

The interpretation of broad diffuse maxima using superspace crystallography

Introducing disorder in superspace

Ella M. Schmidt and Reinhard B. Neder

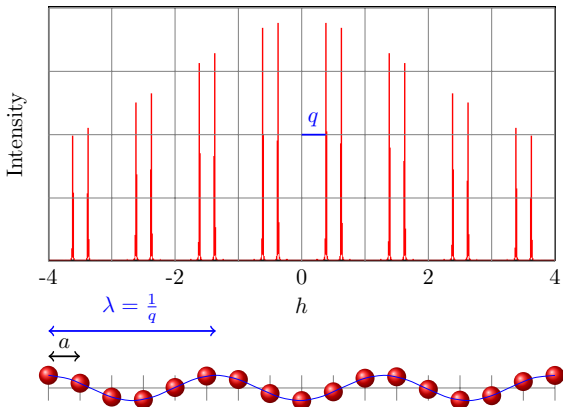
Institute for Crystallography and Structural Physics, FAU Erlangen-Nürnberg

March 20, 2019





Diffraction pattern of a modulated structure



Bragg reflections:

$$h \in \mathbb{Z}$$

Satellite reflections:

$$h \pm mq$$

$$h, m \in \mathbb{Z}$$



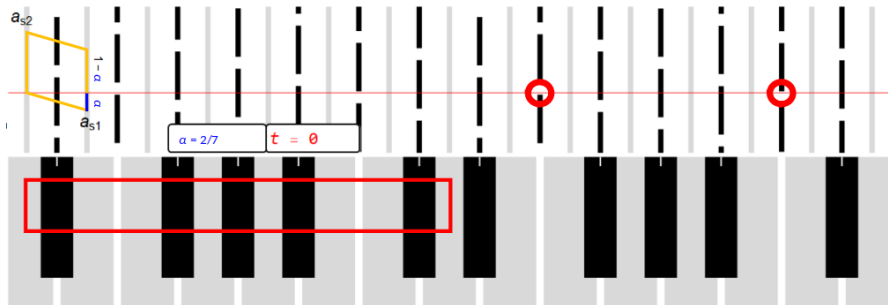
Introduction to superspace crystallography

- Describe long-range ordered, modulated structure with few parameters
- Modulation wave vector $\mathbf{q} = (\alpha, \beta, \gamma)$



Paul B. Klar
FZU Prague

1-dimensional piano structure in two-dimensional superspace:





Introduction to superspace crystallography

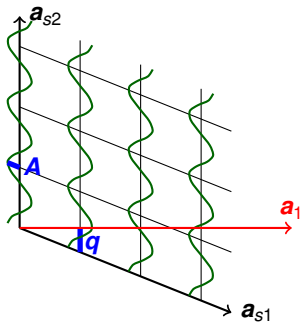
Changing α \leftrightarrow Changing real space periodicity



Paul B. Klar
FZU Prague



Introduction to superspace crystallography



Modulation function:

$$u_+(\mathbf{a}) = A \cos(2\pi \mathbf{q} \mathbf{a})$$

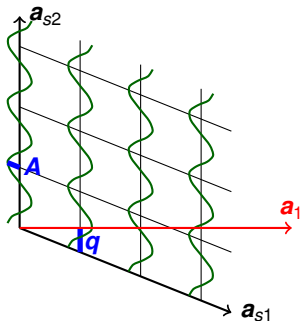
- \mathbf{q} determines periodicity of modulation function
- A determines the amplitude of the modulation function
- Modulation functions for occupancy and/or displacement
- Real space \equiv cut through direct superspace
- Reciprocal space \equiv projection of reciprocal superspace

[1] Van Smaalen, Sander. *Incommensurate crystallography*. Vol. 21. Oxford University Press, 2007.

[2] Janssen, Ted, Gervais Chapuis, and Marc De Boissieu. *Aperiodic Crystals: From Modulated Phases to Quasicrystals: Structure and Properties*. Oxford University Press, 2018.

