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# **Use of the structural programs of the Bilbao Crystallographic Server. Group-subgroup relations. The program SUBGROUPS**

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**Why we have to derive and work with  
SUBGROUPS?: DISTORTED STRUCTURES**

**Symmetry break:  $G \rightarrow F$  (subgroup of G)**

## Structure Utilities

**CELLTRAN**

Transform Unit Cells

**STRAIN**

Strain Tensor Calculation

**WPASSIGN**

Assignment of Wyckoff Positions

**TRANSTRU**

Transform structures.

**SETSTRU**

Alternative Settings for a given Crystal Structure

**EQUIVSTRU**

Equivalent Descriptions for a given Crystal Structure

**CIF2STANDARD**

Transforms a given CIF (in any setting) to that of standard setting

**STRCONVERT**

Convert & Edit Structure Data  
(supports the CIF, mCIF, VESTA, VASP formats -- with magnetic information where available)

**VISUALIZE**

Visualize structures using Jmol

**COMPSTRU**

Comparison of Crystal Structures with the same Symmetry

**STRUCTURE**

Evaluation of structure relationships [transformation matrix] between  
group-subgroup related phases

**RELATIONS**

Pseudosymmetry of a lattice and compatible supergroups

# Structure transformation to a lower symmetry

## TRANSTRU

### Transform Structure

TRANSTRU can transform a structure in two ways:

- To a lower symmetry space group. The transformed structure is given in the low symmetry space group basis, taking care of all possible splittings of the Wyckoff positions.
- With an arbitrary matrix. The structure, including the cell parameters and the atoms in the unit cell, is transformed with an arbitrary matrix introduced by the user.

Only the **default choice** for the conventional setting of the space groups is used.

Structure Data [Examinar...](#) No se ha seleccionado ningún archivo.  
[in CIF format] HINT: [ The option for a given filename is preferential ]

High Symmetry Structure

```
# Space Group ITA number  
221  
# Lattice parameters  
5.0 5.0 5.0 90 90 90  
# Number of independent atoms in the asymmetric unit  
3  
# [atom type] [number] [WP] [x] [y] [z]  
Ba 1 1a 0.0 0.0 0  
Ti 2 1b 0.5 0.5 0.5  
O 3 3c 0.5 0.0 0.5
```

CIF file

BCS format

Transform structure to a subgroup basis

Transform structure with an arbitrary matrix

Show

To transform a structure described in standard setting to a lower symmetry space group or with an arbitrary matrix

# Structure transformation to a lower symmetry

**TRANSTRU**

**Transform Structure**

**Transform Structure**

TRANSTRU transforms the structure to the low symmetry space group basis, taking care of all possible splittings of the Wyckoff positions.

Structure

|     |        |        |          |          |          |          |
|-----|--------|--------|----------|----------|----------|----------|
| 221 | 3.9064 | 3.9064 | 3.9064   | 90.      | 90.      | 90.      |
| 3   |        |        |          |          |          |          |
| La  | 1      | 1a     | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| Co  | 1      | 1b     | 0.500000 | 0.500000 | 0.500000 | 0.500000 |
| 0   | 1      | 3c     | 0.500000 | 0.500000 | 0.500000 | 0.000000 |

Low symmetry Space Group ITA number

Transformation Matrix:

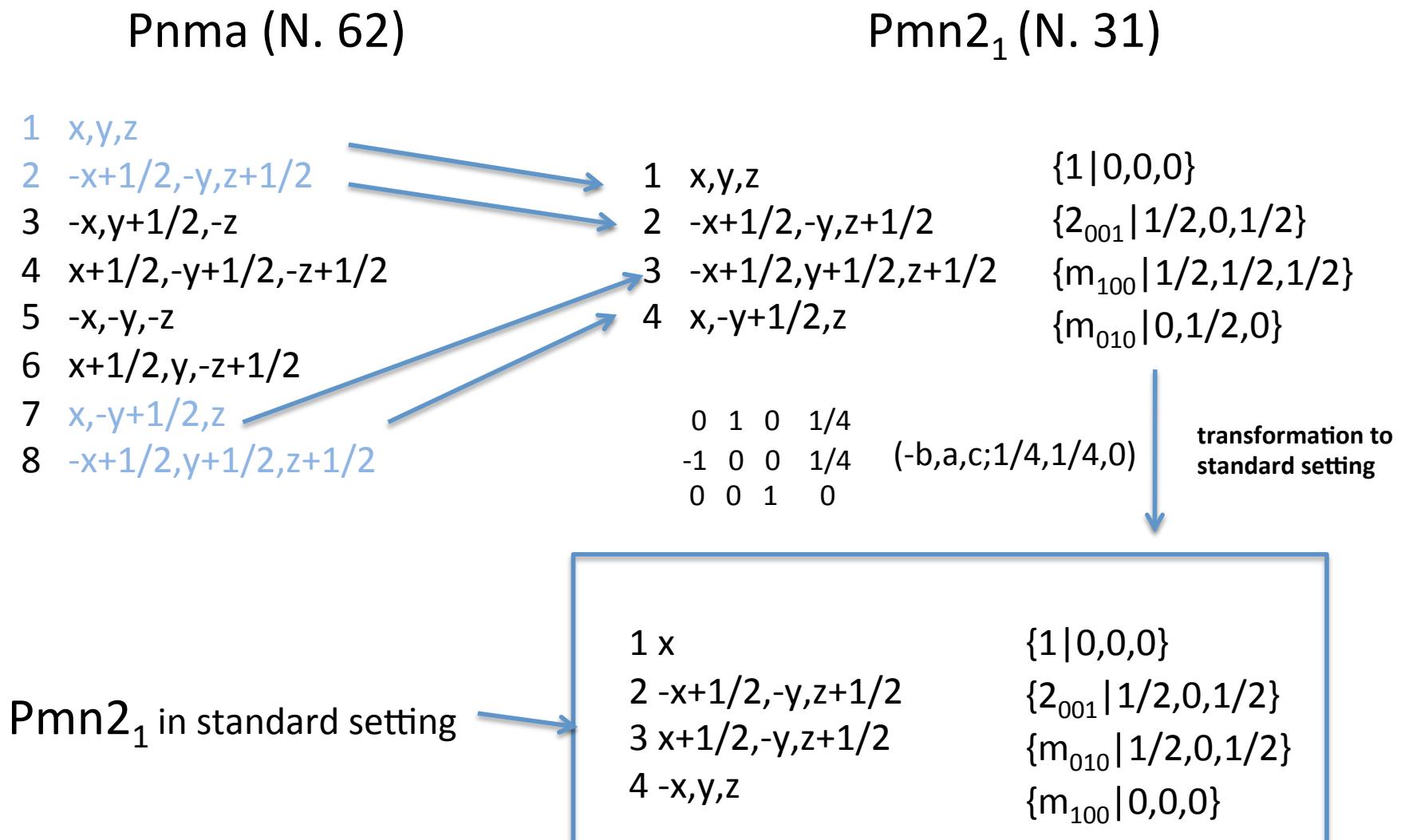
In matrix form:

