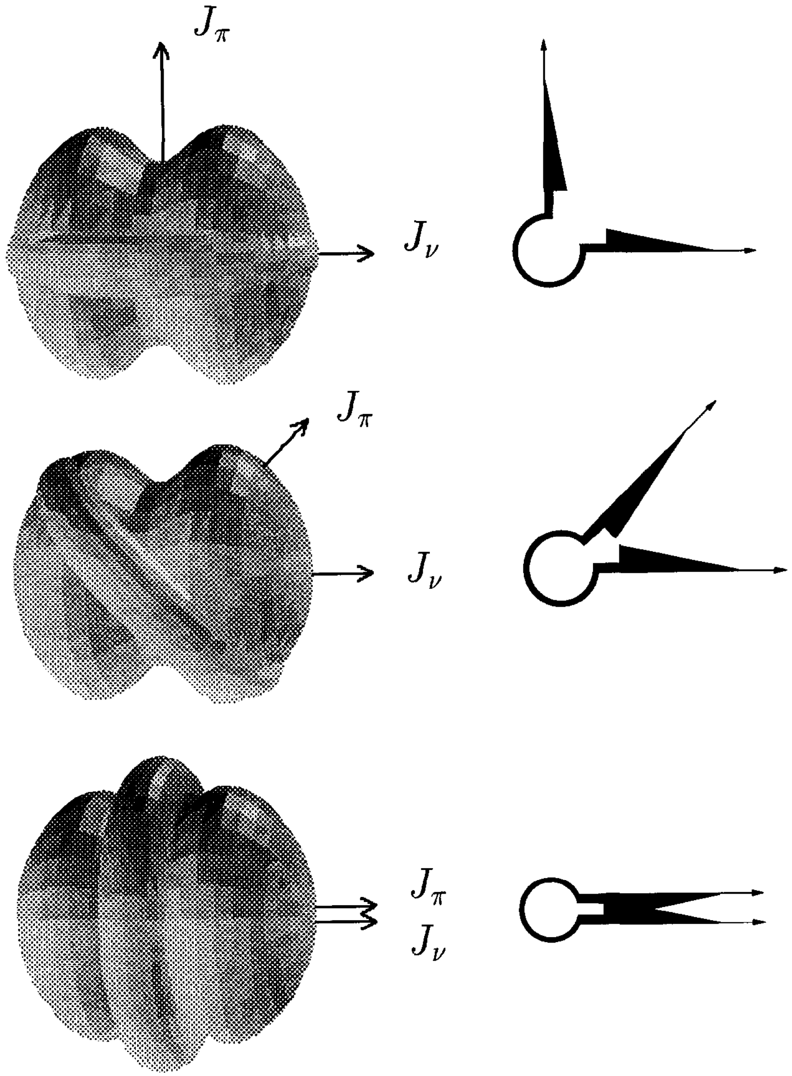
Emergence of new concepts





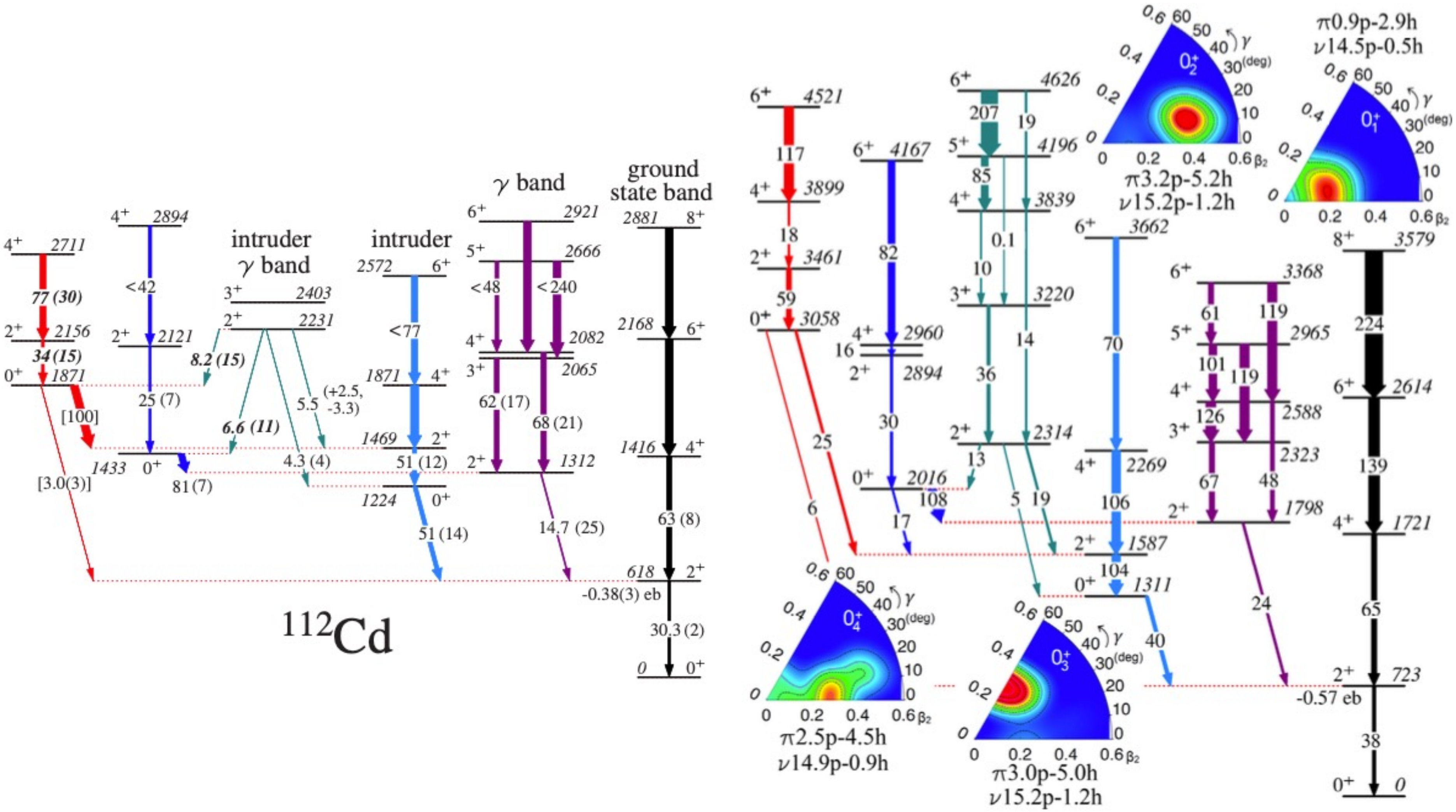
The shears mechanism





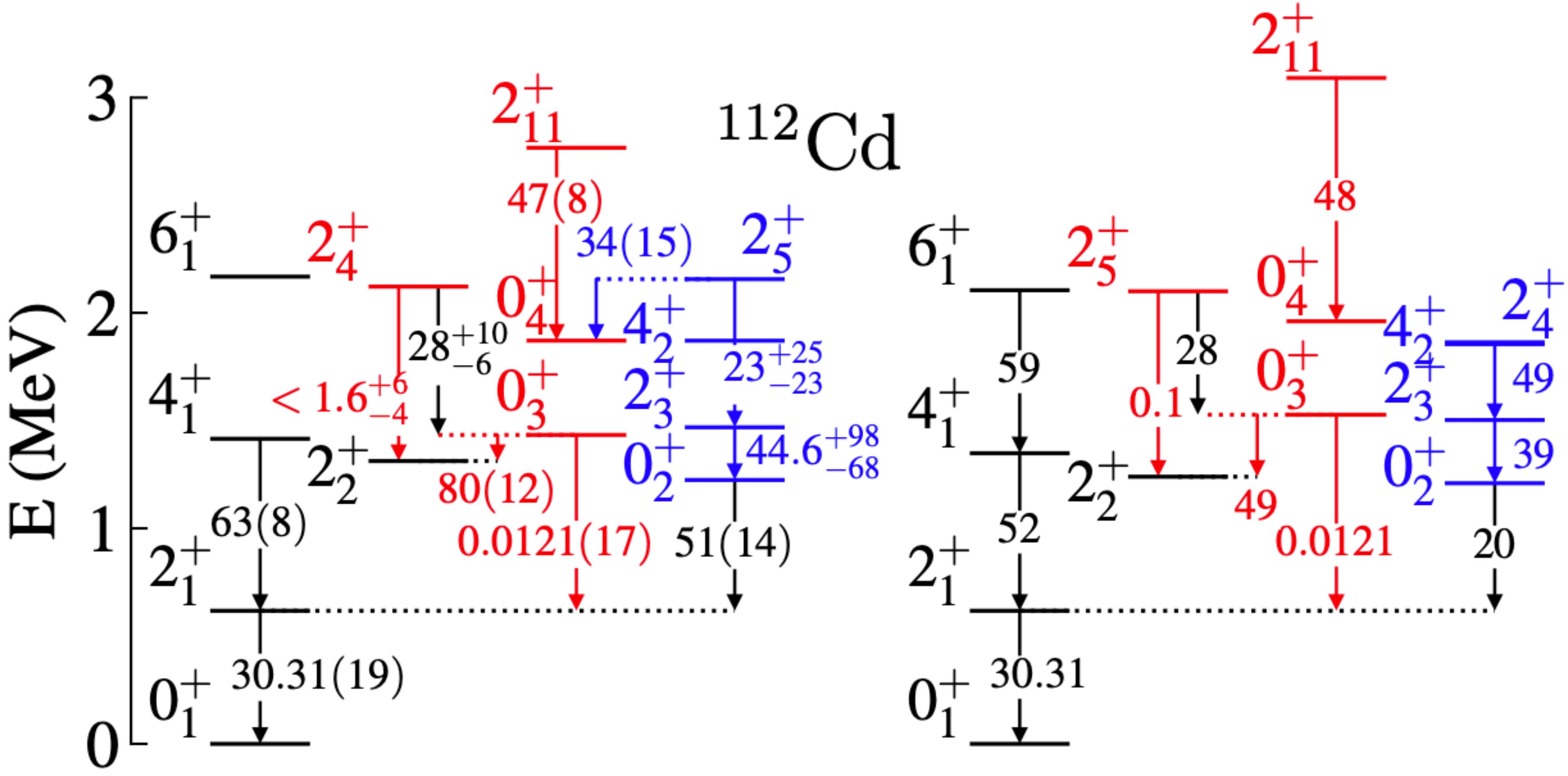
S. Frauendorf, Nucl. Phys. **557** (1993) 259c

Do we need parametrised models?



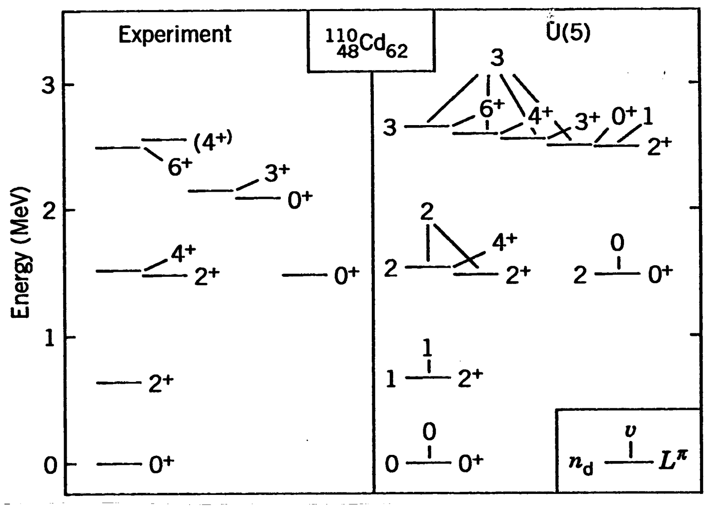
P.E. Garrett *et al.*, Phys. Rev. Lett. **123** (2019) 142502

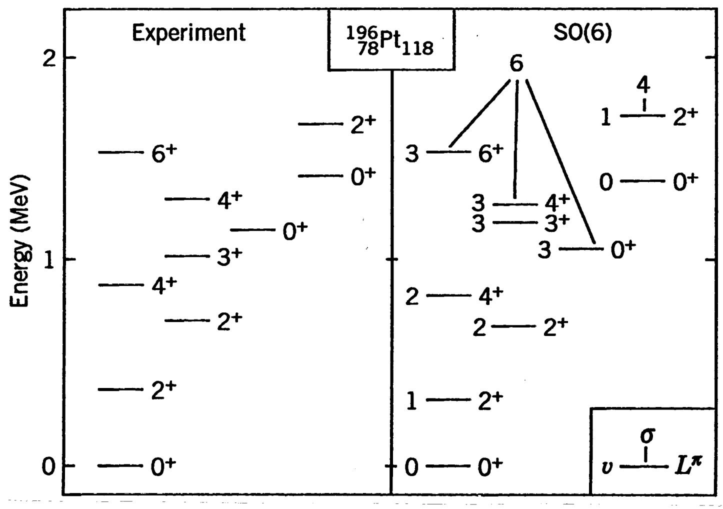
Do we need parametrised models?

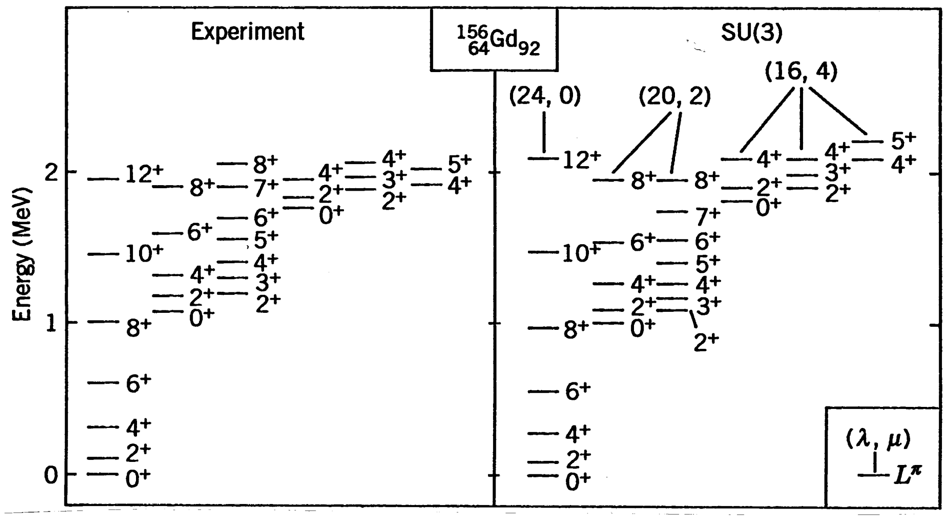


N. Gavrielov *et al.*, to be published

Simplicity through symmetry







F. Iachello & A. Arima, *The Interacting Boson Model*

Symmetries of nuclear models

Heisenberg (1932): isospin SU(2)

Wigner (1937): spin-isospin SU(4)

Racah (1943): seniority SU(2)

Elliott (1958): rotation SU(3)

Arima & Iachello (1976): IBM U(6)

Theory of complex spectra

In the 1940s Racah published a series of seminal papers on the application of group theory to atomic spectra. The third of the series (primarily concerned with coefficients of fractional parentage) contains the first mention of seniority.

PHYSICAL REVIEW VOLUME 63, NUMBERS 9 AND 10 MAY 1 AND 15, 1943

Theory of Complex Spectra. III

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The Hebrew University, Jerusalem, Palestine

(Received February 8, 1943)

The consideration of the phases of the fractional-parentage coefficients allows the extension of the matrix methods to configurations with more than two equivalent electrons. Tables are given for the parentages of the terms of p^n and d^n. Applications are made to the spin-orbit interaction of the d^n terms and to the electrostatic interaction between the configurations d^n, $d^{n-1}s$, and $d^{n-2}s^2$. Errata in Part II are indicated.

