DiffPy-CMI – a software toolbox for real-space structure analysis and Complex Modeling

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Methods for analyzing experimental PDFs

direct readout

- bond distances and their variations

big box modeling

- Reverse Monte Carlo (RMC) [Pusztai & McGreevy, Physica B, 234-236, (1997)]
- ~10^4 atoms in a large box with periodic boundary conditions
- MC position optimization → excellent fit to the experimental PDF
- many degrees of freedom – RMC modeling requires constraints to produce physically feasible structures
- interpretation of 10^4 coordinates – bond length and angle statistics

small box modeling

- up to ~100 atoms in a small cell with periodic boundary conditions
- PDF modeling can be focused to a short, specific length scale
- **simple refinement**
  - start with reasonably accurate initial structure
  - downhill minimization of the model variables to fit observed PDF
- **structure determination**
  - extract experimental pair-distances from the PDF
  - find shape that reproduces the same set of pair distances
- **complex refinement**
  - additional cost terms (atom overlap, bond valence sums,…)
  - mixed contributions from molecules and crystalline phases

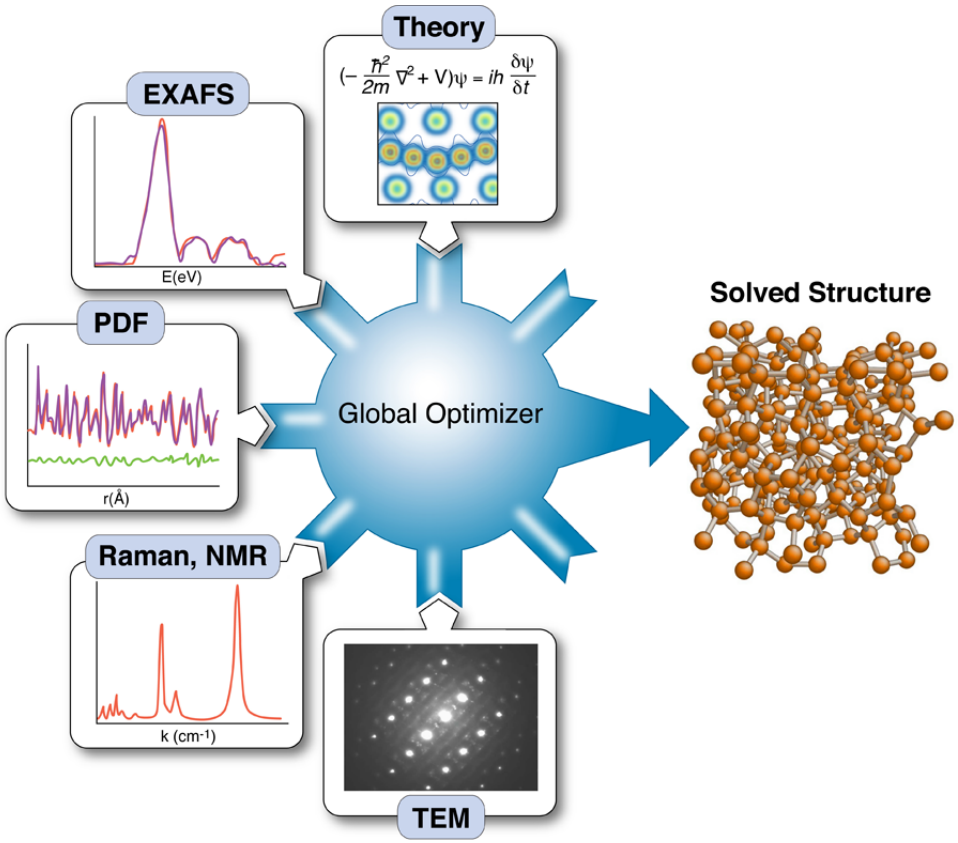
PDFgui limitations

- periodic structures only, no support for isolated particles
- particle size corrections only for spherical shape,
  no option for ellipsoid, cylinder, prism, etc.
- internal structure representation is very basic – no options for molecules,
  rigid groups, bond length/angle restraints, no internal support
  for representation as space group with asymmetric unit
- limited math support within constraint formulas
- no way to adjust PDF calculations by user-defined
  profile functions or improved peak-width formulas
- no option to incorporate other information sources (theory or experiment)
  into optimization
- PDF-calculation engine PDFfit2 is hard to maintain or extend

advantages

- easy to learn, no need to know any programming language
- [simple] simulations are easy to setup and process

Complex modeling



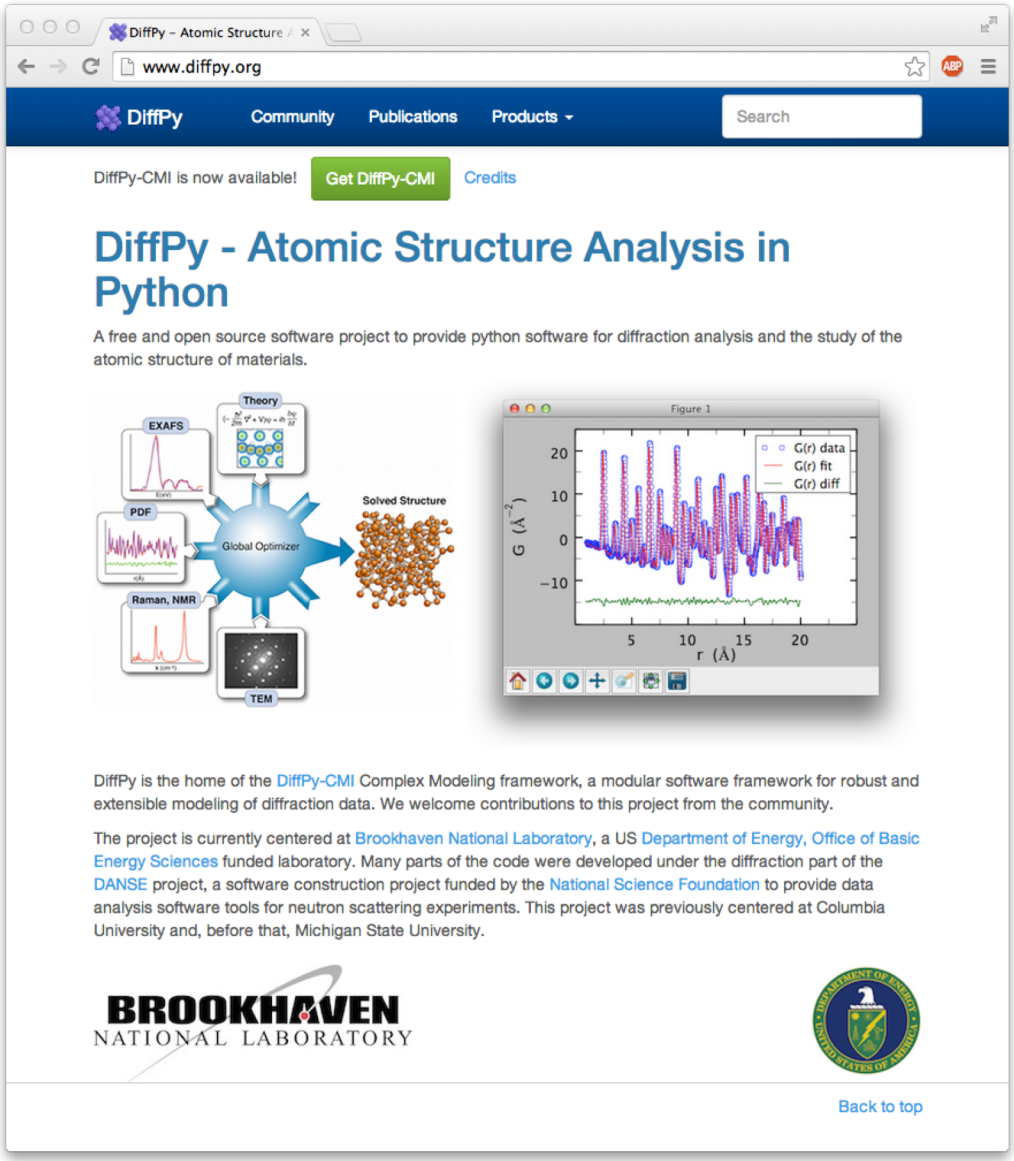
Problem

- not enough information in the available experimental data

Remedy

- collect data from multiple experimental techniques
- use additional knowledge about the studied material - chemical constraints, rigid units, bond-valence sums, energy calculation
- combine all experimental and theoretical inputs about the structure in one optimization scheme
- requires flexible software tools to setup custom models adaptable for specifics of studied materials.

DiffPy-CMI – Complex Modeling Infrastructure



- tools for PDF, BVS, SAS simulations, structure data handling, multi-input optimizations
- Python and C++, object-oriented, reusable, extensible libraries
- available from **https://www.diffpy.org** for Linux, Mac, UNIX systems

upgrade release March 2019

- added support for Python 3

Anaconda Python installation

```
$ conda install –c diffpy diffpy-cmi
```

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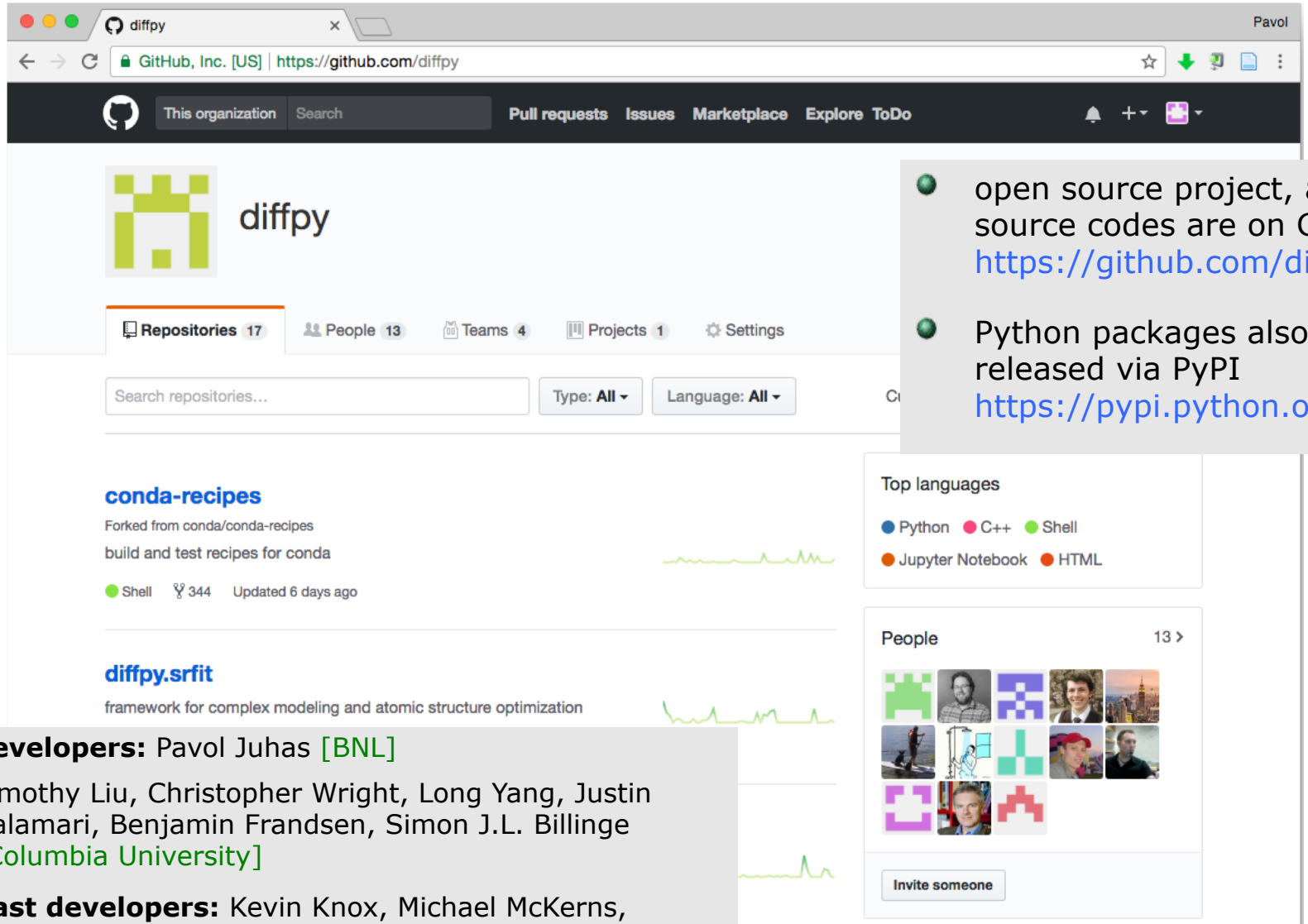
DiffPy-CMI – open source project



- open source project, all source codes are on GitHub https://github.com/diffpy
- Python packages also released via PyPI https://pypi.python.org

developers: Pavol Juhas [BNL]