|             |    |    |              |
|-------------|----|----|--------------|
| Linear part |    |    | Origin Shift |
| 1           | -1 | -1 | 0            |
| 2           | 0  | 0  | 1/2          |
| -1          | -1 | 1  | 1/2          |

Show

Transformation of the set of operations forming the subgroup to standard setting of the space group

## What is a subgroup of a space group and how we define it:



# Structure transformation to a lower symmetry

**SrZrO<sub>3</sub>** Pnma (N. 62)

62  
 5.8206 8.1949 5.8045 90 90 90  
 4  
 Sr 1 4c 0.524000 0.250000 0.004000  
 Zr 1 4a 0.000000 0.000000 0.000000  
 O 1 4c -0.013000 0.250000 -0.069000  
 O 2 8d 0.284000 0.036000 0.215000

7 free parameters

Wyckoff Positions of Group **Pnma** (No. 62)

| Multiplicity | Wyckoff letter | Site symmetry | Coordinates  |
|--------------|----------------|---------------|--|
| 8            | d              | 1             | (x,y,z) (-x+1/2,-y,z+1/2) (-x,y+1/2,-z) (x+1/2,-y+1/2,-z+1/2)<br>(-x,-y,-z) (x+1/2,y,-z+1/2) (x,-y+1/2,z) (-x+1/2,y+1/2,z+1/2) |
| 4            | c              | .m.           | (x,1/4,z) (-x+1/2,3/4,z+1/2) (-x,3/4,-z) (x+1/2,1/4,-z+1/2)  |
| 4            | b              | -1            | (0,0,1/2) (1/2,0,0) (0,1/2,1/2) (1/2,1/2,0)  |
| 4            | a              | -1            | (0,0,0) (1/2,0,1/2) (0,1/2,0) (1/2,1/2,1/2)  |

**SrZrO<sub>3</sub>** Pmn2<sub>1</sub> (N. 31)

031  
 8.194900 5.820600 5.804500 90.000000 90.000000  
 90.000000  
 7  
 Sr 1 2a 0.000000 0.274000 0.004000  
 Sr 1\_2 2a 0.500000 0.226000 0.996000  
 Zr 1 4b 0.250000 0.750000 0.000000  
 O 1 2a 0.000000 0.737000 0.931000  
 O 1\_2 2a 0.500000 0.763000 0.069000  
 O 2 4b 0.214000 0.034000 0.215000  
 O 2\_2 4b 0.714000 0.466000 0.785000

Wyckoff Positions of Group **Pmn2<sub>1</sub>** (No. 31)

| Multiplicity | Wyckoff letter | Site symmetry | Coordinates   |
|--------------|----------------|---------------|---|
| 4            | b              | 1             | (x,y,z) (-x+1/2,-y,z+1/2) (x+1/2,-y,z+1/2) (-x,y,z) |
| 2            | a              | m..           | (0,y,z) (1/2,-y,z+1/2)                              |

# Structure transformation to a lower symmetry

**SrZrO<sub>3</sub>** Pnma (N. 62)

62

5.8206 8.1949 5.8045 90 90 90

4

Sr 1 4c 0.524000 0.250000 0.004000 **2**

Zr 1 4a 0.000000 0.000000 0.000000 **0**

O 1 4c -0.013000 0.250000 -0.069000 **2**

O 2 8d 0.284000 0.036000 0.215000 **6**

**7 free parameters**

**Wyckoff Positions of Group Pnma (No. 62)**

| Multiplicity | Wyckoff letter | Site symmetry | Coordinates  |
|--------------|----------------|---------------|--|
| 8            | d              | 1             | (x,y,z) (-x+1/2,-y,z+1/2) (-x,y+1/2,-z) (x+1/2,-y+1/2,-z+1/2)<br>(-x,-y,-z) (x+1/2,y,-z+1/2) (x,-y+1/2,z) (-x+1/2,y+1/2,z+1/2) |
| 4            | c              | .m.           | (x,1/4,z) (-x+1/2,3/4,z+1/2) (-x,3/4,-z) (x+1/2,1/4,-z+1/2)  |
| 4            | b              | -1            | (0,0,1/2) (1/2,0,0) (0,1/2,1/2) (1/2,1/2,0)  |
| 4            | a              | -1            | (0,0,0) (1/2,0,1/2) (0,1/2,0) (1/2,1/2,1/2)  |