Racah's "seniority number"

In this section we shall classify the terms of the configuration l^n according to the eigenvalues of

$$Q=\sum_{i<j} q_{ij}, \qquad (34)$$

where q_{ij} is a scalar operator which operates on the two equivalent electrons i and j and is defined by the relation

$$(l^2LM|q_{ij}|l^2LM)=(2l+1)\delta(L,0). \qquad (35)$$

It will be shown that to every term of l^n with non-vanishing Q a term of the same kind corresponds in l^{n-2}, and this fact will allow us to assign to each term a "seniority number" according to the value of n for which the term appeared for the first time. Some useful relation between the fractional parentages of corresponding terms will be obtained and it will also be shown that the classification of the terms of l^{2l+1} according to the two possibilities of (76)II depends only on the seniority of the term.

We may thus assign to each term in the QSL scheme a "seniority number" v, which indicates the number of electrons of the first member of its chain; it follows immediately from (45) that Q depends only on n and v and that its values are given by

$$Q(n,v)=\tfrac{1}{4}(n-v)(4l+4-n-v). \qquad (50)$$

Confronting (41) and (50) we see that conjugate terms have the same seniority.

The seniority number suffices for distinguishing the different terms of the same kind in the configurations d^n but not in f^n, since there are in f^n terms of the same kind which have also the same seniority. For such configurations an unspecified parameter α must be maintained besides v; terms corresponding according to (49) will have the same values of v and of α.

Conservation of seniority

Seniority υ is the number of particles not in pairs coupled to J=0 (Racah).

Conditions for the conservation of seniority by a an interaction can be derived in general

Any two-body interaction between identical fermions with spin j conserves seniority if j≤7/2.

A. de-Shalit & I. Talmi, *Nuclear Shell Theory*
I. Talmi, *Simple Models of Complex Nuclei*

Is seniority conserved in nuclei?

The interaction between nucleons is “short range”.

A δ interaction is therefore a reasonable approximation to the nucleon two-body force.

The δ interaction between identical nucleons conserves seniority.

$\therefore$ In semi-magic nuclei seniority is conserved to a good approximation.

Seniority and ph conjugation

The particle-hole conjugation operator Γ transforms a problem of *n* fermions in a *j* shell into one with 2*j*+1−*n* fermions.

A representation of the ph transformation

$$\hat{\Gamma} = \exp\left[\tfrac{1}{2}\pi\left(\hat{S}_+ - \hat{S}_-\right)\right]$$

where $S_\pm$ are the quasi-spin operators

$$\hat{S}_+ = \tfrac{1}{2}\sqrt{2j+1}\left(a_j^+ a_j^+\right)_0^{(0)}, \quad \hat{S}_- = \left(\hat{S}_+\right)^+$$

A. Müller-Arnke, Nucl. Phys. A **215** (1973) 205

A geometric phase

The action of ph conjugation on a seniority state:

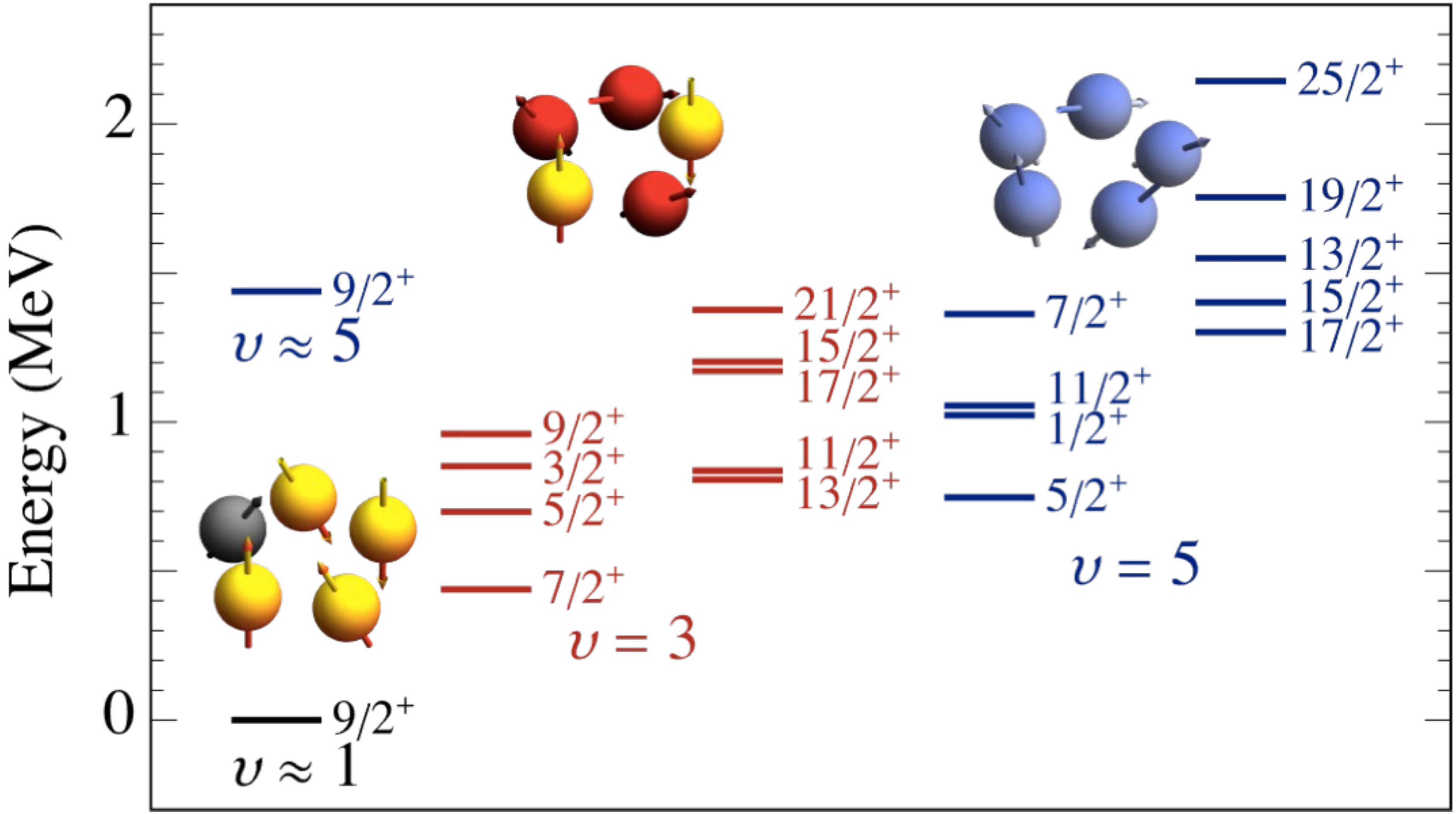
$$\hat{\Gamma}\left|j^n \upsilon J\right\rangle = (-)^{(n-\upsilon)/2}\left|j^{2j+1-n} \upsilon J\right\rangle$$

The sign is without any consequence *except* if the left and right states are the same, that is for a half-filled shell, $n=2j+1-n$.