Timothy Liu, Christopher Wright, Long Yang, Justin Calamari, Benjamin Frandsen, Simon J.L. Billinge [Columbia University]

past developers: Kevin Knox, Michael McKerns, Christopher Farrow, Dmitriy Bryndin, Jiwu Liu, Yingrui Shang, Peng Tian, Wenduo Zhou, Milinda Abeykoon, Emil Bozin, Timur Davis

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DiffPy-CMI software

DiffPy-CMI is

- Complex Modeling Infrastructure – a software toolbox for multi-probe structure analysis
- collection of Python and C++ libraries responsible for tasks needed in structure analysis (structure representation, forward calculators, refinement configuration)
- highly configurable, extensible. Amendable for extensions, such as more calculators and more structure representations. Calculations of structure-derived properties can be tweaked at runtime.

advantages

- allows simulations not viable in PDFgui
- plenty of options for configuring refinements, structure models, simulations, interfacing with other analysis programs…

drawbacks

- steep learning curve
- no GUI, can be only used from Python scripts
- requires fair knowledge of Python

IPython notebooks – data analysis sharing

IPython

- powerfull Python shell with many data analysis features

Jupyter notebooks

- similar to Mathematica notebook format
- data analysis code and its outputs, formatted notes, images, media
- NB rendering and sharing at https://nbviewer.jupyter.org https://github.com

science
Nature |

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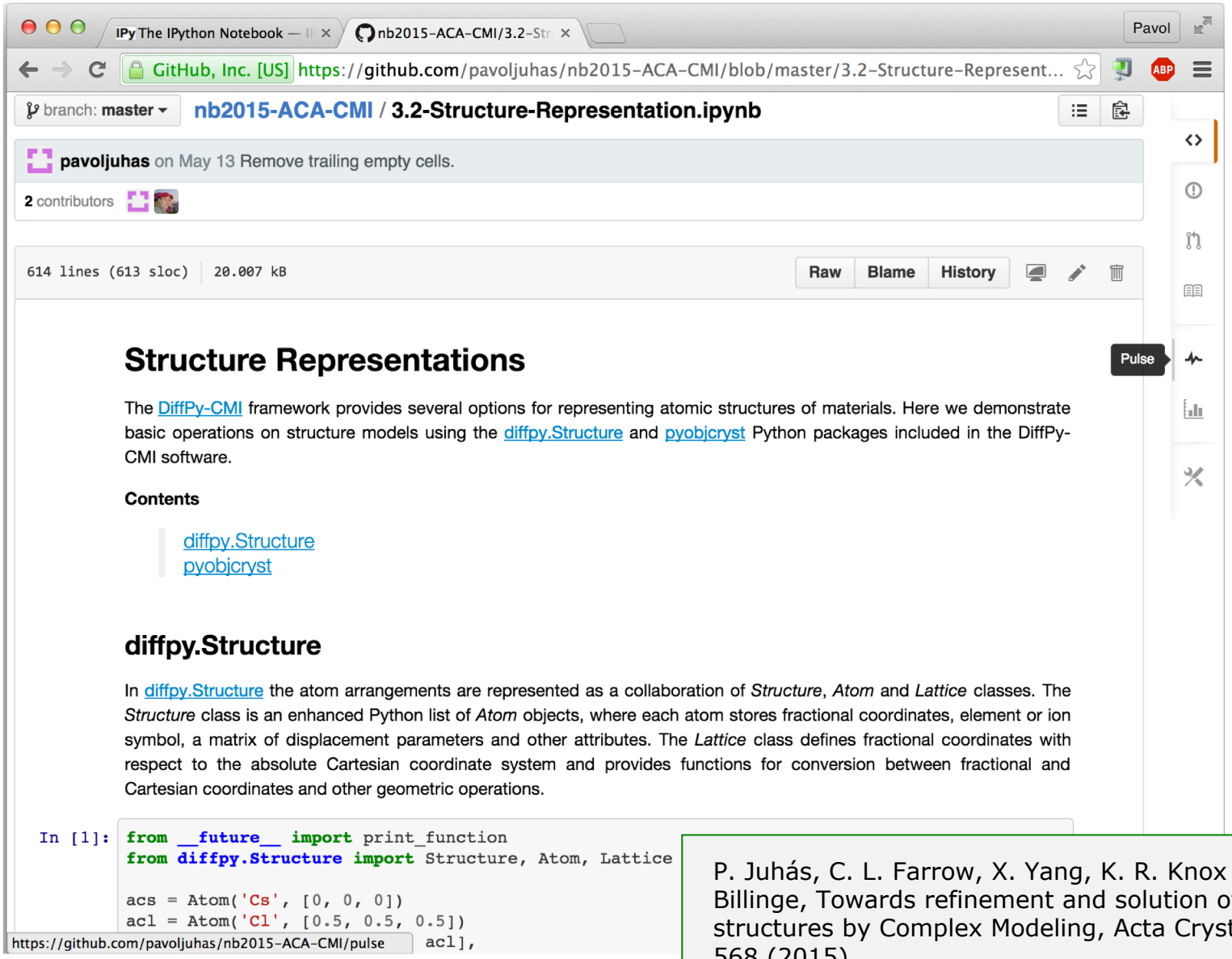
IPython notebooks – data analysis sharing



P. Juhás, C. L. Farrow, X. Yang, K. R. Knox and S. J. L. Billinge, Towards refinement and solution of complicated structures by Complex Modeling, Acta Crystallogr. A 71, 562-568 (2015).

DiffPy-CMI – functionality overview

Structure Representation

- **diffpy.structure** → simple storage of P1 periodic structures, finite clusters, input and ouput for CIF, PDB, xyz, pdffit, discus formats. Space group definitions, symmetry expansion, generation of symmetry-based constraints.
- **pyobjcryst** → advanced structure representations, crystals with space group, crystals containing rigid molecules, bond-length and bond-angle restraints, z-matrix representation. Input and output in custom XML and CIF formats. Python interface to the ObjCryst++ crystallographic library by V. Favre-Nicolin.

Forward Calculators

- **diffpy.srreal** → calculators of pair-interaction-derived quantities, such as PDF, Debye sum, bond lengths, bond valence sums, overlap of empirical atom radii.
- **pyobjcryst** → powder and single-crystal diffraction patterns.
- **srfit-sasview** → selected functions for Small Angle Scattering simulations from the SasView program, http://www.sasview.org

Fit configuration and management

- **diffpy.srfit** → setup and control of general fitting problems, control of constraints and restraints, setup of refinements to multiple data sources, simple analysis of fit results.

C++ libraries

- **libdiffpy** – computationally expensive parts - PDF, BVS, etc.
- **libObjCryst** – free objects for crystallography by Vincent Favre-Nicolin, [J. Appl. Cryst. 35 (2002), 734-743].

PairQuantity

- the base calculator – shared recipe for evaluating physical quantities based on pair-interactions. Used as a blueprint for all calculators.

$$P(r_1, r_2, \ldots, r_N) = \sum_{i,j}^{N} p(r_{ij})$$

- support for partial sums
- optional upper and lower distance bounds
- support for parallel evaluation

related:

- class StructureAdapter
  - translate structure representations to a calculator-compatible format
  - implemented for PDFgui Structure representation and for Crystal and Molecule objects in the ObjCryst C++ library [V. Favre-Nicolin, J. Appl. Cryst. 35 (2002), 734-743]
  - extensible for any structure representation method in Python or C++

BondCalculator

- calculate oriented bond vectors up to a specified distance limit
- optional filtering by atom types, site indices, direction cones

example:

```
>>> from pyobjcryst import loadCrystal
>>> from diffpy.srreal.bondcalculator import BondCalculator
>>> rutile = loadCrystal('TiO2_rutile.cif')
>>> bc = BondCalculator(rmax=2)
>>> bc(rutile)
array([ 1.94720295,  1.94720295,  1.94720295,  1.94720295,  1.94720295,
        1.94720295,  1.98177183,  1.98177183,  1.98177183])
>>> for i in zip(bc.distances, bc.types0, bc.types1, bc.directions):
...     print(i)
...
(1.9472029472402153, 'Ti', 'O', array([-0.8951757,  0.8951757, -1.4795]))
(1.9472029472402153, 'Ti', 'O', array([-0.8951757,  0.8951757,  1.4795]))
(1.9472029472402153, 'Ti', 'O', array([ 0.8951757, -0.8951757, -1.4795]))
(1.9472029472402153, 'Ti', 'O', array([ 0.8951757, -0.8951757,  1.4795]))
(1.9472029472402153, 'O', 'Ti', array([ 0.8951757,  0.8951757, -1.4795]))
(1.9472029472402153, 'O', 'Ti', array([ 0.8951757,  0.8951757,  1.4795]))
(1.9817718303429834, 'Ti', 'O', array([-1.4013243, -1.4013243,  0.    ]))
(1.9817718303429837, 'Ti', 'O', array([ 1.4013243,  1.4013243,  0.    ]))
(1.9817718303429837, 'O', 'Ti', array([-1.4013243, -1.4013243,  0.    ]))
```

OverlapCalculator

- calculate overlap of empirical atom radii
- other results: site square overlap, coordination numbers, coordination histograms, neighborhoods of touching sites, overlap gradients
- related: class *AtomRadiiTable* and its specializations *ConstantRadiiTable*, *CovalentRadiiTable*
  - radius lookup by atom symbol
  - support for custom atom radii

example:

```
>>> from diffpy.structure import loadStructure
>>> from diffpy.srreal.overlapcalculator import OverlapCalculator
>>> sto = loadStructure('SrTiO3.cif')
>>> oc = OverlapCalculator()
>>> oc.atomradiitable.fromString("Sr2+:1.44,  Ti4+:0.605,  O2-:1.35")

>>> oc(sto)
array([  4.89062513e-03,   1.67088000e-05,   1.63577798e-03,
         1.63577798e-03,   1.63577798e-03])
>>> oc.meansquareoverlap
0.0019629335719733893
>>> oc.coordinations
array([ 12.,   6.,   6.,   6.,   6.])
>>> oc.coordinationByTypes(4)
{'Ti4+': 2.0, 'Sr2+': 4.0}
```

BVSCalculator

- bond valence sums – approximate formula for ion valences

Brese, Acta Cryst. B47, 192-197 (1991)

$$v_{ij} = \exp\left[\frac{R_{ij} - d_{ij}}{b}\right]$$

$$V_i = \sum_j v_{ij}$$

- evaluates valence at each site, BVS difference, mean square BVS difference which accounts for partial occupancies and site multiplicities
- related: class BVParametersTable
  - lookup of bond valence parameters, [bvparm2009.cif by I. D. Brown]
  - option to define and revert custom BVS parameters

example:

```
>>> from pyobjcryst import loadCrystal
>>> from diffpy.srreal.bvscalculator import BVSCalculator
>>> sto = loadCrystal('SrTiO3.cif')
>>> bvsc = BVSCalculator()
>>> bvsc(sto)
array([ 2.12652479,  4.16096701, -2.0958306 ])
>>> bvsc.bvdiff
array([-0.12652479, -0.16096701, -0.0958306 ])
>>> bvsc.bvmsdiff
0.013893882037591496
```

PDFCalculator

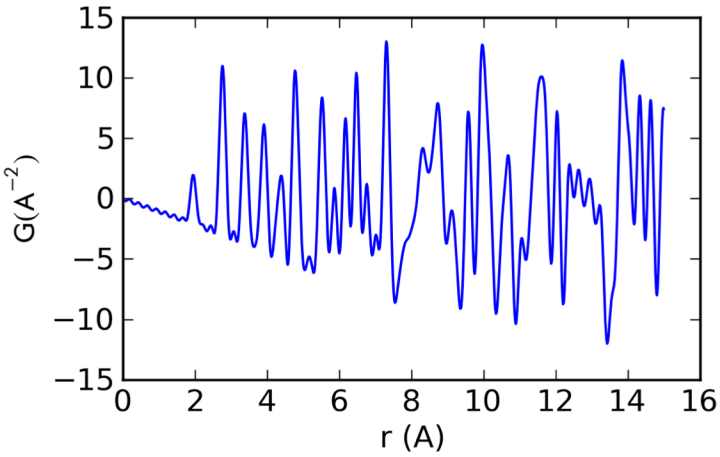
- PDF calculation in real-space
  - suitable for periodic systems
  - one structure per calculator → mixed-phase PDFs obtained by summing single-phase PDFs

$$G(r) = \frac{1}{Nr\langle b\rangle^2} \sum_{i \neq j} b_i b_j \, \delta(r - r_{ij}) \; - \; 4\pi r \rho_0$$

- other results: radial distribution function, partial PDFs, *F*(*Q*)
- class ScatteringFactorTable
  - lookup of xray, netron or electron scattering factors
  - support for custom scattering factors
- class PeakProfile – the profile function for a pair contribution
- class PeakWidthModel – calculates profile width for a given atom pair
- class PDFEnvelope – one or more r-dependent scaling envelopes
- class PDFBaseline – the baseline function, by default $-4\pi\rho_0 r$

example:

```
>>> from diffpy.Structure import Structure
>>> from diffpy.srreal.pdfcalculator import PDFCalculator
>>> sto = Structure(filename='SrTiO3.cif')
>>> pdfc = PDFCalculator(rmax=15, qmax=25)
>>> r, g = pdfc(sto)
>>> import pylab
>>> pylab.plot(r, g)
```