**SrZrO<sub>3</sub>** Pmn2<sub>1</sub> (N. 31)

031

8.194900 5.820600 5.804500 90 90 90

7

Sr 1 2a 0.000000 0.274000 0.004000 **2**

Sr 1\_2 2a 0.500000 0.226000 0.996000 **2**

Zr 1 4b 0.250000 0.750000 0.000000 **3**

O 1 2a 0.000000 0.737000 0.931000 **2**

O 1\_2 2a 0.500000 0.763000 0.069000 **2**

O 2 4b 0.214000 0.034000 0.215000 **3**

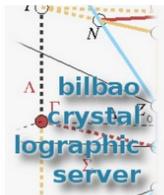
O 2\_2 4b 0.714000 0.466000 0.785000 **3**

**17 free parameters**

**Wyckoff Positions of Group Pmn2<sub>1</sub> (No. 31)**

| Multiplicity | Wyckoff letter | Site symmetry | Coordinates   |
|--------------|----------------|---------------|---|
| 4            | b              | 1             | (x,y,z) (-x+1/2,-y,z+1/2) (x+1/2,-y,z+1/2) (-x,y,z) |
| 2            | a              | m..           | (0,y,z) (1/2,-y,z+1/2)                              |

[www.cryst.ehu.es](http://www.cryst.ehu.es)



**Crystallography Online:  
Workshop on the use of  
the structural and  
magnetic tools of the  
Bilbao Crystallographic  
Server**  
27 June - 1 July 2022, Leioa (Spain)

Forthcoming schools and  
workshops

**News:**

- **Space-group symmetry**  
05/2022: The monoclinic and tetragonal ITA-settings database has been completed.
- **New Article**  
04/2022: Regnault *et al.* "Catalogue of flat-band stoichiometric materials". *Nature* (2022) **603**, 824-828
- **New version of B-IncStrDB**  
02/2022: New version of the data-base of incommensurate structures.
- **New upload option in MAGNDATA**  
10/2021: New feature that permits anyone to submit to this database any published magnetic structure not yet included in the collection.

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Space-group symmetry

Magnetic Symmetry and Applications

Group-Subgroup Relations of Space Groups

Representations and Applications

Solid State Theory Applications

Structure Utilities

Topological Quantum Chemistry

Subperiodic Groups: Layer, Rod and Frieze Groups

Structure Databases

Raman and Hyper-Raman scattering

Point-group symmetry

Plane-group symmetry

Double point and space groups



## Group-Subgroup Relations of Space Groups

|                  |   |
|------------------|---|
| SUBGROUPGRAPH    | Lattice of Maximal Subgroups  |
| HERMANN          | Distribution of subgroups in conjugated classes   |
| COSETS           | Coset decomposition for a group-subgroup pair   |
| WYCKSPLIT        | The splitting of the Wyckoff Positions  |
| MINSUP           | Minimal Supergroups of Space Groups   |
| SUPERGROUPS      | Supergroups of Space Groups   |
| CELLSUB          | List of subgroups for a given k-index.  |
| CELLSUPER        | List of supergroups for a given k-index.  |
| NONCHAR          | Non Characteristic orbits.  |
| COMMONSUBS       | Common Subgroups of Space Groups  |
| COMMONSUPER      | Common Supergroups of Two Space Groups  |
| INDEX            | Index of a group subgroup pair  |
| <b>SUBGROUPS</b> | Subgroups of a space group consistent with some given supercell, propagation vector(s) or irreducible representation(s) |

B-IncStrDB  
02/2022: New version of the

Determine and explore online all possible symmetries that can result from the distortion of a parent structure of higher symmetry

database any published magnetic structure not yet included in the collection.

Double point and space groups

## SUBGROUPS: [https://www.cryst.ehu.es/cgi-bin/cryst/programs/subgrmag1\\_cell.pl](https://www.cryst.ehu.es/cgi-bin/cryst/programs/subgrmag1_cell.pl)

**Subgroups: Subgroups compatible with a given supercell or some propagation vector(s).**

The program *Subgroups* provides the possible subgroups of a space group which are possible for a given **supercell**. The program provides a list of the set of space groups or a graph showing the group-subgroup hierarchy, grouped into conjugacy classes. More optional information about the classes or subgroups is also given.

Other alternatives for the input of the program:

- Instead of the whole set of subgroups, the output can be limited to subgroups having a chosen common subgroup of lowest symmetry, common point group of lowest symmetry, or groups which belong to a specific crystal class.
- Instead of a supercell, a set of modulation wave vectors can be given, including complete or partial wave-vectors stars.
- The subgroups compatible with intermediate unit cells between the unit cell of the parent space group and the given supercell (or the supercell determined by the given wave vector(s) when the previous option is used) can be included.
- When a set of wave-vectors is used as input, the output can be further refined introducing the Wyckoff positions of the atoms and/or a set of irreducible representations.