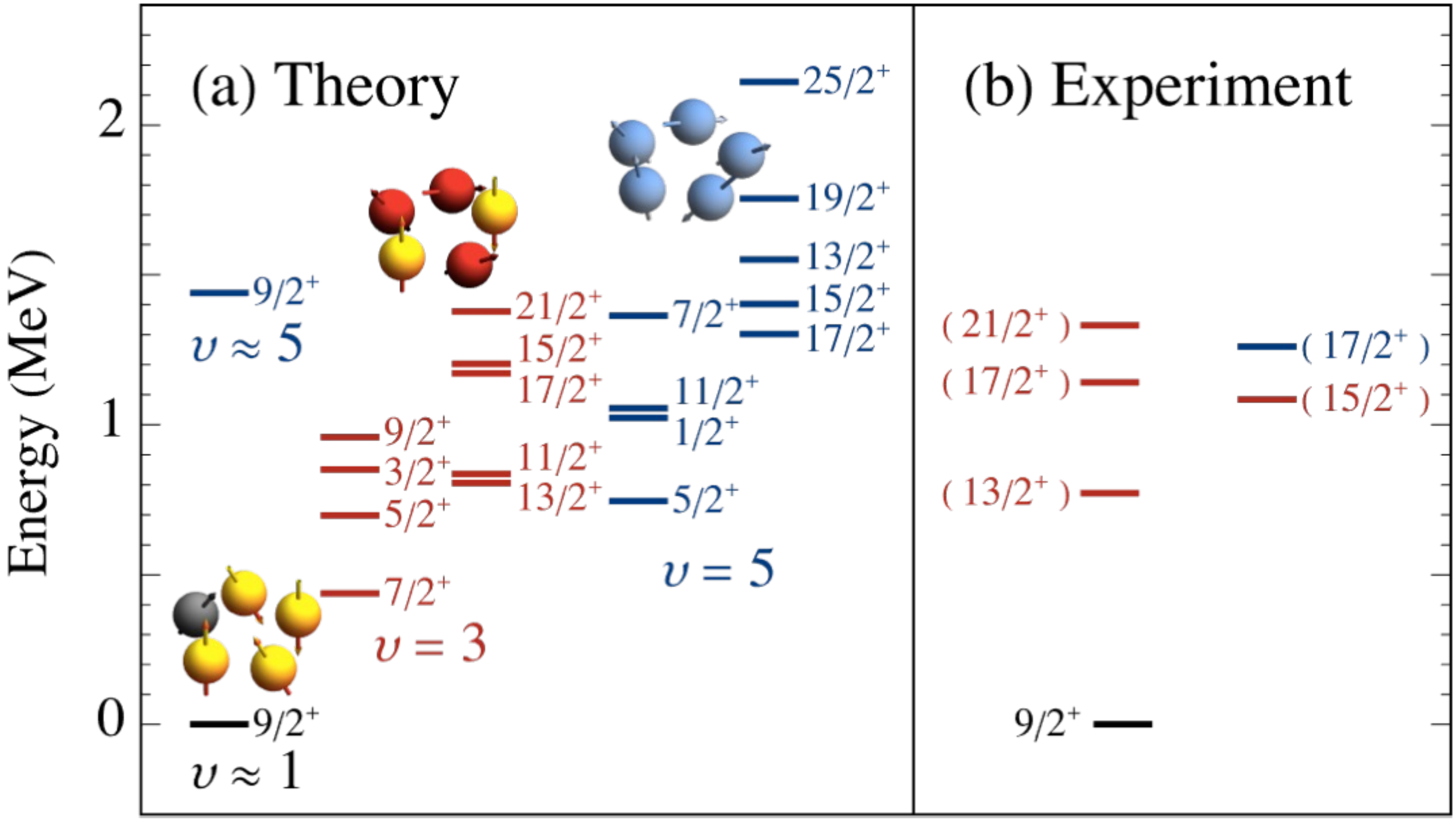
The observable consequence of this phase is that $\Delta\upsilon=\pm 2$ seniority mixing is forbidden.

R.D. Lawson, *Theory of the Nuclear Shell Model*

Five nucleons in a j=9/2 shell

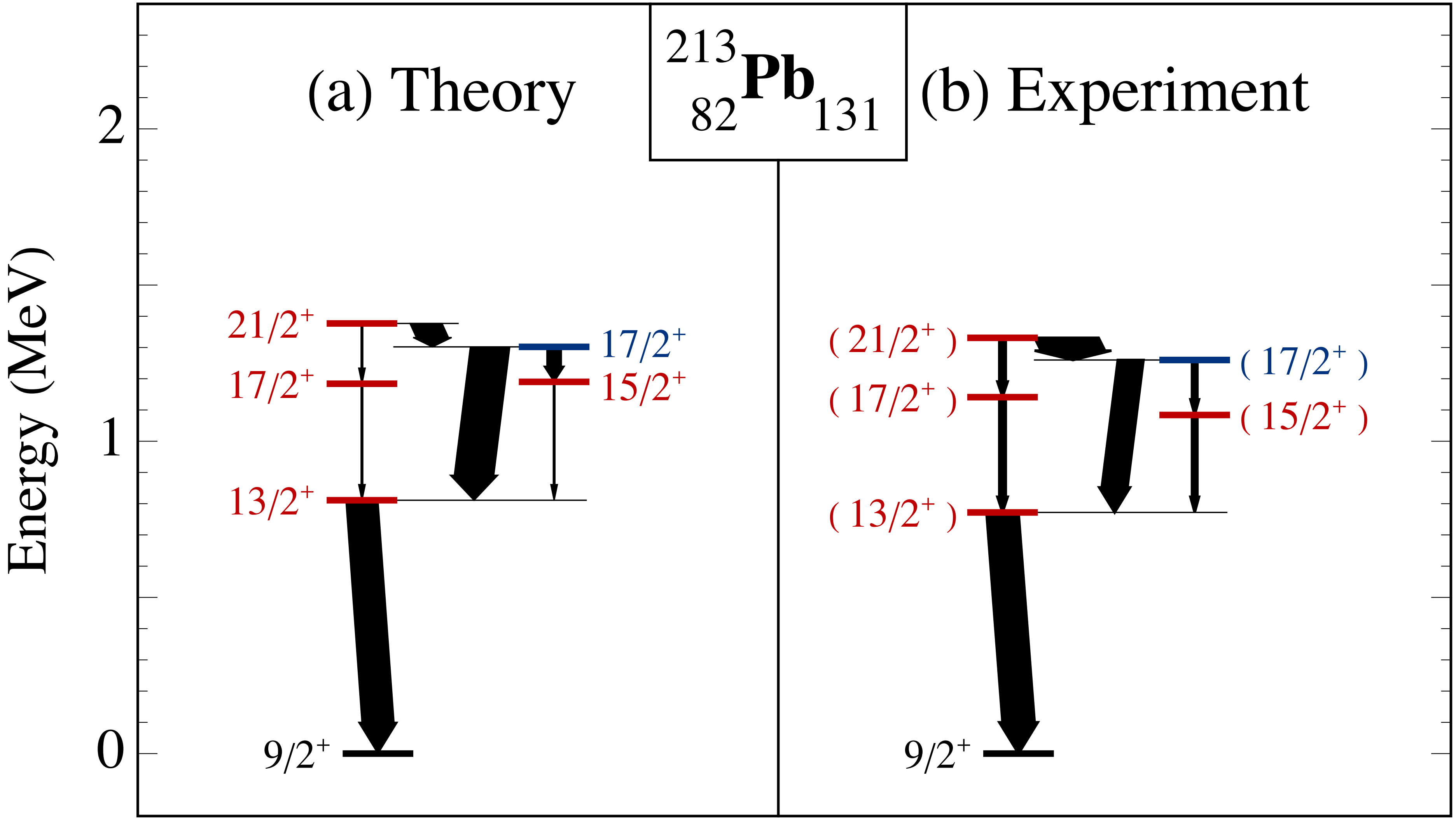


Spectrum of ^{213}Pb



J.J. Valiente-Dobón *et al.*, Phys. Lett. B **816** (2021) 136183

E2 transitions in ^{213}Pb



J.J. Valiente-Dobón *et al.*, Phys. Lett. B **816** (2021) 136183

Effective charges

One kind of nucleon in a single-*j* shell

$$\hat{T}_1(E2) = e_{\text{eff}}\left(a_j^+\tilde{a}_j\right)^{(2)}$$

Matrix elements between *n*-nucleon states are

$$\left\langle j^n\alpha' J'\left\|\hat{T}_1(E2)\right\|j^n\alpha J\right\rangle = \frac{n}{n-1}(-)^{j+J}\sqrt{(2J+1)(2J'+1)}$$

$$\times\sum_{\tilde{\alpha}R\tilde{\alpha}'R'} c_{n\alpha J}^{\tilde{\alpha}R} c_{n\alpha' J'}^{\tilde{\alpha}'R'}\begin{Bmatrix} J & J' & 2 \\ R' & R & j \end{Bmatrix}\left\langle j^{n-1}\tilde{\alpha}'R'\left\|\hat{T}_1(E2)\right\|j^{n-1}\tilde{\alpha}R\right\rangle$$

Calculate recursively until $\left\langle j\left\|\hat{T}_1(E2)\right\|j\right\rangle = e_{\text{eff}}\sqrt{5}$

→ All B(E2) values within a single-*j* shell depend on one effective charge.