DebyePDFCalculator

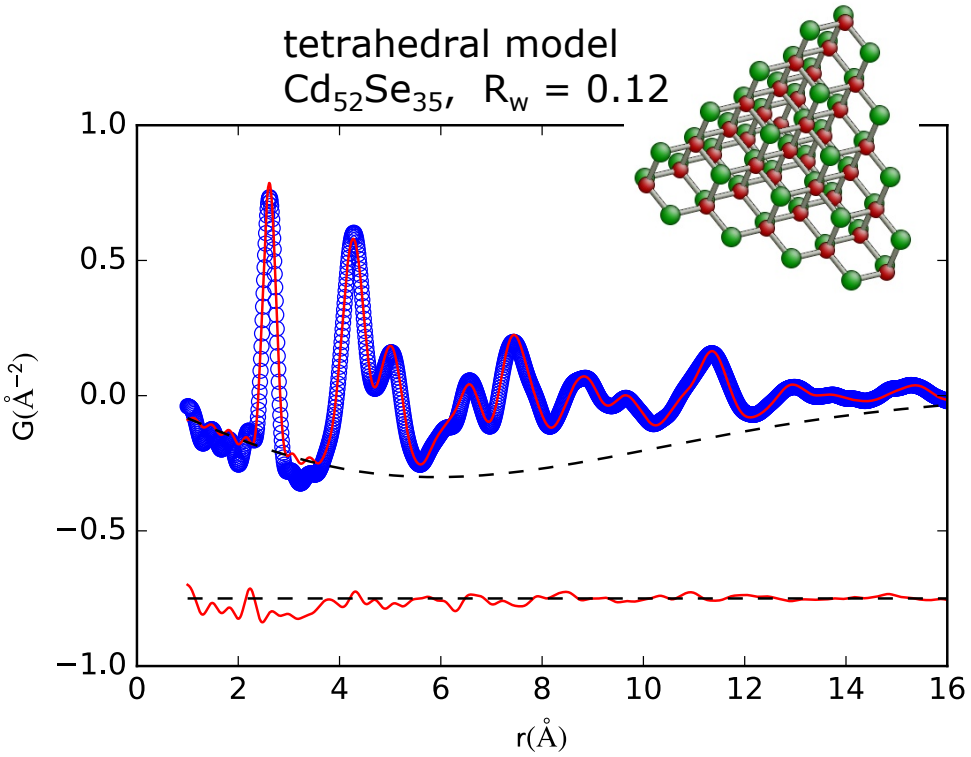
- PDF calculation in *Q*-space – *F*(*Q*) calculated by Debye scattering equation and Fourier transformed to *G*(*r*)

$$F(Q) = \frac{1}{N\langle f(Q)\rangle^2} \sum_{i,j} f_i(Q) f_j(Q) \frac{\sin Qr_{ij}}{r_{ij}} \exp\left[-\frac{1}{2}\sigma_{ij}^2 Q^2\right] \qquad G(r) = \frac{2}{\pi} \int_{Q_{\min}}^{Q_{\max}} F(Q) \sin Qr \, \mathrm{d}Q$$

- suitable for molecules or nano-clusters
- PDF baseline simulated by Q_{min} cutoff in the calculated *S*(*Q*)

example:

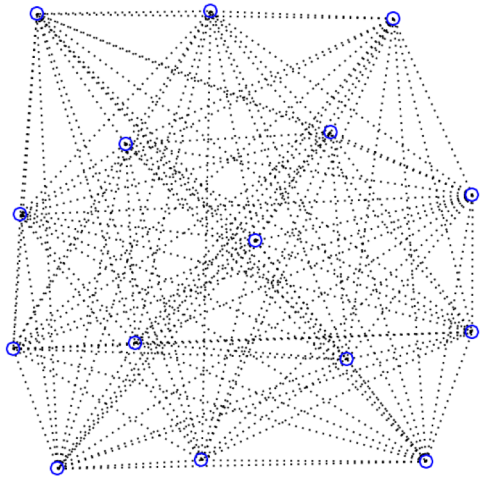
- structure refinement of CdSe quantum dots
- simulated PDF reflects particle shape in amplitude dampening and baseline shape

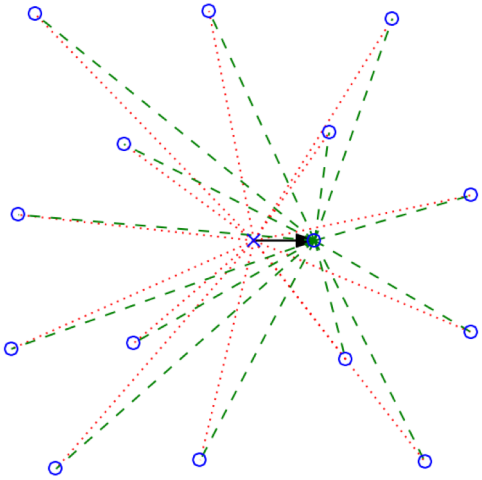


A. Beecher, J. Am. Chem. Soc., 2014, 136 (30), 10645–10653

Optimized PDF evaluation

- PDF is calculated from a sum of N^2 pair contributions → it is computationally expensive for larger models

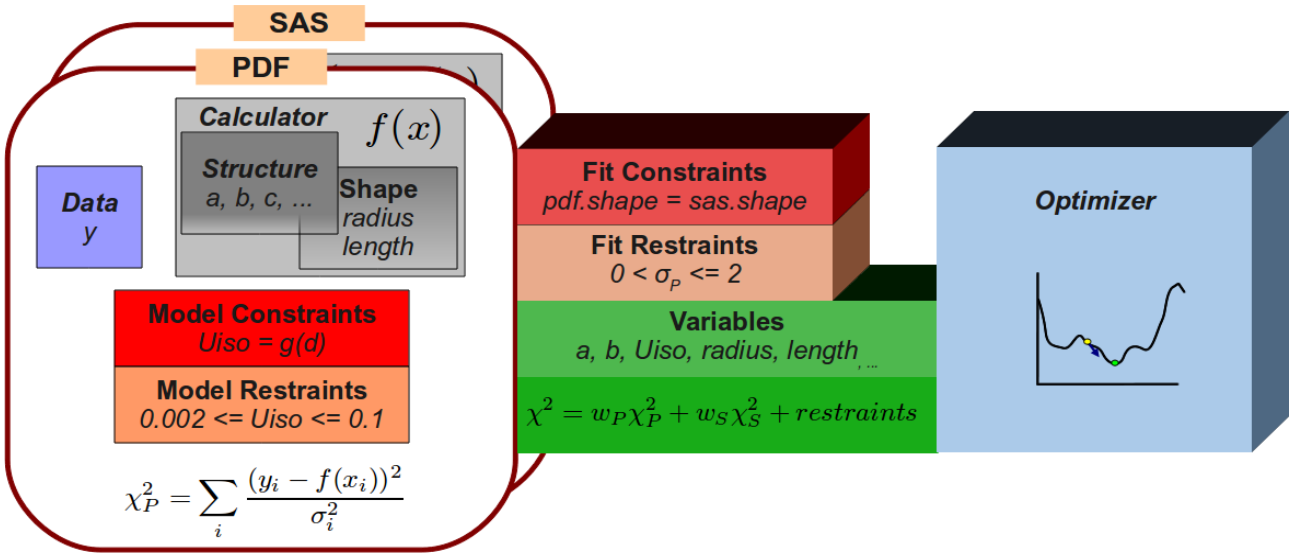




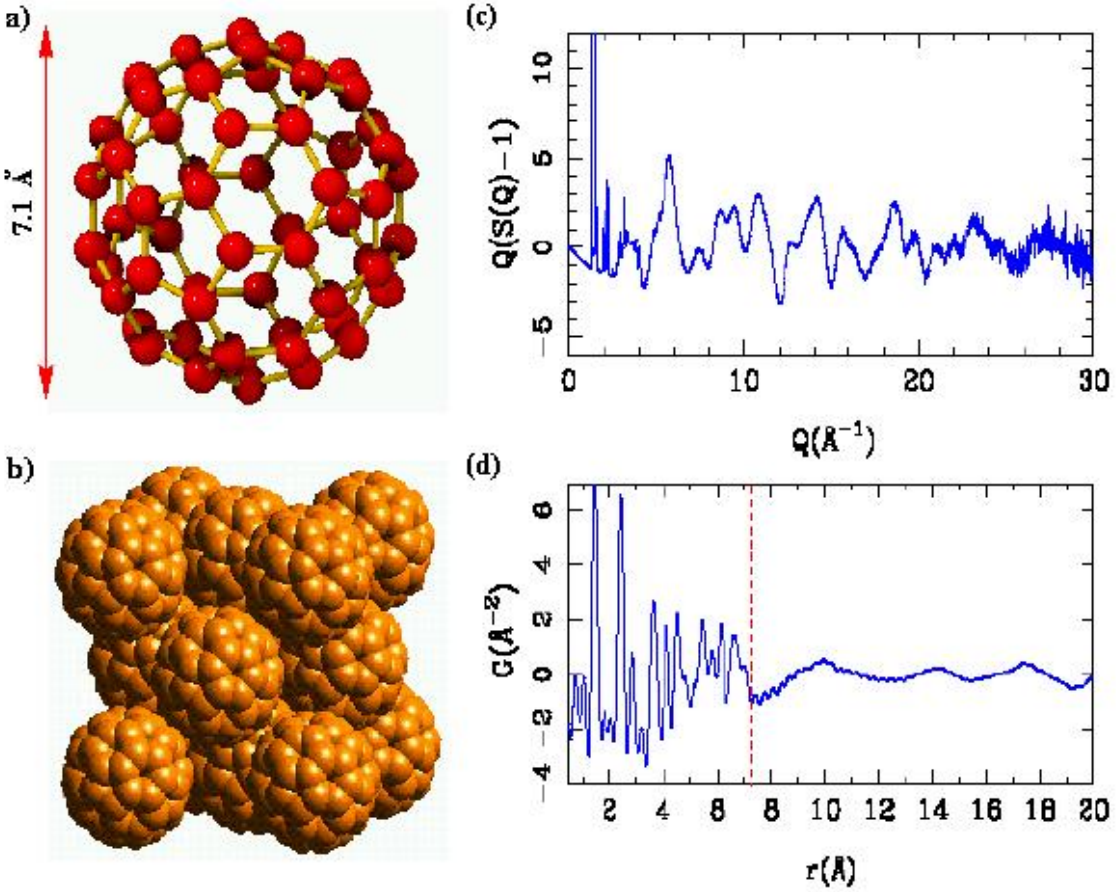
- real-space and Debye PDF calculators support optimized PDF calculation.
- when few atoms change, their old contributions are subtracted and new ones added to the PDF. Computational cost is reduced from N^2 → N.
- optimized PDF calculation is about 1000 faster for one-atom updates in 10,000-atom structure.

SrFit – multi-component fit manager

- Python module for general multi-component data refinement
- construct FitContribution by associating observed data with simulation
  - models can be defined with built-in calculators, math expressions, Python functions
  - model parameters are exposed to SrFit. Parameters can be constrained or restrained, e.g., *"a = b = c"* for cubic structure
- FitContributions are combined to a single total cost function (residual vector or scalar value) with interface suitable for optimization routines
- control functions to fix/free variables, define constraints, restraints, hook functions
- post-processing to generate fit result reports – partial costs per each contribution, error estimates and correlations of the fit variables.



PDF modeling of fcc-C_{60}

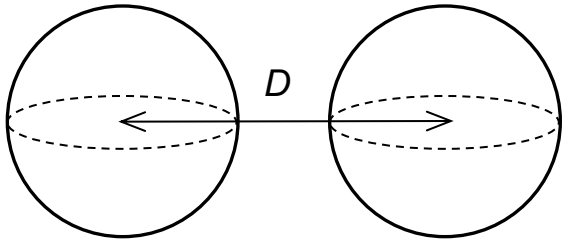


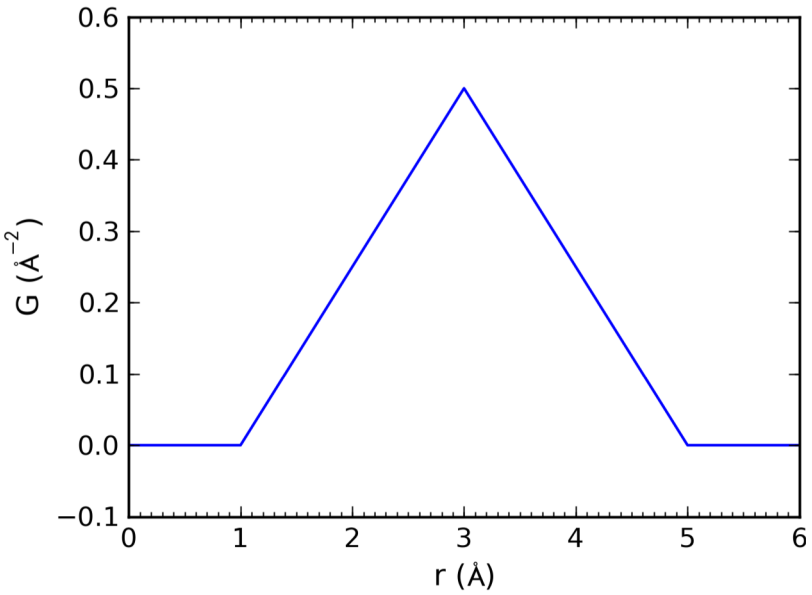
- neutron PDF measured on C_{60} fcc structure [GLAD IPNS, E. Bozin]
- low-*r* sharp peaks – correlations within C_{60}
- high-*r* broad peaks – correlations between randomly oriented balls

Can we simulate PDF on a full measured range?