Tutorial \_SUBGROUPS: [download](#)

See the [Help](#) for details.

Enter the serial number of the space group:

**Introduce the supercell**

Alternatively give the modulation wave-vectors

|                                  |                                  |                                  |
|----------------------------------|----------------------------------|----------------------------------|
| $a_s =$                          | $b_s =$                          | $c_s =$                          |
| <input type="text" value="1"/> a | <input type="text" value="0"/> a | <input type="text" value="0"/> a |
| +                                | +                                | +                                |
| <input type="text" value="0"/> b | <input type="text" value="1"/> b | <input type="text" value="0"/> b |
| +                                | +                                | +                                |
| <input type="text" value="0"/> c | <input type="text" value="0"/> c | <input type="text" value="1"/> c |

The supercell is centred:

Include the subgroups compatible with intermediate cells.  
(It is not applied when only the maximal subgroups are calculated)

Optional: refine further the subgroups of the output giving the Wyckoff positions of the atoms  
Give the Wyckoff positions

Optional: Show only subgroups that can be the result of a Landau-type transition (single irrep order parameter).

Possible limitations of the subgroup list.  
(Check only one option on the left and the specific value on the right)  
(Check only one option on the left and the specific value on the right)

Lowest space group to consider

Lowest point group to consider

Lowest crystal system to consider

Only maximal subgroups

Further limitations considering physical properties of the point groups

- Only centrosymmetric/non centrosymmetric groups
- Only polar/non polar groups
- Only proper ferroelastic phase transitions

List of subgroups  Graph of subgroups

Parent space group

Supercell or modulation wave-vector

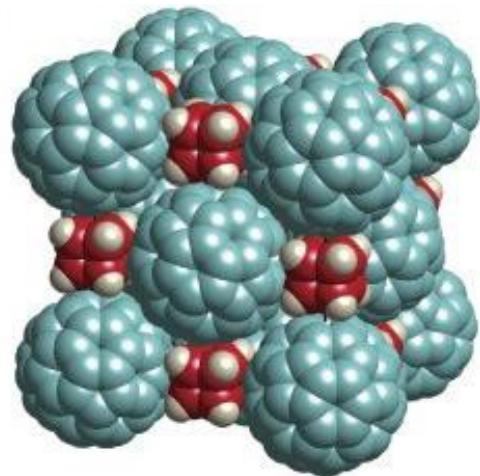
Minimal input

Other alternatives to filtered the results of the program

# Example : Fullerene-cubane crystal

High-temperature phase

Fm-3m(No. 225)



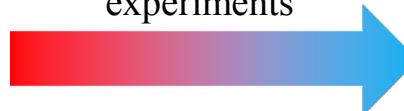
*Nature Mat.* **4**, 764 (2005)

Disordered fullerenes molecules **4a** (0,0,0)

Disordered cubane molecules **4b** (1/2,1/2,1/2)

Low-temperature phase

Power diffraction  
experiments



?

Orthorhombic structure

$$a_o \approx b_o \approx \frac{a_c}{\sqrt{2}} ; c_o \approx 2a_c$$

*phys. stat. sol. (b)* 243, 2999 (2006)

*J. Phys. Chem. B* **113**, 2042 (2009)

Possible symmetry (space group) of the detected low symmetry phase?

# Example : Fullerene-cubane crystal

**Subgroups:** Subgroups compatible with a given supercell or some propagation vector(s).

**Subgroups**

The program *Subgroups* provides the possible subgroups of a space group which are possible for a given [supercell](#). The program provides a list of the set of space groups or a graph showing the group-subgroup hierarchy, grouped into conjugacy classes. More optional information about the classes or subgroups is also given.

Other alternatives for the input of the program:

- Instead of the whole set of subgroups, the output can be limited to subgroups having a chosen common subgroup of lowest symmetry, common point group of lowest symmetry, or groups which belong to a specific crystal class.
- Instead of a supercell, a set of modulation wave vectors can be given, including complete or partial

Enter the serial number of the space group:  choose it

Introduce the supercell

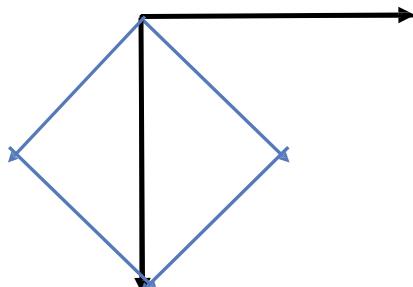
Alternatively give the modulation wave-vectors

|  |   |   |
|--|---|---|
| $a_s =$  | $b_s =$   | $c_s =$   |
| $\begin{matrix} 1/2 \\ + \\ -1/2 \\ + \\ 0 \end{matrix} a$ | $\begin{matrix} 1/2 \\ + \\ 1/2 \\ + \\ 0 \end{matrix} b$ | $\begin{matrix} 0 \\ + \\ 0 \\ + \\ 2 \end{matrix} c$ |

The supercell is centred:

...