A. de-Shalit & I. Talmi, *Nuclear Shell Theory*
I. Talmi, *Simple Models of Complex Nuclei*

State-dependent effective charges

In a single-*j* shell

$$\hat{T}_1(E2) = e_{\text{eff}}(J,J')\left(a_j^+\tilde{a}_j\right)^{(2)}$$

Matrix elements between *n*-body states are calculated recursively until

$$\left\langle j^2 J' \left\| \hat{T}_1(E2) \right\| j^2 J \right\rangle$$

$$= -\sqrt{20(2J+1)(2J'+1)}\begin{Bmatrix} j & j & 2 \\ J & J' & j \end{Bmatrix} e_{\text{eff}}(J,J')$$

→ Many effective charges.

State-dependent effective charges

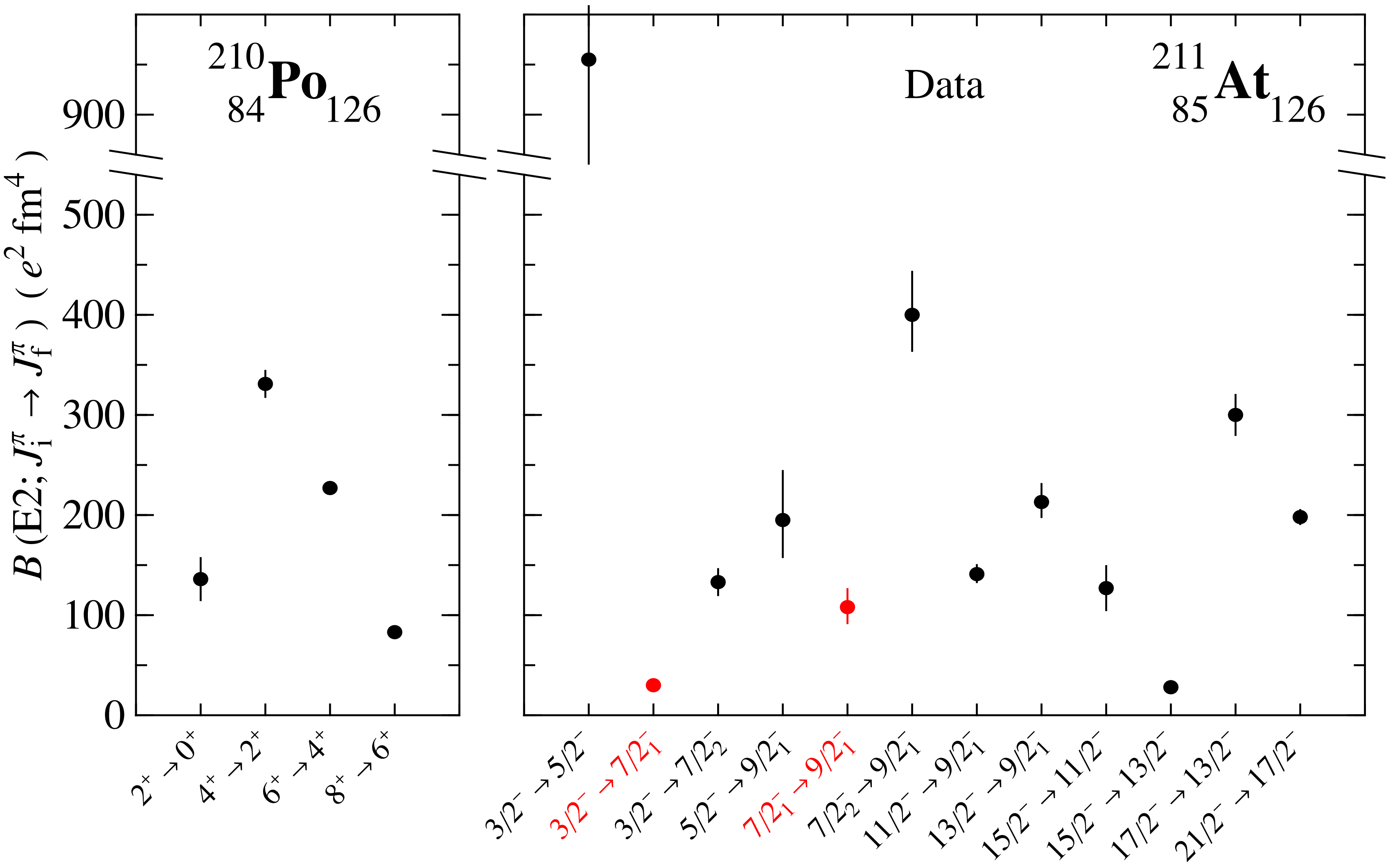
- B(E2) values depend on all effective charges $e_{\text{eff}}(J,J-2)$ for $n > 2$.
- Example: the $13/2^+ \to 9/2^+$ E2 transition for $n=3$.
  - *Constant effective charge:*

$$\left\langle j^3\, {}^9\!/\!{}_2 \left\| \hat{T}_1(E2) \right\| j^3\, {}^{13}\!/\!{}_2 \right\rangle = \sqrt{\tfrac{70}{11}}\, e_{\text{eff}}$$