- calculate as a sum of single particle PDF and PDF from a lattice of spherical shells

PDF peak profile for spherical shells





- PDF of two spherical shells can be calculated analytically

$$G(r) = \frac{1}{S_1 S_2 r} \iint_{S_1 S_2} \delta(r - r_{12})\, \mathrm{d}S_1 \mathrm{d}S_2$$

  triangular profile centered at spheres' distance *D*

- cluster of spherical shells → PDF calculation requires <u>triangular profile function</u>
- non-standard PDF profile requires
  - definition of the profile function
  - select it for a PDFCalculator instance

Custom PDF peak profile

profile defined in C++

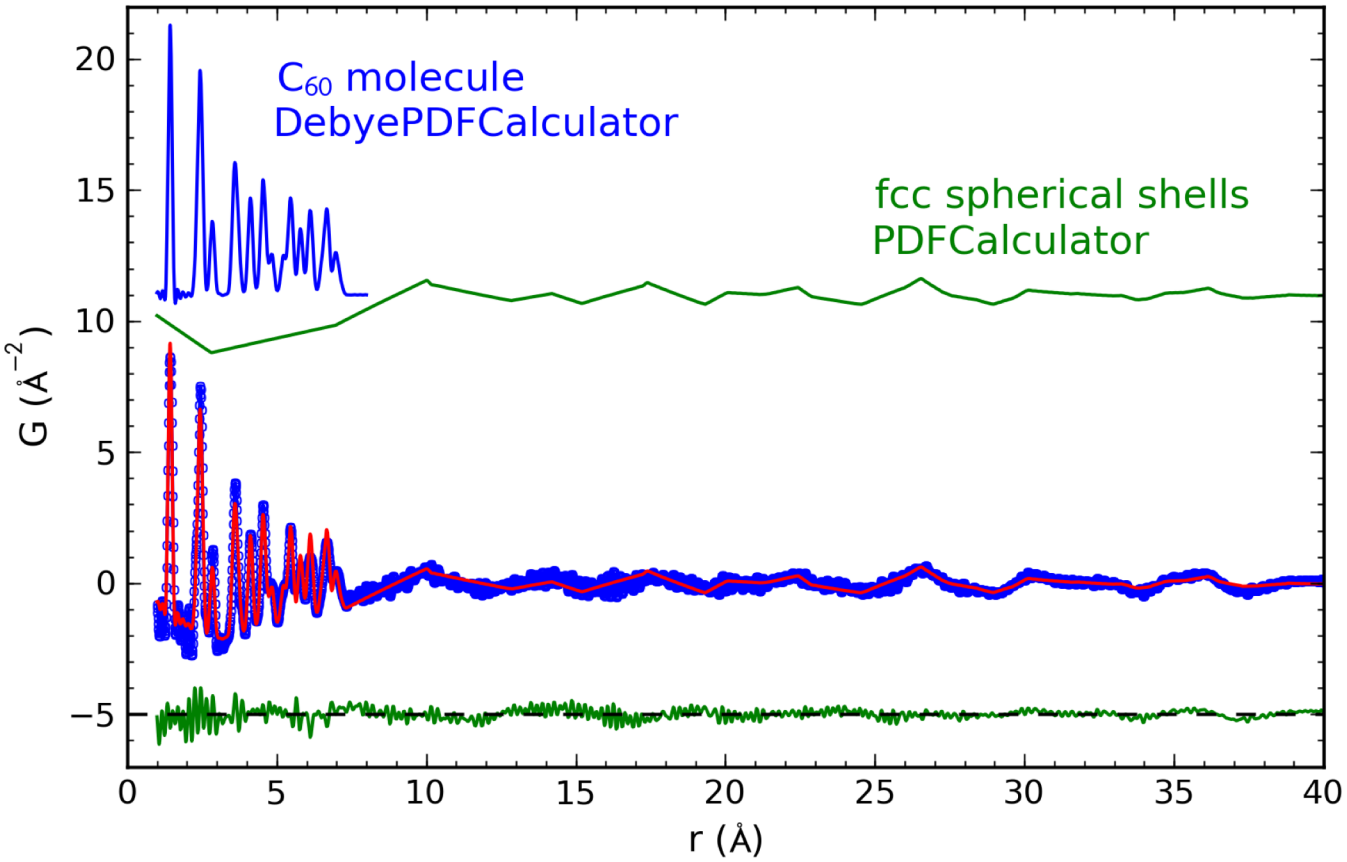
```
#include <cmath>
#include <diffpy/srreal/PeakProfile.hpp>

using diffpy::srreal::PeakProfile;
using diffpy::srreal::PeakProfilePtr;

class SphericalShellsProfile : public PeakProfile {
public:
    PeakProfilePtr create() const {
        return PeakProfilePtr(new SphericalShellsProfile());
    }

    PeakProfilePtr clone() const {
        return PeakProfilePtr(new SphericalShellsProfile(*this));
    }

    const std::string& type() const {
        static std::string tp = "sphericalshells-cpp";
        return tp;
    }

    double yvalue(double x, double fwhm) const
    {
        if (fabs(x) > fwhm)  return 0.0;
        double rv = (fwhm - fabs(x)) / (1.0 * fwhm * fwhm);
        return rv;
    }

    double xboundlo(double fwhm) const   { return -fwhm; }

    double xboundhi(double fwhm) const   { return +fwhm; }
};

bool reg_SawToothProfile = SphericalShellsProfile().registerThisType();
```

profile used in Python

```
>>> from diffpy.srreal.pdfcalculator import PeakProfile, PDFCalculator
>>> PeakProfile.getRegisteredTypes()
set(['croppedgaussian', 'gaussian'])
>>> import ctypes
>>> ctypes.cdll.LoadLibrary('./sphericalshells-cpp.so')
>>> PeakProfile.getRegisteredTypes()
set(['sphericalshells-cpp', 'croppedgaussian', 'gaussian'])
>>> pdfcalc = PDFCalculator()
>>> pdfcalc.setPeakProfileByType('sphericalshells-cpp')
```

- new profile functions can be defined either in Python or C++
- for C++ the profile function is compiled as a dynamic link library sphericalshells-cpp.so
- on loading the library adds new profile to the global registry → profile ready for use in Python
- no need to compile any other C++ sources related to PDFCalculator
- no need to write any Python wrappers for the new profile function

PDF refinement of fcc C_{60}

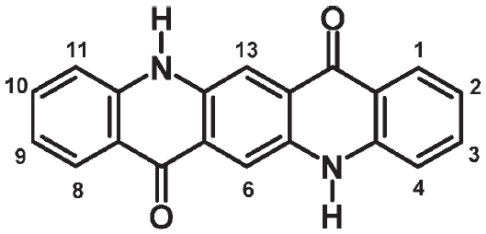


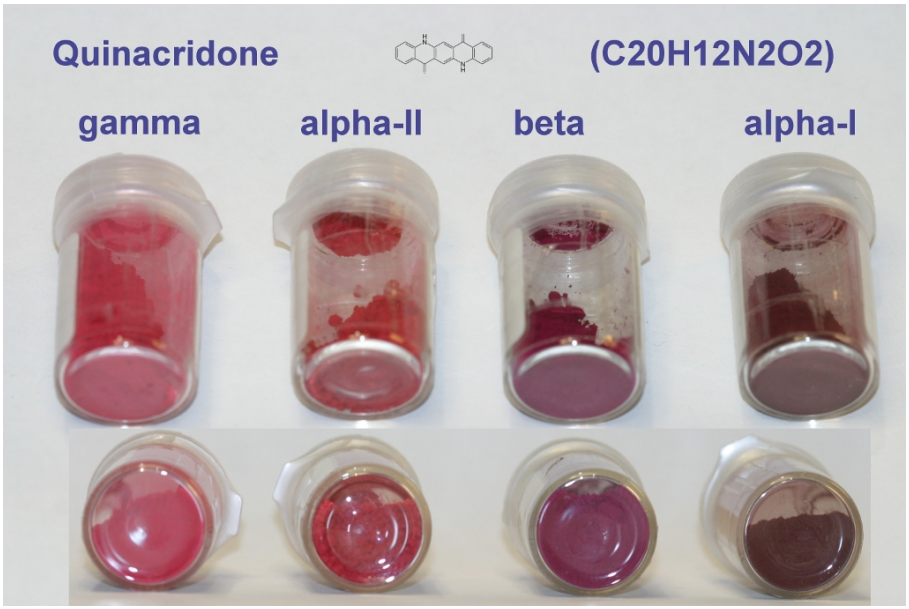
fit residuum R_w = 0.26 scale ratio = 60.0(1) U_{iso} = 0.00323(4)

molecule diameter = 7.113(2) shell diameter = 7.22(4)

- PDF from fcc C_{60} can be refined on the full measured range accounting for both intra and inter-molecular correlations

PDF analysis of organic crystals





quinacridone – $C_{20}H_{12}N_2O_2$

- industrially important pigments, red and violet paints
- can form many phases, some (alpha-II) do not crystallize and have unknown structure
- experimental PDFs measured at APS ANL beamline 11-ID-B and NSLS BNL beamline X17A
- standard refinement with PDFgui is of poor quality even for the known β-phase

collaboration with Prof. Martin U. Schmidt and
Dr. Dragica Prill, Goethe Universität, Frankfurt am Main

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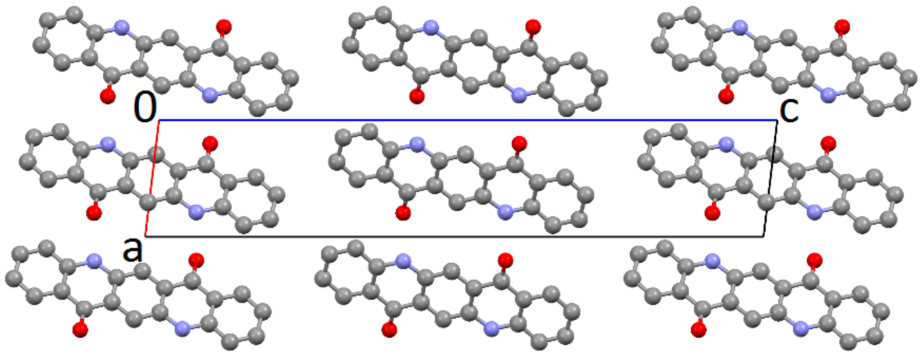
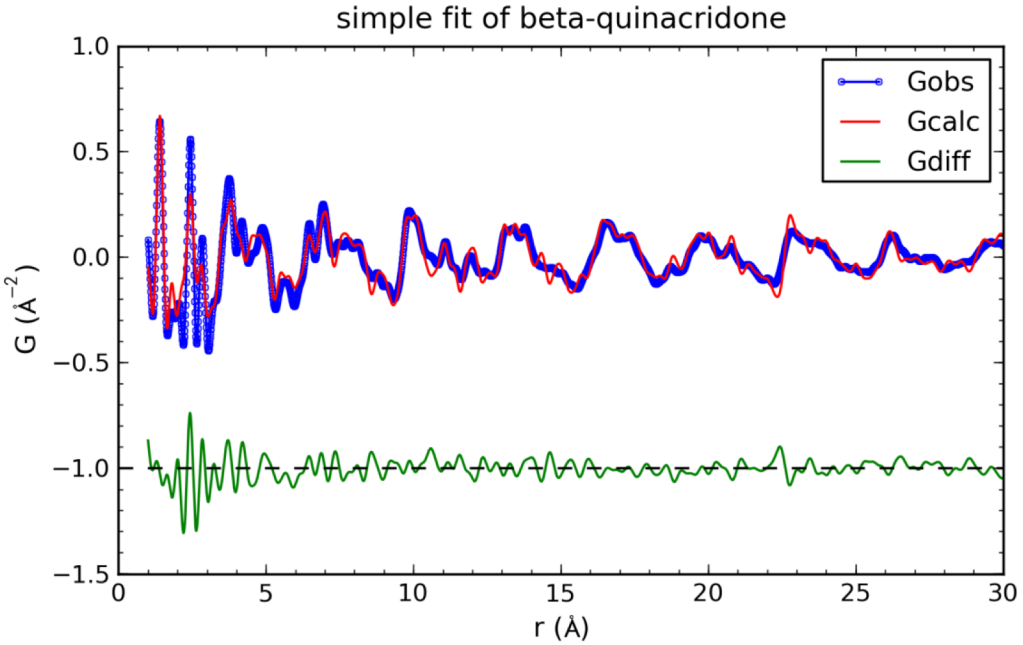
PDF modeling of β-quinacridone