List of subgroups       Graph of subgroups



# Example : Fullerene-cubane crystal

List of subgroups that fulfill the given conditions

| N  | Group Symbol           | Transformation matrix   | Group-Subgroup index | Other members of the Conjugacy Class           | irreps                                    |
|----|------------------------|---|----------------------|--|---|
| 1  | $P4_2/nmc$ (No. 138)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$   | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 2  | $P4_2/nmc$ (No. 137)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/4 \end{pmatrix}$    | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 3  | $P4_2/mcm$ (No. 132)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$      | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 4  | $P4_2/mmc$ (No. 131)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix}$        | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 5  | $P4/ncc$ (No. 130)     | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/4 \end{pmatrix}$    | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 6  | $P4/nmm$ (No. 129)     | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$   | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 7  | $P4/mcc$ (No. 124)     | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix}$        | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 8  | $P4/mmm$ (No. 123)     | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$      | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 9  | $P\bar{4}c2$ (No. 116) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 1/4 \\ 0 & 0 & 2 & 1/4 \end{pmatrix}$  | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 10 | $P\bar{4}c2$ (No. 116) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix}$        | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 11 | $P\bar{4}m2$ (No. 115) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$ | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |

99 subgroups

Input data

Subgroups of the space group :  
Lowest space group to consider:  
Supercell given by:  
Centred supercell:

$Fm\bar{3}m$  (N. 225)  
 $P1$  (N. 1)  
(1/2,-1/2,0),(1/2,1/2,0),(0,0,2)  
No

|    |                    |   |          |  |   |
|----|--------------------|---|----------|--|---|
| 90 | $Pm$ (No. 6)       | $\begin{pmatrix} 1/2 & 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & -2 & 0 & 1/2 \end{pmatrix}$     | 96=4x24  | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 91 | $C2$ (No. 5)       | $\begin{pmatrix} 1 & 0 & 0 & 1/4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1/4 \end{pmatrix}$             | 96=4x24  | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 92 | $C2$ (No. 5)       | $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$               | 96=4x24  | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 93 | $P2_1$ (No. 4)     | $\begin{pmatrix} 1/2 & 1/2 & 0 & -1/8 \\ -1/2 & 1/2 & 0 & 1/8 \\ 0 & 0 & 2 & 1/4 \end{pmatrix}$ | 96=4x24  | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 94 | $P2_1$ (No. 4)     | $\begin{pmatrix} 1/2 & 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$       | 96=4x24  | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 95 | $P2$ (No. 3)       | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$      | 96=4x24  | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 96 | $P2$ (No. 3)       | $\begin{pmatrix} 1/2 & 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$       | 96=4x24  | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 97 | $P\bar{1}$ (No. 2) | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$   | 96=4x24  | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 98 | $P\bar{1}$ (No. 2) | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$      | 96=4x24  | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 99 | $P1$ (No. 1)       | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix}$        | 192=4x48 | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |

# Example: Fullerene-cubane crystal

Go back to the input page

Enter the serial number of the space group:  225

**Introduce the supercell**

Alternatively give the modulation wave-vectors

|  |   |   |
|--|---|---|
| $a_s =$  | $b_s =$   | $c_s =$                                       |
| $\begin{pmatrix} 1/2 \\ -1/2 \\ 0 \end{pmatrix} a$ | $\begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix} b$ | $\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} c$ |
| $+ \begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix}$  | $+ \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$     | $+ \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$ |

The supercell is centred:  P

**Possible limitations of the subgroup list.**

(Check only one option on the left and the specific value on the right)

(Check only one option on the left and the specific value on the right)

- Lowest space group to consider  1
- Lowest point group to consider  -----
- Lowest crystal system to consider  Orthorhombic
- Only maximal subgroups

| N | Group Symbol         | Transformation matrix   | Group-Subgroup index | Other members of the Conjugacy Class   | irreps |
|---|----------------------|---|----------------------|--|--------|
| 1 | $P4_2/nmc$ (No. 138) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$ | 12=4x3               | <input type="button" value="Conjugacy Class"/> <input type="button" value="Get irreps"/> |        |
| 2 | $P4_2/nmc$ (No. 137) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/4 \end{pmatrix}$  | 12=4x3               | <input type="button" value="Conjugacy Class"/> <input type="button" value="Get irreps"/> |        |
| 3 | $P4_2/mcm$ (No. 132) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$    | 12=4x3               | <input type="button" value="Conjugacy Class"/> <input type="button" value="Get irreps"/> |        |
| 4 | $P4_2/mmc$ (No. 131) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix}$      | 12=4x3               | <input type="button" value="Conjugacy Class"/> <input type="button" value="Get irreps"/> |        |
| 5 | $P4/ncc$ (No. 130)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/4 \end{pmatrix}$  | 12=4x3               | <input type="button" value="Conjugacy Class"/> <input type="button" value="Get irreps"/> |        |
| 6 | $P4/nmm$ (No. 129)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$ | 12=4x3               | <input type="button" value="Conjugacy Class"/> <input type="button" value="Get irreps"/> |        |

The list of subgroups is reduced from 99 to 62

Most of them can be discarded symmetry higher than orthorhombic

|    |                      |   |         |  |
|----|----------------------|---|---------|--|
| 60 | $P2_{12}12$ (No. 18) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$ | 48=4x12 | <input type="button" value="Conjugacy Class"/> <input type="button" value="Get irreps"/> |
| 61 | $P222_1$ (No. 17)    | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$  | 48=4x12 | <input type="button" value="Conjugacy Class"/> <input type="button" value="Get irreps"/> |
| 62 | $P222$ (No. 16)      | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$  | 48=4x12 | <input type="button" value="Conjugacy Class"/> <input type="button" value="Get irreps"/> |

Orthorhombic point groups: 222, mm2 or mmm

# Example: Fullerene-cubane crystal

List of subgroups that fulfill the given conditions

[Get the subgroup-graph](#)

Go back to the input page

Enter the serial number of the space group:  225

Introduce the supercell

Alternatively give the modulation wave-vectors

|  |   |   |
|--|---|---|
| $a_s =$  | $b_s =$   | $c_s =$                                       |
| $\begin{pmatrix} 1/2 \\ -1/2 \\ 0 \end{pmatrix}$ | $\begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix}$ | $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$   |
| $+ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$    | $+ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$   | $+ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ |
| $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$      | $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$     | $\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$   |

The supercell is centred:

Possible limitations of the subgroup list.