  - *State-dependent effective charges:*

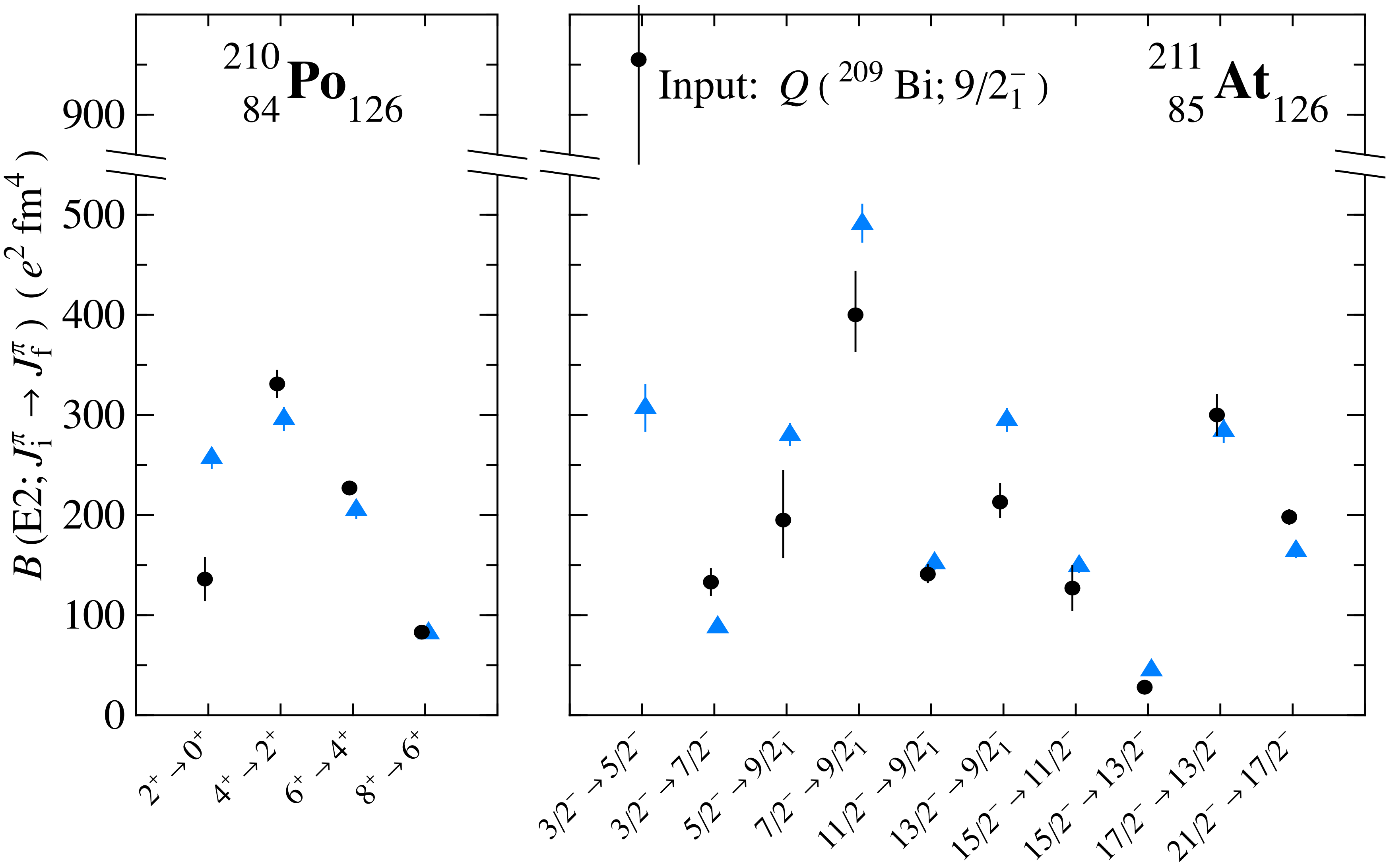
$$\left\langle j^3\, {}^9\!/\!{}_2 \left\| \hat{T}_1(E2) \right\| j^3\, {}^{13}\!/\!{}_2 \right\rangle = \sqrt{\tfrac{7}{110}} \Big(\tfrac{54}{11} e_{\text{eff}}(2,0) + \tfrac{453}{1573} e_{\text{eff}}(4,2) + \tfrac{3067}{1573} e_{\text{eff}}(6,4) + \tfrac{408}{143} e_{\text{eff}}(8,6) \Big)$$

E2 data in ^{210}Po and ^{211}At



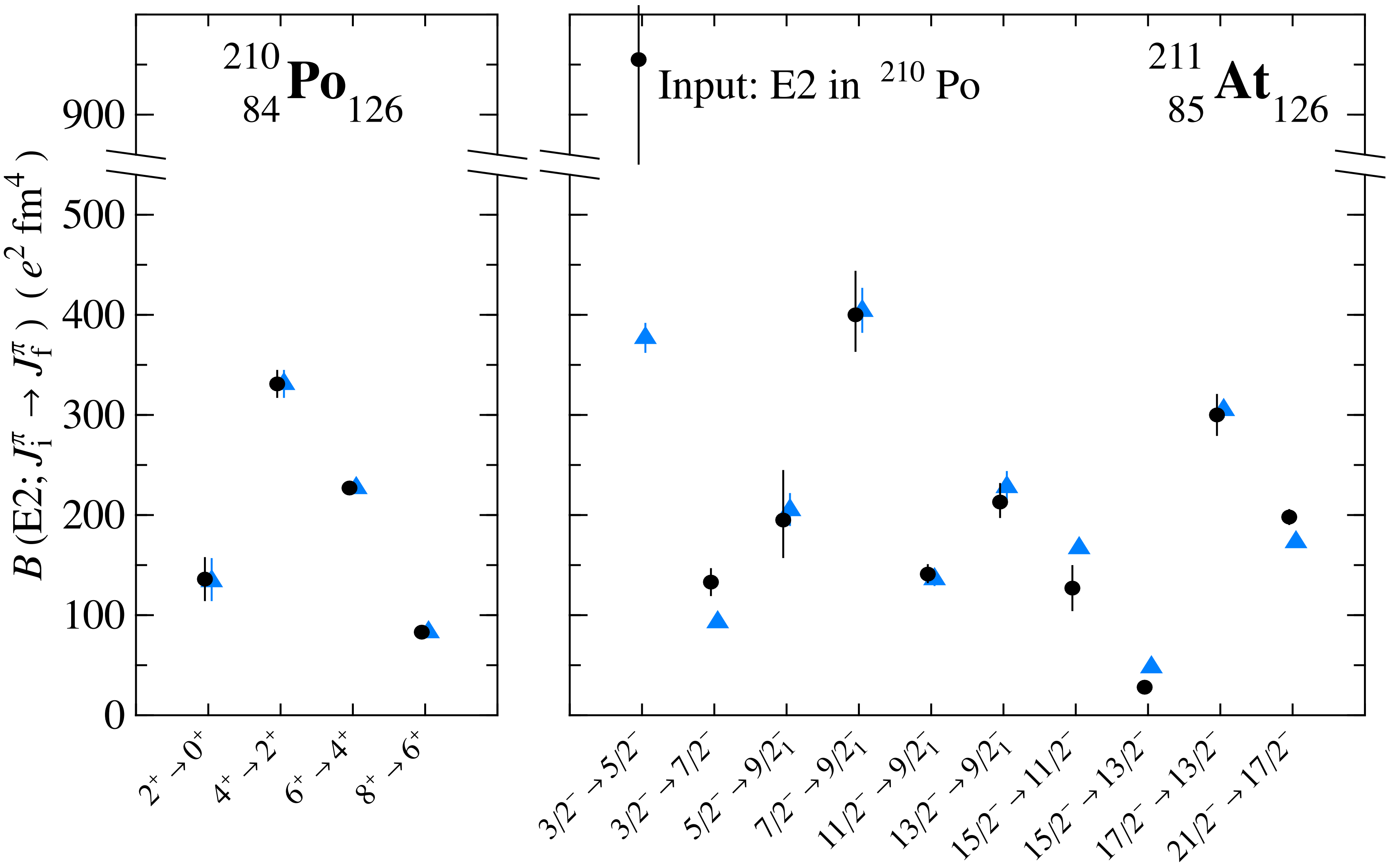
V. Karayonchev *et al.*, Phys. Rev. C **99** (2019) 024326
V. Karayonchev *et al.*, Phys. Rev. C **106** (2022) 044321