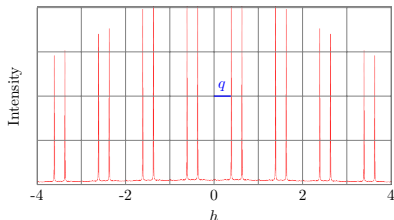


Introduction to superspace crystallography



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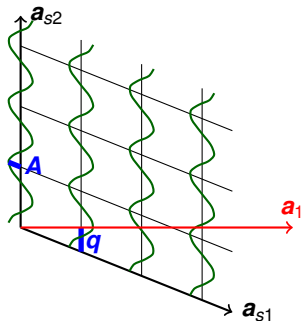


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(1+1)D ordered superspace

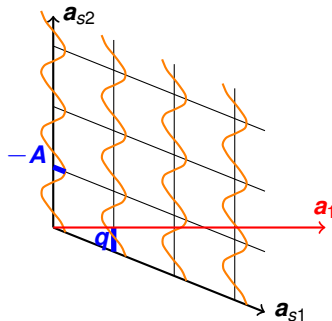


Modulation function:

$$u_+(\mathbf{a}) = A \cos(2\pi \mathbf{q} \mathbf{a})$$

→ Sharp satellite reflections at $h \pm q$.

→ $u_+(\mathbf{a})$ and $u_-(\mathbf{a})$ cannot be distinguished experimentally.

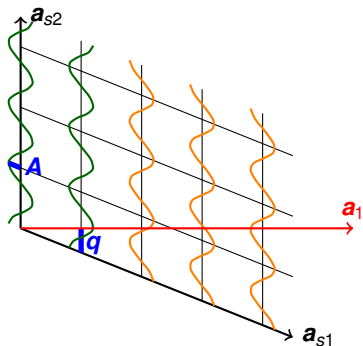


Modulation function:

$$u_-(\mathbf{a}) = -A \cos(2\pi \mathbf{q} \mathbf{a})$$



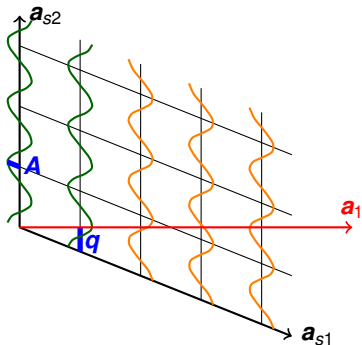
(1+1)D disordered superspace



- Phase domains in superspace
- Positive correlation along a_{s1}
- Warren-Cowley short range order parameter $\alpha_1^s > 0$



(1+1)D disordered superspace



- Phase domains in superspace
- Positive correlation along \mathbf{a}_{s1}
- Warren-Cowley short range order parameter $\alpha_1^s > 0$
- What happens to the sharp satellite reflections?



(1+1)D disordered superspace - Occupational disorder

- Au/Ag 1:1 crystal
- $q = \frac{\sqrt{7}}{7} \approx 0.378$
- Modulation functions:
$$\rho_+(a) = 0.5 + 0.5 \cos(2\pi qa)$$
$$\rho_-(a) = 0.5 - 0.5 \cos(2\pi qa)$$
- Create disordered super space structure with
 - positive correlation ($\alpha_1^s = 0.85$)
 - $m_{p_+} = m_{p_-} = 0.5$