(Check only one option on the left and the specific value on the right)

(Check only one option on the left and the specific value on the right)

- Lowest space group to consider  1
- Lowest point group to consider
- Lowest crystal system to consider
- Only maximal subgroups

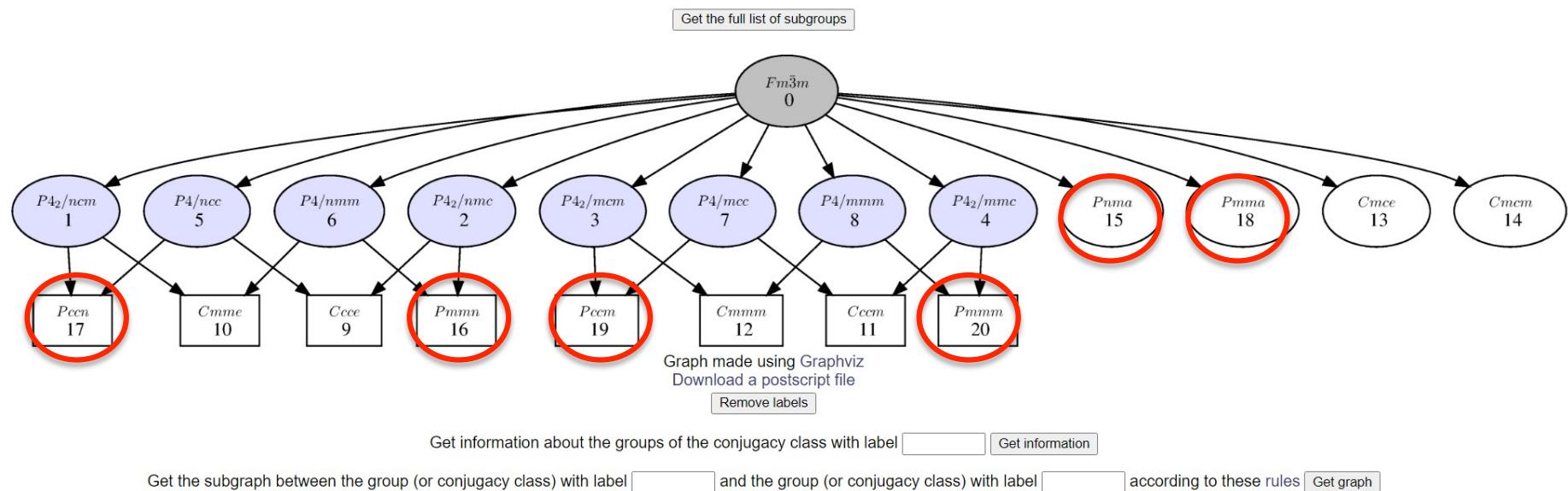
| N  | Group Symbol         | Transformation matrix  | Group-Subgroup index | Other members of the Conjugacy Class           | irreps                                    |
|----|----------------------|--|----------------------|--|---|
| 1  | $P4_2/ncm$ (No. 138) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 2  | $P4_2/nmc$ (No. 137) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 3  | $P4_2/mcm$ (No. 132) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 4  | $P4_2/mmc$ (No. 131) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 5  | $P4/ncc$ (No. 130)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 6  | $P4/nmm$ (No. 129)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 7  | $P4/mcc$ (No. 124)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 8  | $P4/mmm$ (No. 123)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 9  | $Ccce$ (No. 68)      | $\begin{pmatrix} 1 & 0 & 0 & 1/4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$           | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 10 | $Cmme$ (No. 67)      | $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$           | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 11 | $Cccm$ (No. 66)      | $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$              | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 12 | $Cmmm$ (No. 65)      | $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$              | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 13 | $Cmce$ (No. 64)      | $\begin{pmatrix} 0 & 1 & 0 & 1/4 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$          | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 14 | $Cmcn$ (No. 63)      | $\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$             | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 15 | $Pnma$ (No. 62)      | $\begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 1/4 \\ -2 & 0 & 0 & -1/4 \end{pmatrix}$ | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 16 | $Pmmn$ (No. 59)      | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 17 | $Pccn$ (No. 56)      | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 18 | $Pmma$ (No. 51)      | $\begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 0 \\ -2 & 0 & 0 & 1/2 \end{pmatrix}$    | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 19 | $Pccm$ (No. 49)      | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |
| 20 | $Pmmm$ (No. 47)      | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |

# Example: Fullerene-cubane crystal

## Input data

Subgroups of the space group :  $Fm\bar{3}m$  (N. 225)  
Lowest point group to consider:  $mmm$  (N. 8)  
Supercell given by:  $(1/2, -1/2, 0), (1/2, 1/2, 0), (0, 0, 2)$   
Centred supercell: No