One-body E2

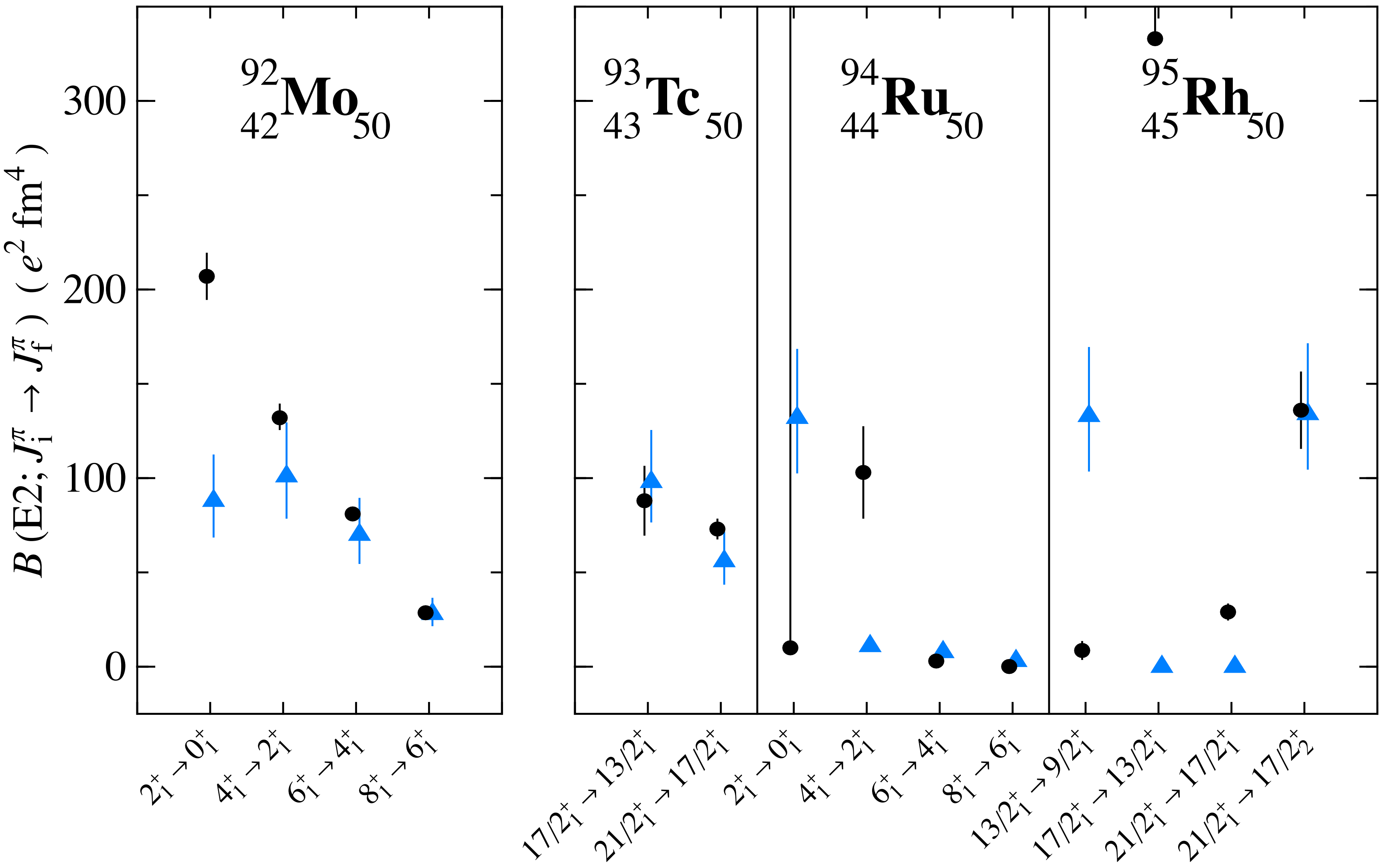


$^{210}_{84}\mathbf{Po}_{126}$
$^{211}_{85}\mathbf{At}_{126}$
Input: $Q\,(^{209}\mathrm{Bi};\,9/2^-_1)$
$B\,(\mathrm{E2};\,J^\pi_\mathrm{i} \to J^\pi_\mathrm{f})\ (e^2\,\mathrm{fm}^4)$
0
100
200
300
400
500
900
$2^+ \to 0^+$
$4^+ \to 2^+$
$6^+ \to 4^+$
$8^+ \to 6^+$
$3/2^- \to 5/2^-$
$3/2^- \to 7/2^-$
$5/2^- \to 9/2^-_1$
$7/2^- \to 9/2^-_1$
$11/2^- \to 9/2^-_1$
$13/2^- \to 9/2^-_1$
$15/2^- \to 11/2^-$
$15/2^- \to 13/2^-$
$17/2^- \to 13/2^-$
$21/2^- \to 17/2^-$