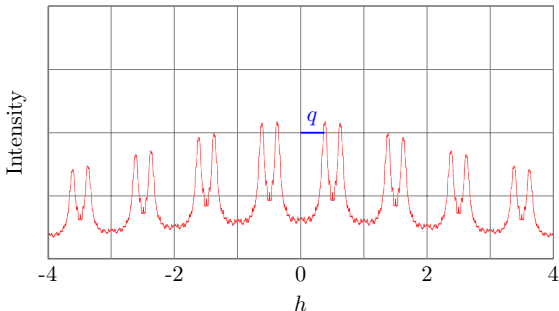


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$$q = \frac{\sqrt{7}}{7}$$

$$\alpha_1^s = 0.85$$





(1+1)D disordered superspace - Peak position tuning

1D occupational disorder

- $\rho_+(a) = 0.5 + 0.5 \cos(2\pi qa)$
- $\rho_-(a) = 0.5 - 0.5 \cos(2\pi qa)$



(1+1)D disordered superspace - Peak width tuning

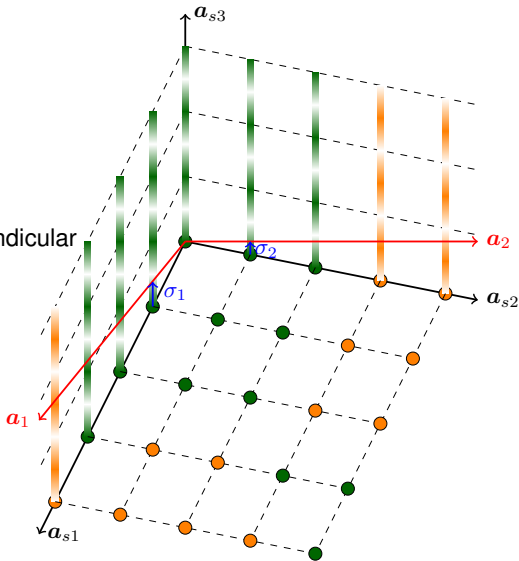
1D occupational disorder

- $p_+(a) = 0.5 + 0.5 \cos(2\pi qa)$
- $p_-(a) = 0.5 - 0.5 \cos(2\pi qa)$



(2+1)D superspace for occupational modulations

- 2D basic structure
- $\mathbf{q} = (\sigma_1, \sigma_2)$
- Internal dimension \mathbf{a}_{s3} perpendicular to external, physical space spanned by \mathbf{a}_1 and \mathbf{a}_2



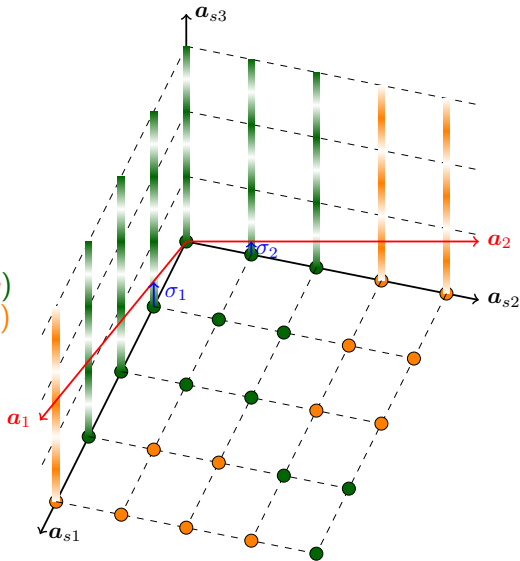


(2+1)D disordered superspace - Occupational disorder

- 2D basic structure
- $\mathbf{q} = (\sigma_1, \sigma_2)$
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$$\rho_+(\mathbf{a}) = 0.5 + 0.5 \cos(2\pi \mathbf{q} \mathbf{a})$$

$$\rho_-(\mathbf{a}) = 0.5 - 0.5 \cos(2\pi \mathbf{q} \mathbf{a})$$





(2+1)D disordered superspace - Occupational disorder

- Au/Ag 1:1 crystal
- Plane space group $p1$
- One atom per unit-cell at $(0, 0)$



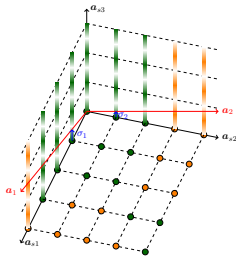
(2+1)D disordered superspace - Occupational disorder

- Au/Ag 1:1 crystal
- Plane space group $p1$
- One atom per unit-cell at $(0, 0)$
- $\mathbf{q} = \left(\frac{1}{2}, \frac{\sqrt{7}}{7}\right)$
- Modulation functions:
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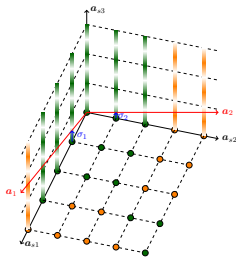
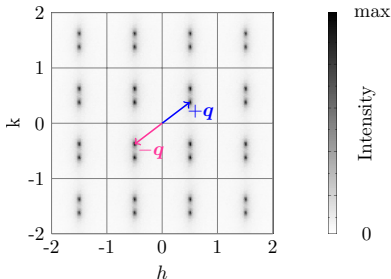
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- Create disordered super space structure with
 - positive correlations along \mathbf{a}_{s1} and \mathbf{a}_{s2} :
 $\alpha_{(1,0)}^s = 0.7$
 $\alpha_{(0,1)}^s = 0.5$
 - $m_{p_+} = m_{p_-} = 0.5$