Graph of subgroups that fulfill the given conditions



Graph showing the group-subgroup hierarchy of these 20 subgroups

# Example : Fullerene-cubane crystal

| N  | Group Symbol         | Transformation matrix  | Group-Subgroup index | Other members of the Conjugacy Class | irreps |
|----|----------------------|--|----------------------|--------------------------------------|--------|
| 1  | $P4_2/nmc$ (No. 138) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 12=4x3               | [Conjugacy Class] [Get irreps]       |        |
| 2  | $P4_2/nmc$ (No. 137) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 12=4x3               | [Conjugacy Class] [Get irreps]       |        |
| 3  | $P4_2/mcm$ (No. 132) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 12=4x3               | [Conjugacy Class] [Get irreps]       |        |
| 4  | $P4_2/mmc$ (No. 131) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 12=4x3               | [Conjugacy Class] [Get irreps]       |        |
| 5  | $P4/ncc$ (No. 130)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 12=4x3               | [Conjugacy Class] [Get irreps]       |        |
| 6  | $P4/nmm$ (No. 129)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 12=4x3               | [Conjugacy Class] [Get irreps]       |        |
| 7  | $P4/mcc$ (No. 124)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 12=4x3               | [Conjugacy Class] [Get irreps]       |        |
| 8  | $P4/mmm$ (No. 123)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 12=4x3               | [Conjugacy Class] [Get irreps]       |        |
| 9  | $Ccce$ (No. 68)      | $\begin{pmatrix} 1 & 0 & 0 & 1/4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$           | 24=4x6               | [Conjugacy Class] [Get irreps]       |        |
| 10 | $Cmme$ (No. 67)      | $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$           | 24=4x6               | [Conjugacy Class] [Get irreps]       |        |
| 11 | $Cccm$ (No. 66)      | $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$              | 24=4x6               | [Conjugacy Class] [Get irreps]       |        |
| 12 | $Cmmm$ (No. 65)      | $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$              | 24=4x6               | [Conjugacy Class] [Get irreps]       |        |
| 13 | $Cmce$ (No. 64)      | $\begin{pmatrix} 0 & 1 & 0 & 1/4 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$          | 24=4x6               | [Conjugacy Class] [Get irreps]       |        |
| 14 | $Cmcm$ (No. 63)      | $\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$             | 24=4x6               | [Conjugacy Class] [Get irreps]       |        |
| 15 | $Pnma$ (No. 62)      | $\begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 1/4 \\ -2 & 0 & 0 & -1/4 \end{pmatrix}$ | 24=4x6               | [Conjugacy Class] [Get irreps]       |        |
| 16 | $Pmmn$ (No. 59)      | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 24=4x6               | [Conjugacy Class] [Get irreps]       |        |
| 17 | $Pccn$ (No. 56)      | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 24=4x6               | [Conjugacy Class] [Get irreps]       |        |
| 18 | $Pmma$ (No. 51)      | $\begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 0 \\ -2 & 0 & 0 & 1/2 \end{pmatrix}$    | 24=4x6               | [Conjugacy Class] [Get irreps]       |        |
| 19 | $Pccm$ (No. 49)      | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 24=4x6               | [Conjugacy Class] [Get irreps]       |        |
| 20 | $Pmmm$ (No. 47)      | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 24=4x6               | [Conjugacy Class] [Get irreps]       |        |

Tetragonal subgroups

C centered orthorhombic

The unit cell of the LS-phase is known to be primitive orthorhombic

6 possible symmetries

# Example : Fullerene-cubane crystal

|    |                      |  |        |                 |            |
|----|----------------------|--|--------|-----------------|------------|
| 15 | <i>Pnma</i> (No. 62) | $\begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 1/4 \\ -2 & 0 & 0 & -1/4 \end{pmatrix}$ | 24=4x6 | Conjugacy Class | Get irreps |
| 16 | <i>Pmmn</i> (No. 59) | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 24=4x6 | Conjugacy Class | Get irreps |
| 17 | <i>Pccn</i> (No. 56) | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 24=4x6 | Conjugacy Class | Get irreps |
| 18 | <i>Pmma</i> (No. 51) | $\begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 0 \\ -2 & 0 & 0 & 1/2 \end{pmatrix}$    | 24=4x6 | Conjugacy Class | Get irreps |
| 19 | <i>Pccm</i> (No. 49) | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 24=4x6 | Conjugacy Class | Get irreps |
| 20 | <i>Pmmm</i> (No. 47) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 24=4x6 | Conjugacy Class | Get irreps |

The irreducible representations of the parent structure that are compatible with this specific symmetry for the distorted structure

List of physically irreducible representations and order parameters between a parent group and a given subgroup.