- monoclinic $P2_1/c$, 2 molecules per unit cell
- refinement in PDFgui gives poor $\boldsymbol{R_w = 0.41}$
  - low-*r* peaks too wide
  - high-*r* peaks too sharp
- simple model assumes independent isotropic thermal vibrations
- peak widths depend strongly on *r*
  - sharp peaks for intra-molecular atom pairs
  - broad peaks for inter-molecular correlations
- PDF model has to use different displacement factors for pairs in the same molecule and inter-molecular pairs
- molecule must not deform when cell parameters change

D. Prill, *J. Appl. Cryst.* **48**, 171-178 (2015).

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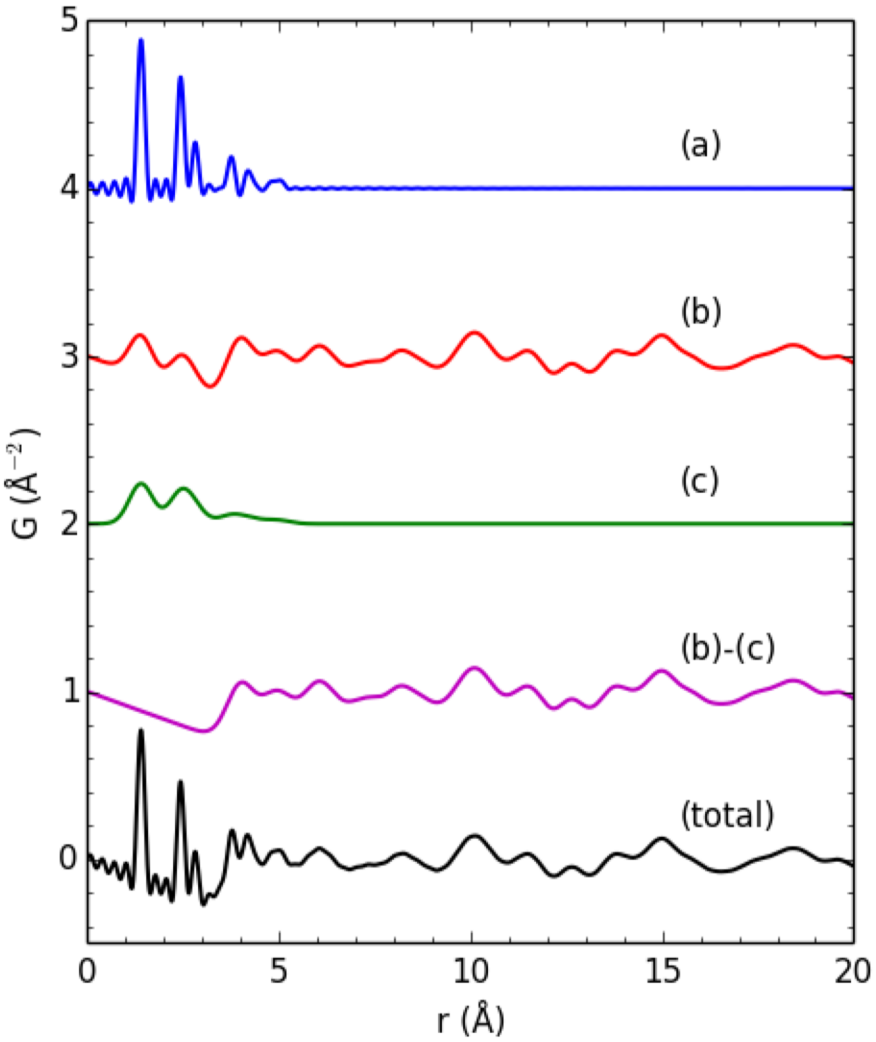
PDF modeling of β-quinacridone



PDF calculation with separate intra- and inter-molecular contributions

(a) PDF from a single-molecule, small atom displacements U_{intra}

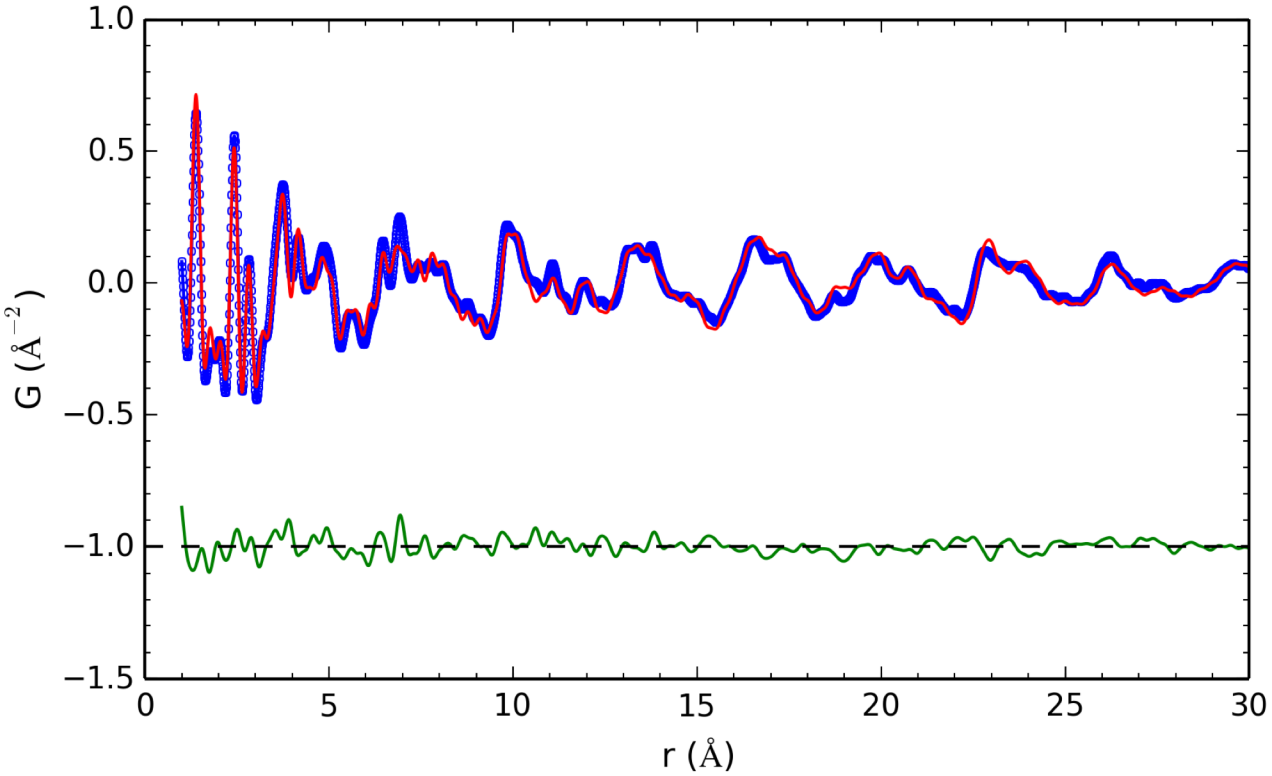
(b) PDF from a crystal with large displacements U_{inter}

(c) PDF from a molecule with large displacements U_{inter}

(b) – (c) PDF from inter-molecular interactions only

(a) + (b) – (c) total PDF reflecting both displacements U_{intra}, U_{inter}

PDF modeling of β-quinacridone



- refined unit cell, data scale and displacement factors
  U_{inter} = 0.0014(2) Å^2 U_{intra} = 0.023(2) Å^2
- significant fit improvement R_w = 0.41 → **R_w = 0.28**
- remnant fit difference due to anisotropic molecule displacements, displacement anisotropy can be studied with improved models

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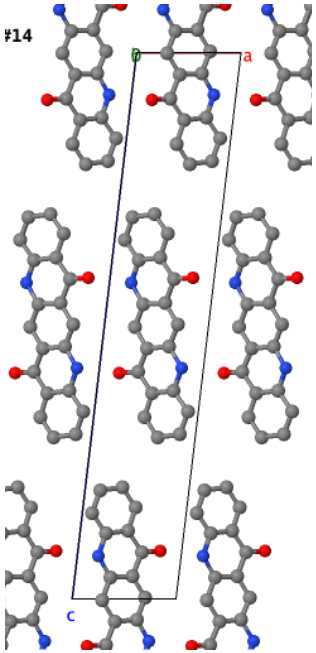
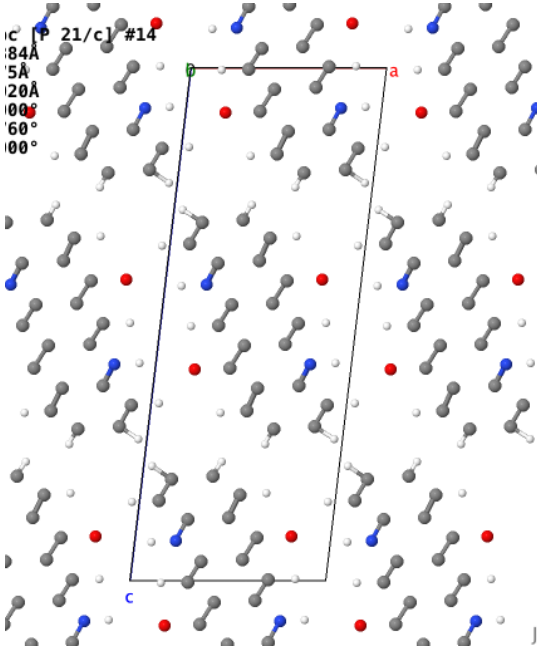
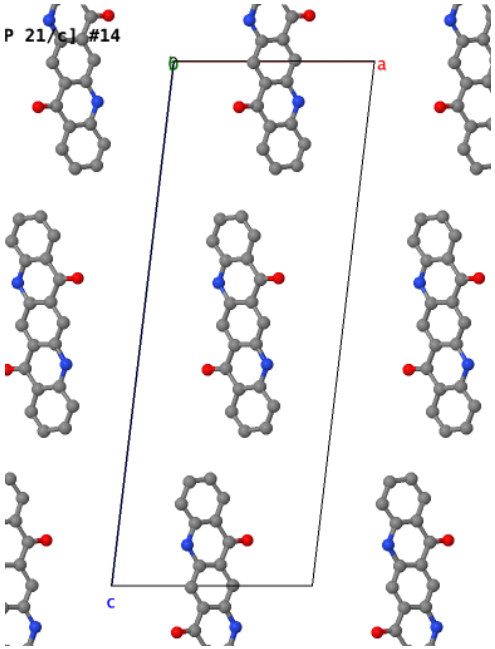
Structure representation with rigid molecules