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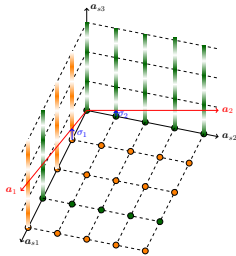
(2+1)D disordered superspace - Diffuse rods

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(2+1)D disordered superspace - Diffuse rods

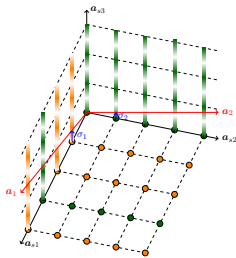
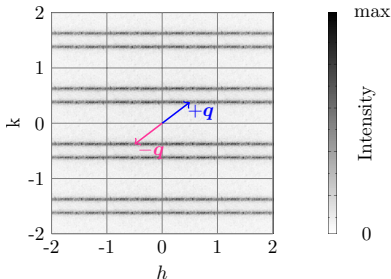
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- Create disordered super space structure with
 - positive correlation along \mathbf{a}_{s2} :
 $\alpha_{(1,0)}^S = 0.0$
 $\alpha_{(0,1)}^S = 0.95$
 - $m_{p+} = m_{p-} = 0.5$





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(2+1)D disordered superspace - Reflection condition

- Basic structure shows symmetry
- Superspace group $pg1\left(\frac{1}{2}, \beta\right)$
 - Site A: $(+0.1, 0.0)$
 - Site B: $(-0.1, 0.5)$



(2+1)D disordered superspace - Reflection condition

- Basic structure shows symmetry

- Superspace group $pg1\left(\frac{1}{2}, \beta\right)$

Site A: $(+0.1, 0.0)$

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- Bragg-reflection condition:

$$0k : \quad k = 2n$$

- $\mathbf{q} = \left(\frac{1}{2}, \frac{\sqrt{7}}{7}\right)$

- $\alpha_{(1,0)}^s = \alpha_{(0,1)}^s = 0.7$

- Modulation functions:

$$\text{Site A: } \rho_A(\mathbf{a}_A) = 0.5 \pm 0.5 \cos(2\pi(+\sigma_1 a_{1A} + \sigma_2 a_{2A}))$$

$$\text{Site B: } \rho_B(\mathbf{a}_B) = 0.5 \pm 0.5 \cos(2\pi(-\sigma_1 a_{1B} + \sigma_2 a_{2B} + a_{2B}))$$

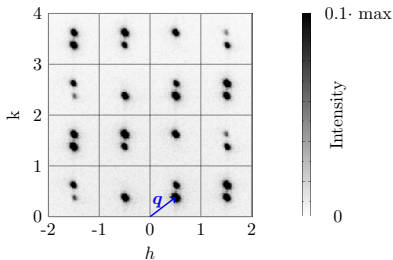


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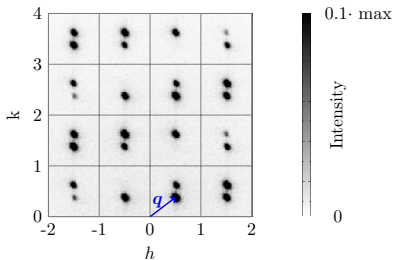
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→ Diffuse satellites obey reflection condition as well



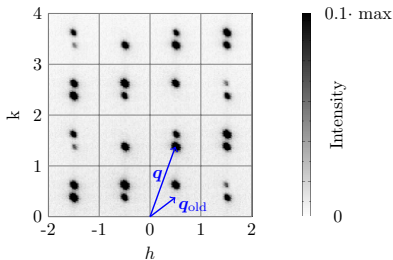


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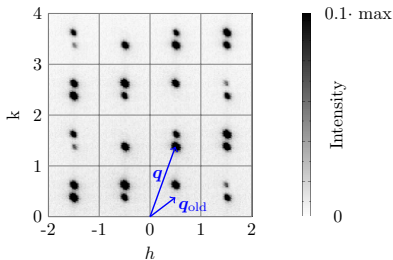
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→ Diffuse satellites obey reflection condition as well