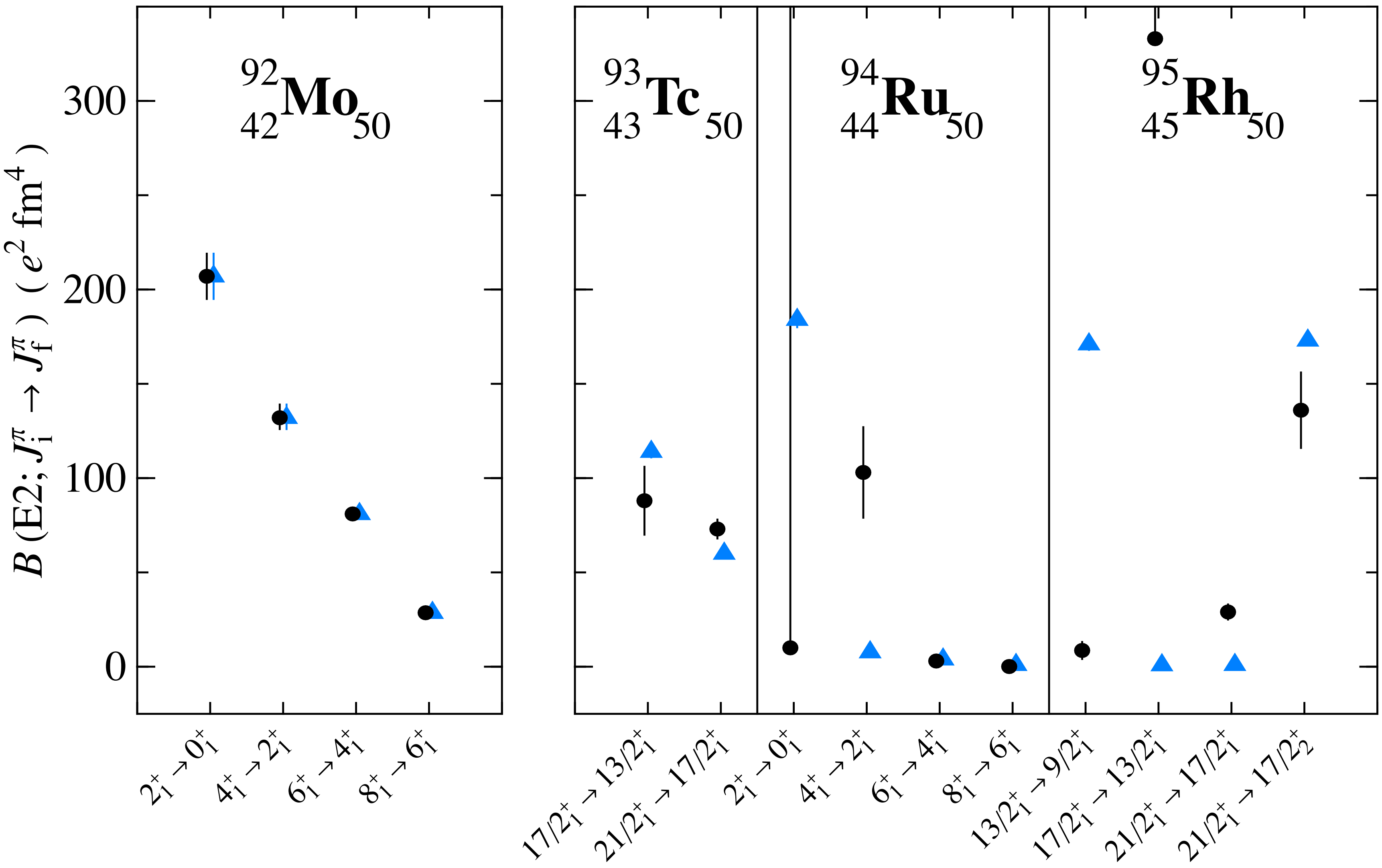
State-dependent one-body E2



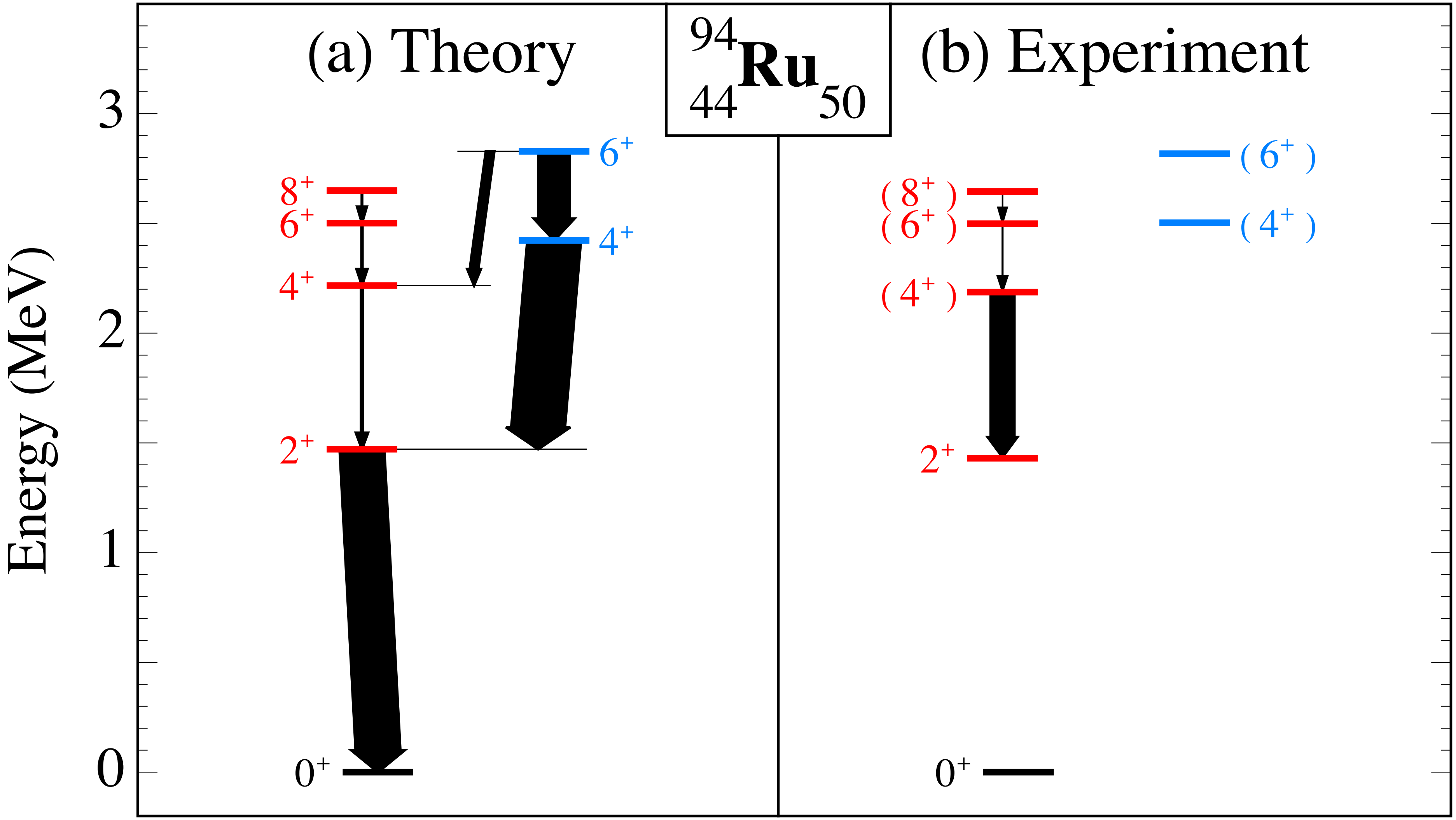
E2 properties of N=50 isotones



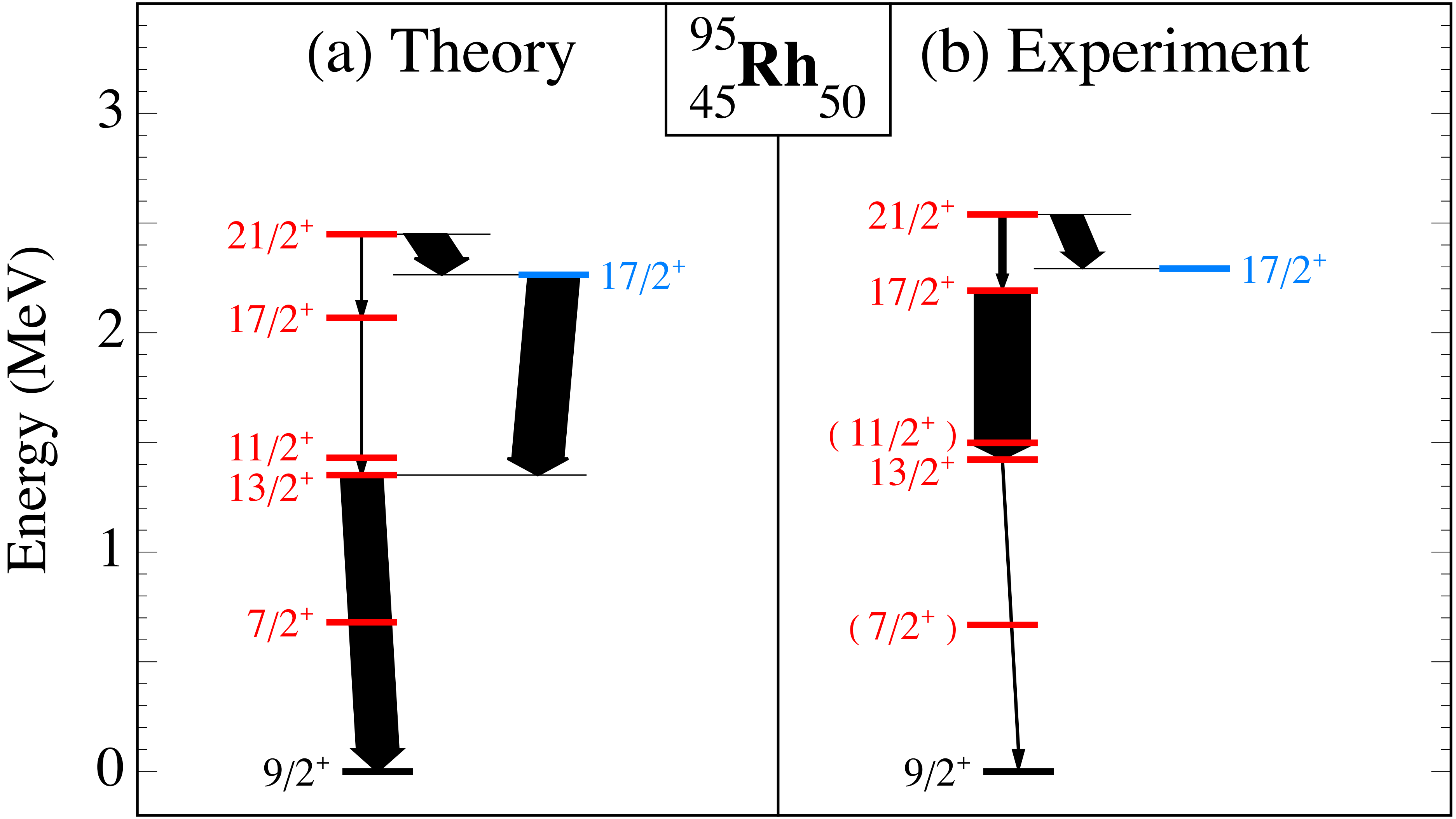
E2 properties of N=50 isotones



E2 transitions in ^{94}Ru



E2 transitions in ^{95}Rh



B. Das *et al.*, to be published

Conclusions

Despite a rapid progress in *ab initio* calculations and many-body methods there is still a need for simple models of complex nuclei.

Symmetry methods grounded in group theory provide such simple models.

Unexpected and unexplained experimental results are the most interesting ones.