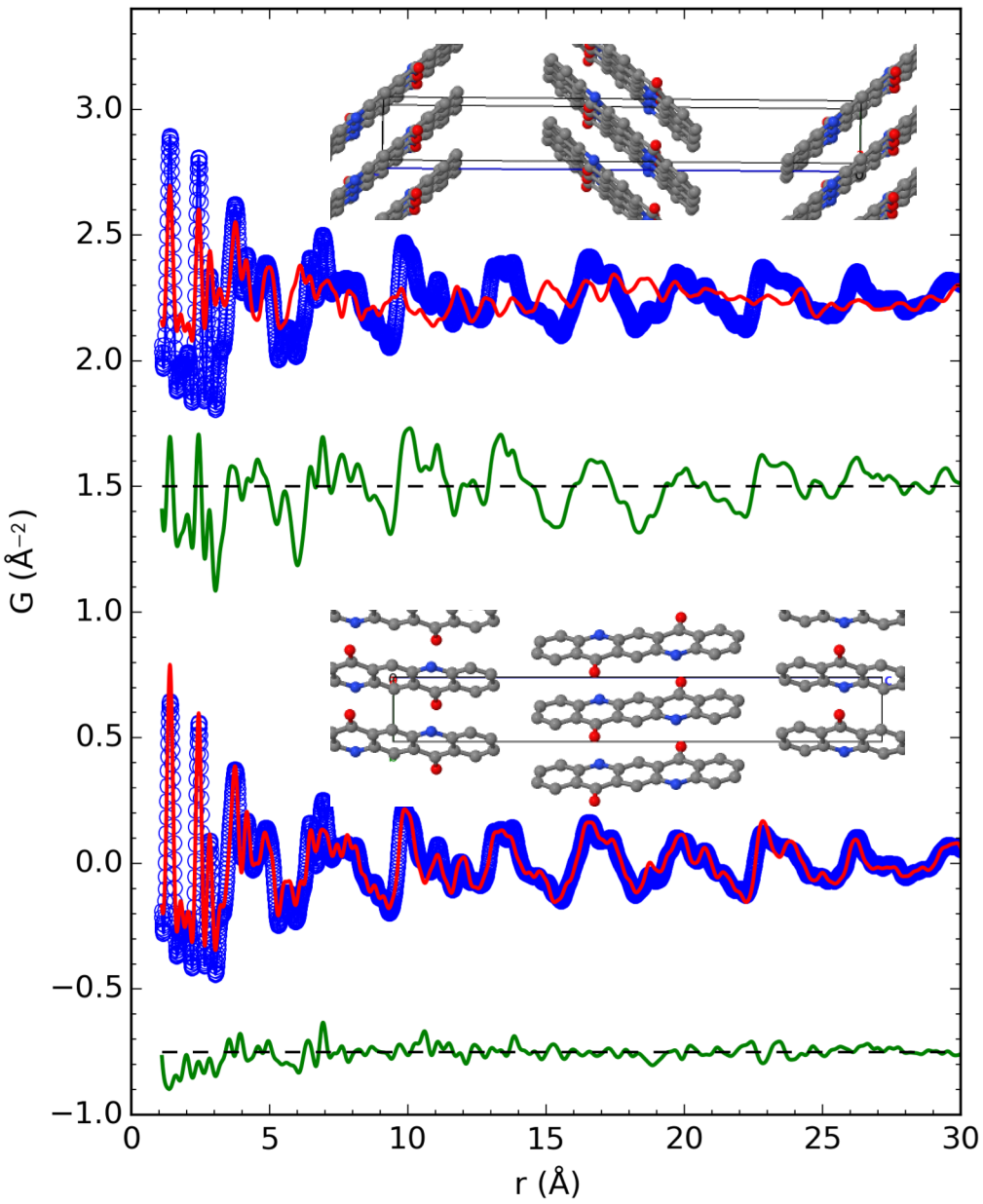




- structure representation needs to handle molecules as a rigid unit within crystal lattice → pyobjcryst in DiffPy-CMI, an interface to ObjCryst++
- molecule retains its shape when cell parameters change
- molecule placement is defined by its center-of-mass and orientation quaternion

V. Favre-Nicolin, J. Appl. Cryst. 35, 734-743 (2002)

Solution of molecule orientation in β-quinacridone



- apply random rotation on the quinacridone molecule in asymmetric unit
- preserve $P2_1/c$ symmetry
  - molecule position fixed at inversion center
  - use symmetric rotation for the second molecule in the UC
- refine molecule orientation, cell parameters, ADPs:
  $R_w = 0.83 \rightarrow R_w = 0.27$
- optimized structure converged to β-quinacridone

D. Prill *et al.*, Acta Crystallogr. A 72, 62-72 (2016)

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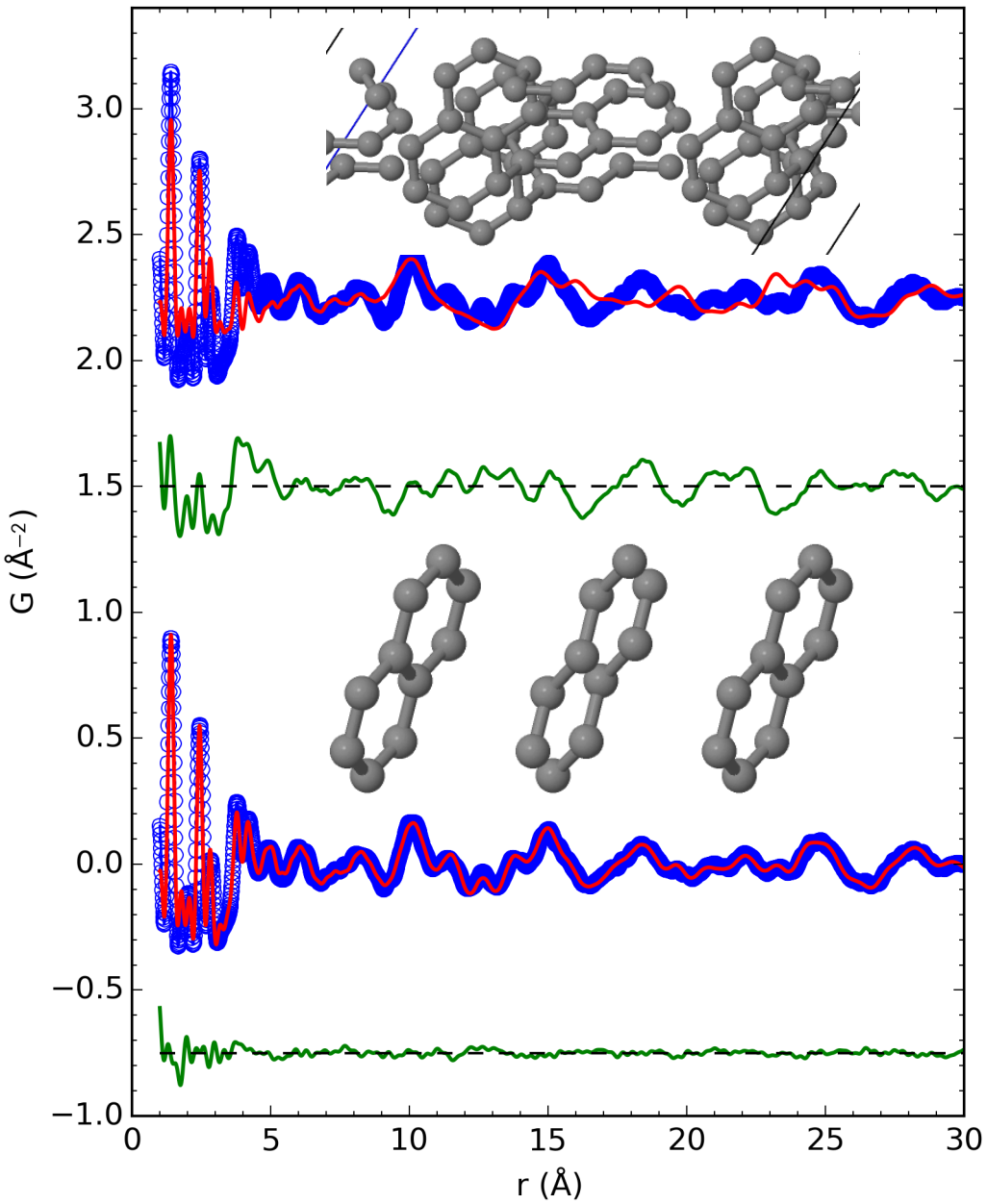
Solving molecule placement in naphthalene



- structure solution in P1
- 2 molecules in the unit cell with independent positions and orientations, 9 DOF
- start from a random initial placement of the UC molecules
- refine orientations, position, 6 cell parameters, ADPs: $R_w = 0.61 \rightarrow R_w = 0.16$
- optimized structure converged to $P2_1/a$ naphthalene

D. Prill *et al.*, Acta Crystallogr. A 72, 62-72 (2016)

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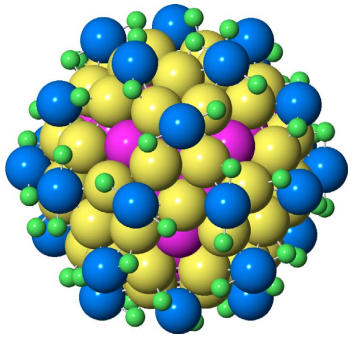
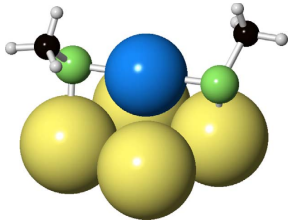
Structure solutions of molecular crystals

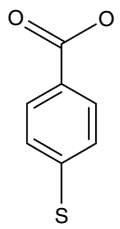
sample	symmetry	molecules	DOF	success rate
β-quinacridone	$P2_1/c$	1	3	6.4% (14/220)
naphthalene	$P2_1/a$	1	3	33% (79/240)
naphthalene	P1	2	9	2.5% (7/280)
allopurinol	$P2_1/c$	1	3	3.6% (12/329)
allopurinol	P1	4	21	0.4% (2/534)

- structure determination of molecular crystals
  - start from random initial placement of molecule(s) in the unit cell
  - fit the PDF, optimize rotation, position, cell parameters, intra- and inter-molecular ADPs
  - repeat to search for best convergence
- correct solutions were found also at lowered symmetry and enlarged DOF for molecular placement

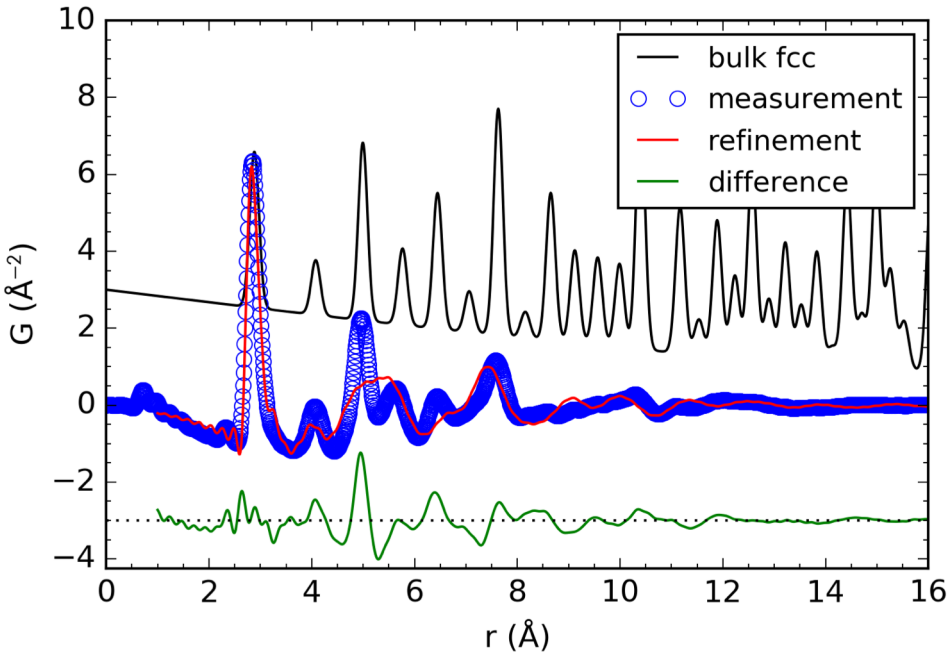
Polymorphism in $Au_{144}(SR)_{60}$







p-MBA



- $Au_{144}(SR)_{60}$ is ultra-stable (magic size), easy to prepare nanocluster system
- does not crystallize, published NMR+DFT studies claim icosahedral structure [Bahena *et al.*, J. Phys. Chem. Lett., 4, 975-981 (2013)
- Au_{144}(p-MBA)$_{60}$ produced by Prof. Ackerson group, Colorado State Univ., Fort Collins.
- X-ray PDF from Au_{144}(p-MBA)$_{60}$ is dramatically different and inconsistent with icosahedral model.
- measured peaks line-up with prominent peaks in bulk Au-*fcc* → actual structure should contain close-packed motifs.