→ "Virtual" satellite reflection condition $0k : k = 2n + 1$





(2+1)D disordered superspace - Reflection condition

- Basic structure shows symmetry
- Superspace group $pm1(0, \beta)s_0$ with internal translation
Site A: $(+0.1, 0.0)$
Site B: $(-0.1, 0.0)$

- Satellite-reflection condition^a:

$$hk : \quad h \neq 0$$

- $\mathbf{q} = \left(0, \frac{\sqrt{7}}{7}\right)$

- $\alpha_{(1,0)} = \alpha_{(0,1)} = 0.7$

- Modulation functions:

$$\text{Site A: } \rho_A(\mathbf{a}_A) = 0.5 \pm 0.5 \cos(2\pi(\sigma_2 a_{2A}))$$

$$\text{Site B: } \rho_B(\mathbf{a}_B) = 0.5 \pm 0.5 \cos(2\pi(\sigma_2 a_{2B} + \frac{1}{2}))$$

^a For occupational modulation functions with only one cosine-term.



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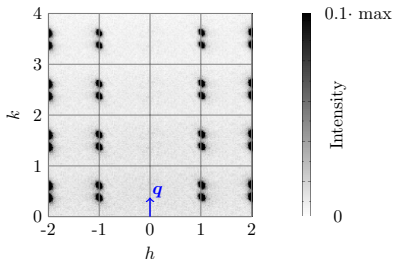
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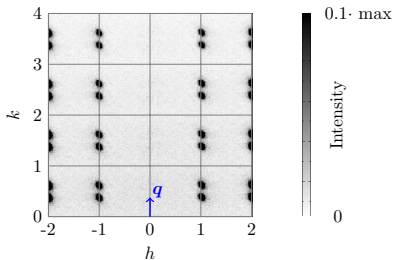
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→ Diffuse satellites obey superspace reflection condition as well

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Possible applications

Diffuse rods showing 'extinction conditions'



research papers

Acta Crystallographica Section B
**Structural
Science**
ISSN 0108-7681

Symmetry analysis of extinction rules in diffuse-scattering experiments

R. L. Withers,^a M. I. Aroyo,^{b,c}
J. M. Perez-Mato^b and D.
Orbenga^{a*}

Structured diffuse-scattering intensities, whether of compositional or of pure displacive origin, static or dynamic, contain important information about the symmetry of the individual compositional and/or displacive modes responsible for the observed intensities. However, the interpretation of the

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Sharp satellites on top of diffuse scattering



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research papers

Exploiting superspace to clarify vacancy and Al/Si ordering in mullite

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Phase transitions: Diffuse scattering turns into sharp satellite reflections

Inorganic Chemistry

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Article

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The Mystery of the Auln 1:1 Phase and Its Incommensurate Structural Variations

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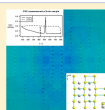
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ABSTRACT: In this communication, the Auln 1:1 phase (Naturwissenschaften, 1953, 40, 437; DOI: 10.1007/BF00390353), and its ordering behavior at various temperatures is investigated. To enable the growth of a X-ray suitable specimen, a tempering routine was established by the interpretation of a differential scanning calorimetry (DSC) study. In this way, good quality single crystals were grown and measured at the Crystal Beamline at Synchrotron SOLEIL. From the acquired data, three variations of this structure could be found at temperatures of 400 °C and 300 °C and room temperature, with differing degrees of incommensurate modulation. Diffuse scattering found at 400 °C was interpreted with the help of a three-dimensional difference pair distribution function (3D-ΔPDF) study.





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- Unified model for diffuse maxima at different positions in reciprocal space
- Superspace symmetry allows direct access to extinction rules



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- Possible peak shape tuning by introducing size-effect like relaxations?
- Possibility of curved diffuse features?



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