K. Jensen and P. Juhas *et al*, Nat. Commun., 7:11859 (2016)

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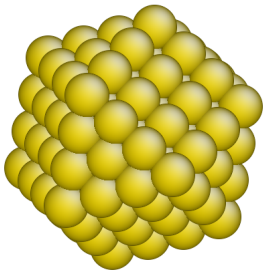
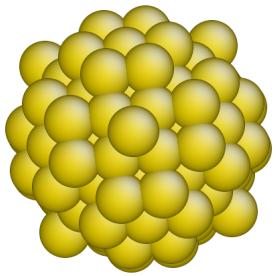
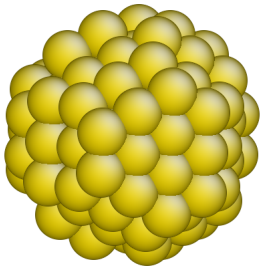
Polymorphism in $Au_{144}(SR)_{60}$



Au_{147} cuboctahedron

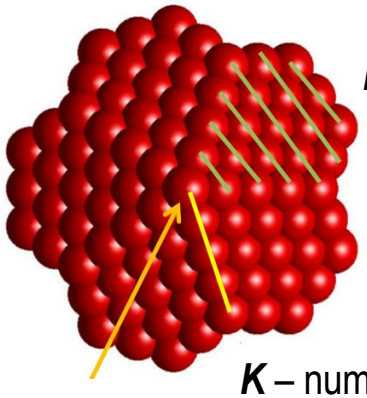


Au_{141} *fcc*



Au_{147} *hcp*

Marks decahedron (*N, M, K, T*)



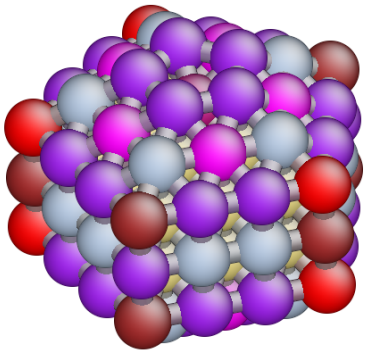
M – number of {002} shells

T – number of planes truncated from the top and bottom of decahedron

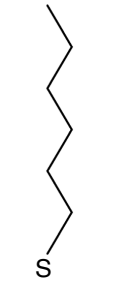
K – number of columns along the twin boundary

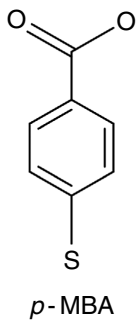
N – atoms in the central column

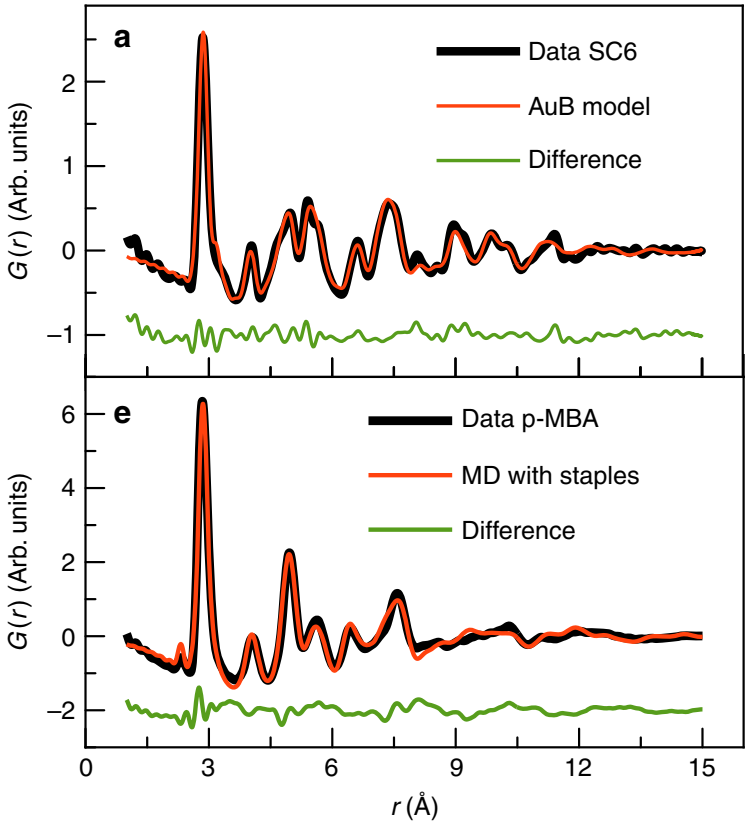
- DiffPy-CMI used for PDF refinement of gold clusters (expansion ratio, isotropic displacements for core and surface atoms)
- PDF fit evaluated for simple closed packed structures and a series of generated Marks decahedra (MD)
- very good PDF fit for 144-atom MD6441. Close PDF fit also after stripping to 114-atom MD6341, which has room for 30 S-Au-S surface "staples".

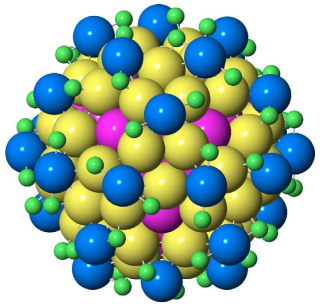


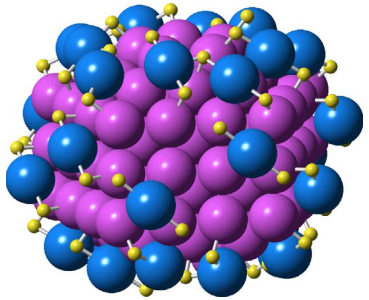
Polymorphism in $Au_{144}(SR)_{60}$











- PDF has not enough resolution to determine surface structure, i.e., staples-ligand placement. The 114-atom MD core is required for good fit.
- follow-up PDF measurements on $Au_{144}(SR)_{60}$ samples produced with different ligands showed the previous icosahedral phase and a mixture of icosahedral and MD phases → polymorphism happens also for nanoclusters.

K. Jensen and P. Juhas *et al*, Nat. Commun., 7:11859 (2016)

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MPDF – magnetic Pair Distribution Function

- developed by Benjamin Frandsen, Columbia University (now Brigham Young University)
  [*Acta Crystallogr. A* **70**, 3-11 (2014)]
- **magpdf** – extension to DiffPy-CMI for simulation and refinement of magnetic PDFs
- magpdf can be obtained from
  https://github.com/benfrandsen/diffpy.magpdf

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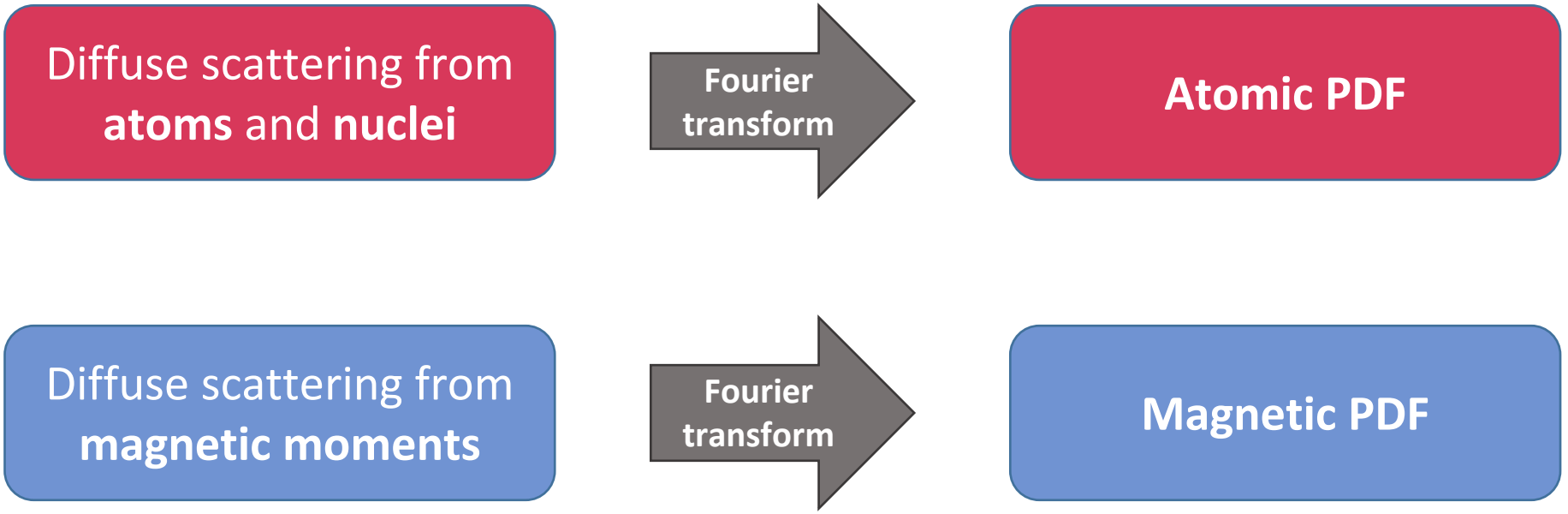
How can we analyze diffuse magnetic scattering?



Expression for the magnetic PDF

Magnetic PDF is defined analogously to the atomic PDF:

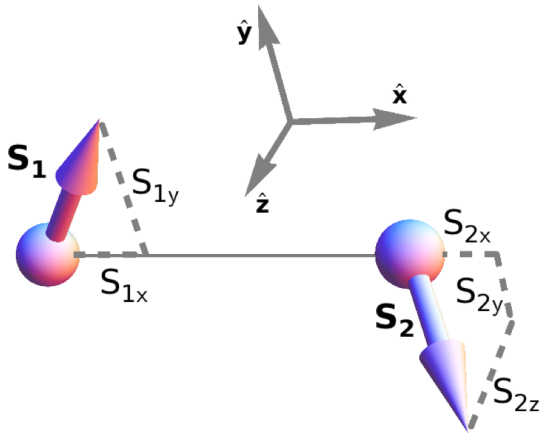
$$f(r) = \frac{2}{\pi}\int_0^\infty Q\,(S(Q)-1)\sin Qr\mathrm{d}Q$$

where $S(Q)$ is the *magnetic* structure function, given by

$$S(Q) = 1 + \frac{1}{N}\frac{3}{2S(S+1)}\sum_{i\neq j}\left[A_{ij}\frac{\sin Qr_{ij}}{Qr_{ij}} + B_{ij}\left(\frac{\sin Qr_{ij}}{(Qr_{ij})^3} - \frac{\cos Qr_{ij}}{(Qr_{ij})^2}\right)\right]$$

$$A_{ij} = S_i^y S_j^y$$

$$B_{ij} = 2S_i^x S_j^x - S_i^y S_j^y$$



This results in the following expression for the mPDF:

$$f(r) = \frac{1}{N}\frac{3}{2S(S+1)}\sum_{i\neq j}\left(\frac{A_{ij}}{r}\delta(r-r_{ij}) + B_{ij}\frac{r}{r_{ij}^3}\Theta(r_{ij}-r)\right)$$

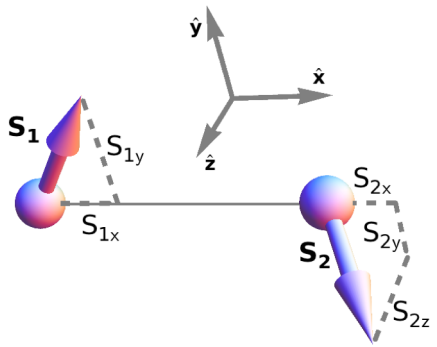
Simple examples of the mPDF

ATOMIC $$f(r) = \frac{1}{N\langle b\rangle^2}\sum_{i\neq j}\frac{b_i b_j}{r}\delta(r-r_{ij})$$

MAGNETIC $$f(r) = \frac{1}{N}\frac{3}{2S(S+1)}\sum_{i\neq j}\left(\frac{A_{ij}}{r}\delta(r-r_{ij}) + B_{ij}\frac{r}{r_{ij}^3}\Theta(r_{ij}-r)\right)$$

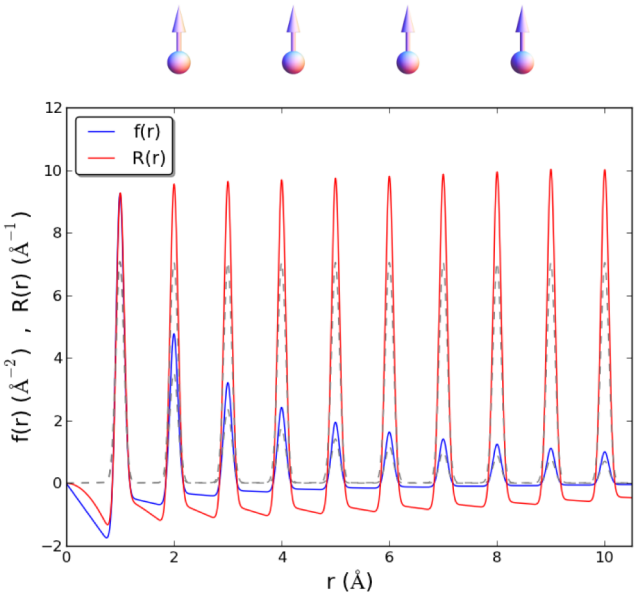
$$A_{ij} = S_i^y S_j^y$$

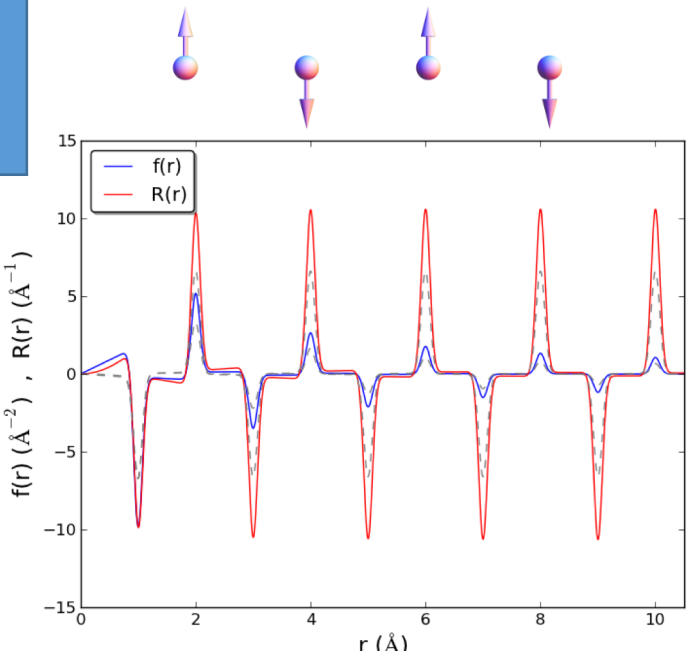
$$B_{ij} = 2S_i^x S_j^x - S_i^y S_j^y$$



Positive peaks for *ferromagnetic* correlations

Negative peaks for *antiferromagnetic* correlations

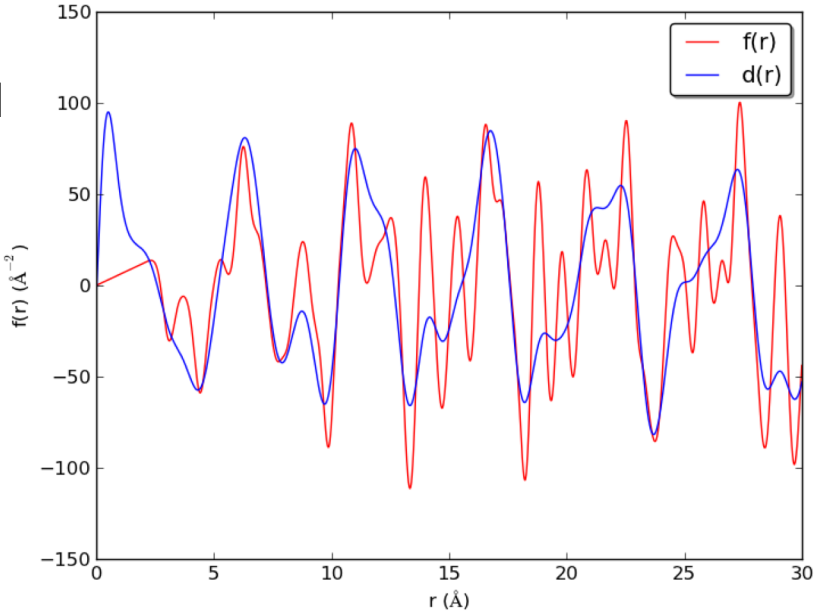




Frandsen et al, *Acta A* **70**, 3-11 (2014)

What we get in a standard atomic PDF measurement

In a typical neutron PDF measurement, the **magnetic and nuclear scattering are mixed together.** Standard data reduction routines normalized by the average squared nuclear scattering length, but not the magnetic form factor. As a result, the **mPDF obtained from the standard procedures is a *broadened form* of the true expression:**



$$G_{\mathrm{mag}} \approx f(r) * S(r)$$

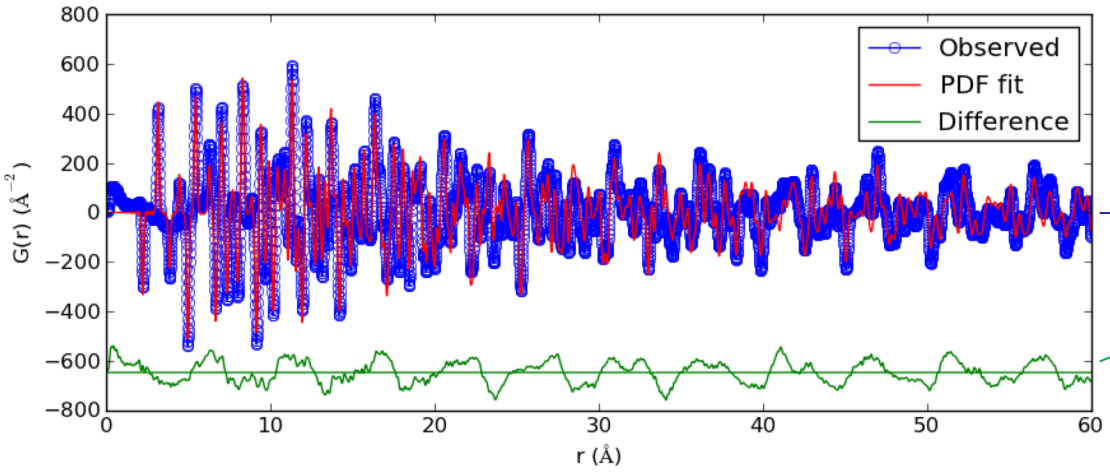
Proper mPDF

Convolution

Approximately $\sqrt{2}\times$ the real-space extent of the magnetic moment

Red = proper mPDF
Blue = broadened mPDF

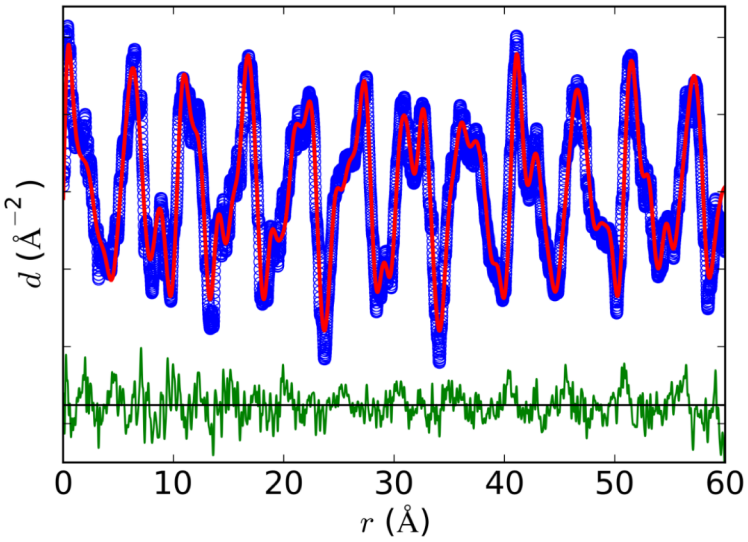
Example: Antiferromagnetic MnO



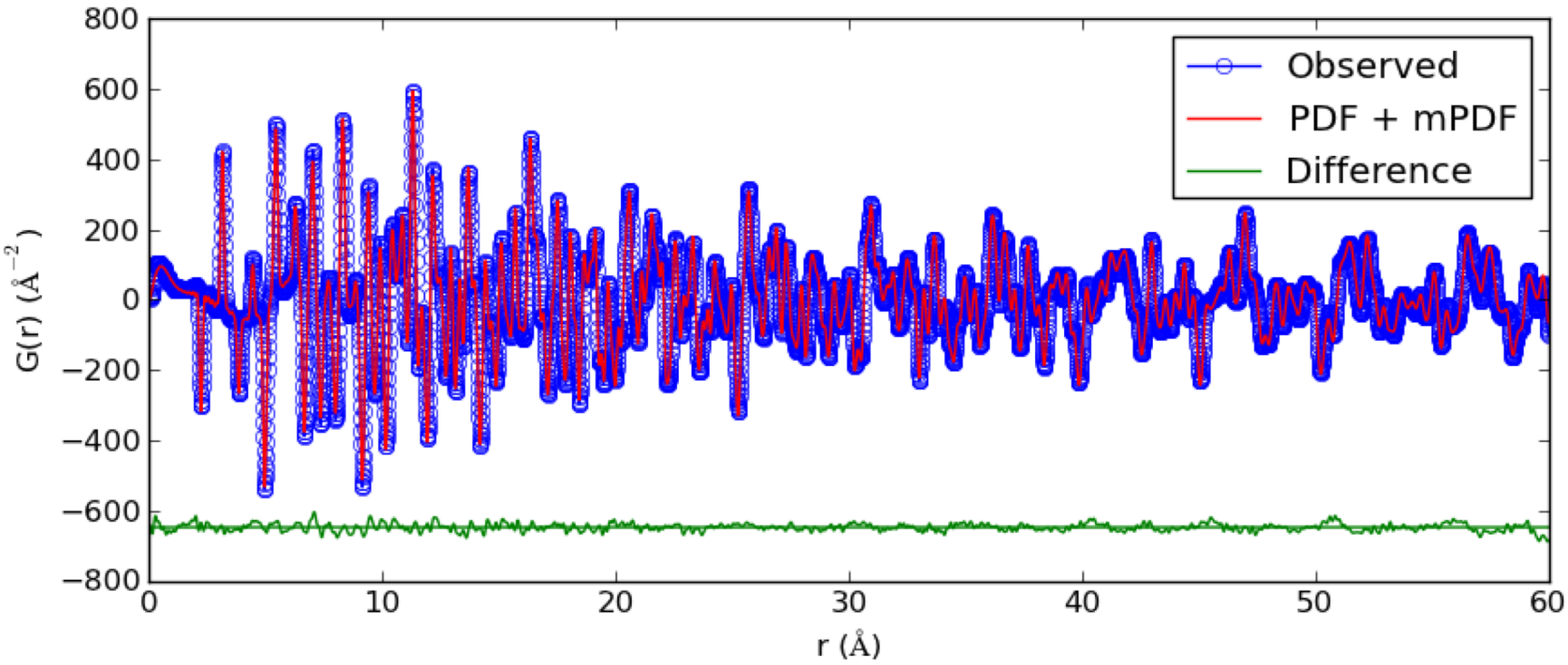
Fit includes only the atomic PDF.

Fit residual contains the mPDF.

Known antiferromagnetic structure of MnO provides excellent fit to the mPDF.

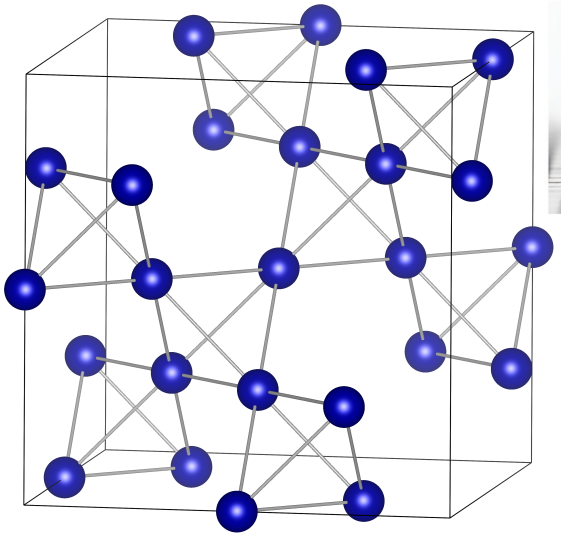


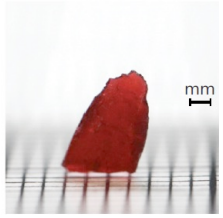
Example: Antiferromagnetic MnO

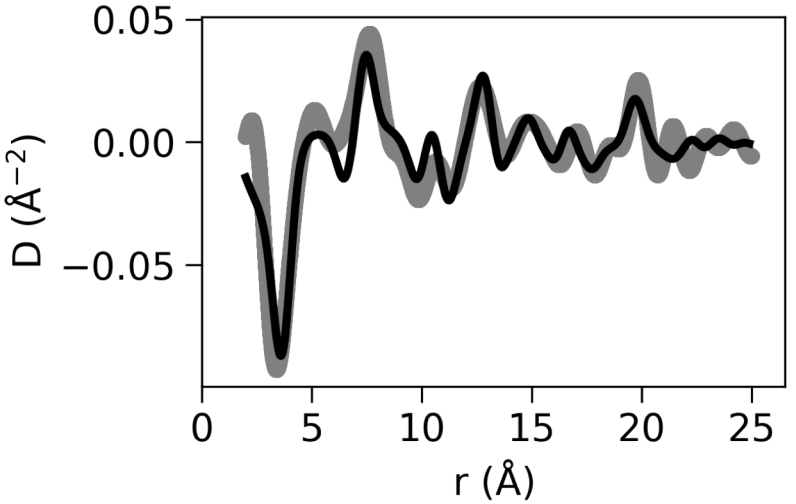


Total PDF fit (atomic + magnetic)

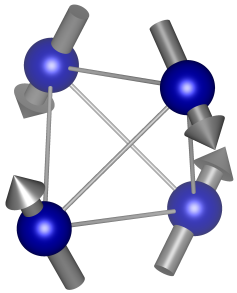
Exotic example: New frustrated pyrochlore system $NaCaCo_2F_7$







mPDF analysis reveals a non-trivial, non-collinear magnetic configuration with a correlation length of ≈2 nm.



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Summary

- DiffPy-CMI – software framework for complex modeling, structure representations and manipulations, calculators for PDF, BVS, SAS, multi-component fit management, add-on for magnetic PDF analysis
- **resources:** https://www.diffpy.org – project webpage
  diffpy-users Google Group – users mailing list
- **installation on Linux or Mac**
  - download Anaconda Python – https://anaconda.com
  - install diffpy-cmi from "conda" package manager

```
conda install –c diffpy diffpy–cmi
```

- **installation on Windows** – requires Linux Virtual Machine

DiffPy-CMI tutorial

https://github.com/diffpy/add2019-diffpy-cmi

- **what to expect:**
  - IPython notebooks that proceed from basic functions to refinements not possible in PDFgui.
- **goals:**
  - learn about the main components of the framework and understand how they work together to define refinements.
  - perform structure manipulations and simple calculations
  - learn what structure analysis problems are suitable for DiffPy-CMI