Input data

| Group→subgroup                             | Transformation matrix  |
|--|--|
| <i>Fm</i> $\bar{3}m (N. 225)→Pnma (N. 62)$ | $\begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 1/4 \\ -2 & 0 & 0 & -1/4 \end{pmatrix}$ |

Representations and order parameters

Show the graph of isotropy subgroups

| k-vectors                       | irreps and order parameters             | isotropy subgroup transformation matrix                                       | link to the irreps     |
|---------------------------------|---|---|------------------------|
| GM: (0,0,0)                     | GM <sub>1</sub> <sup>+</sup> : (a)      | <i>Fm</i> $\bar{3}m (No. 225)a,b,c;0,0,0$                                     |                        |
|                                 | GM <sub>3</sub> <sup>+</sup> : (a,0)    | <i>I</i> 4/ <i>m</i> <i>mm</i> (No. 139)<br>a/2-b/2,a/2+b/2,c;0,0,0           | matrices of the irreps |
|                                 | GM <sub>5</sub> <sup>+</sup> : (a,0,0)  | <i>I</i> <i>m</i> <i>mm</i> (No. 71)<br>a/2+b/2,-1/2a+b/2,c;0,0,0             |                        |
| DT: (0,1/2,0)(1/2,0,0)(0,0,1/2) | DT <sub>5</sub> : (0,0,0,0,0,0,a,0,0,a) | <i>Pnma</i> (No. 62)<br>-2c,a/2+b/2,a/2-b/2;0,1/4,-1/4                        | matrices of the irreps |
| X: (0,1,0)(1,0,0)(0,0,1)        | X <sub>2</sub> <sup>-</sup> : (0,0,a)   | <i>P</i> 4 <sub>2</sub> / <i>nmc</i> (No. 137)<br>a/2-b/2,a/2+b/2,c;0,1/4,1/4 |                        |
|                                 | X <sub>3</sub> <sup>-</sup> : (0,0,a)   | <i>P</i> 4/ <i>nmm</i> (No. 129)<br>a/2-b/2,a/2+b/2,c;1/4,0,1/4               | matrices of the irreps |

The symmetry break  
*Fm*-3*m* → *Pnma*  
can be realized through a  
Landau type phase transition

# Example : Fullerene-cubane crystal

List of physically irreducible representations and order parameters between a parent group and a given subgroup.

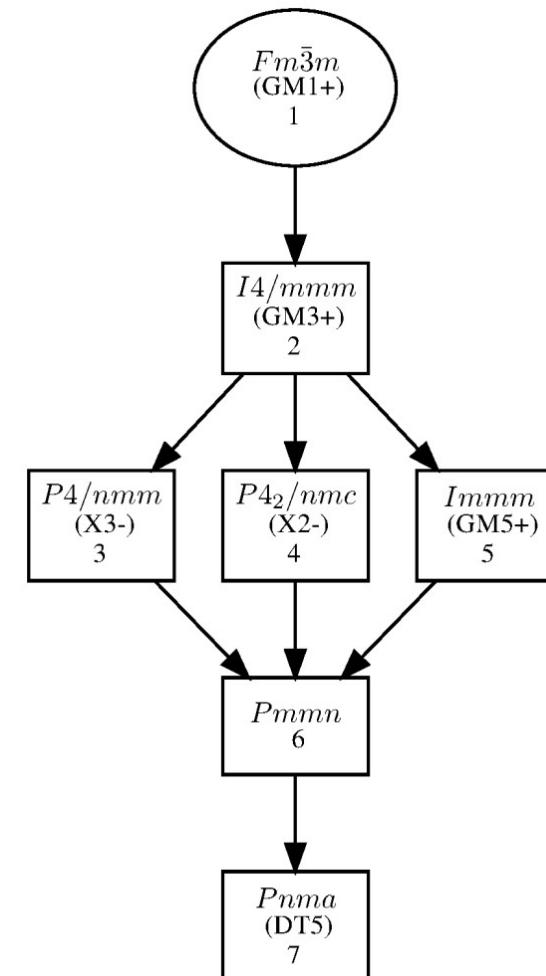
Input data

| Group→subgroup                        | Transformation matrix  |
|---------------------------------------|--|
| $Fm\bar{3}m$ (N. 225)→ $Pnma$ (N. 62) | $\begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 1/4 \\ -2 & 0 & 0 & -1/4 \end{pmatrix}$ |

Representations and order parameters

Show the graph of isotropy subgroups

| k-vectors                       | irreps and order parameters               | isotropy subgroup transformation matrix             | link to the irreps     |
|---------------------------------|---|---|------------------------|
| GM: (0,0,0)                     | GM <sub>1</sub> <sup>+</sup> : (a)        | $Fm\bar{3}m$ (No. 225)<br>a,b,c;0,0,0               |                        |
|                                 | GM <sub>3</sub> <sup>+</sup> : (a,0)      | $I4/mmm$ (No. 139)<br>a/2-b/2,a/2+b/2,c;0,0,0       |                        |
|                                 | GM <sub>5</sub> <sup>+</sup> : (a,0,0)    | $Immm$ (No. 71)<br>a/2+b/2,-1/2a+b/2,c;0,0,0        |                        |
| DT: (0,1/2,0)(1/2,0,0)(0,0,1/2) | DT <sub>5</sub> : (0,0,0,0,0,0,0,a,0,0,a) | $Pnma$ (No. 62)<br>-2c,a/2+b/2,a/2-b/2;0,1/4,-1/4   | matrices of the irreps |
| X: (0,1,0)(1,0,0)(0,0,1)        | X <sub>2</sub> <sup>-</sup> : (0,0,a)     | $P4_2/nmc$ (No. 137)<br>a/2-b/2,a/2+b/2,c;0,1/4,1/4 | matrices of the irreps |
|                                 | X <sub>3</sub> <sup>-</sup> : (0,0,a)     | $P4/nmm$ (No. 129)<br>a/2-b/2,a/2+b/2,c;1/4,0,1/4   | matrices of the irreps |



# Example : Fullerene-cubane crystal

List of physically irreducible representations and order parameters between a parent group and a given subgroup.

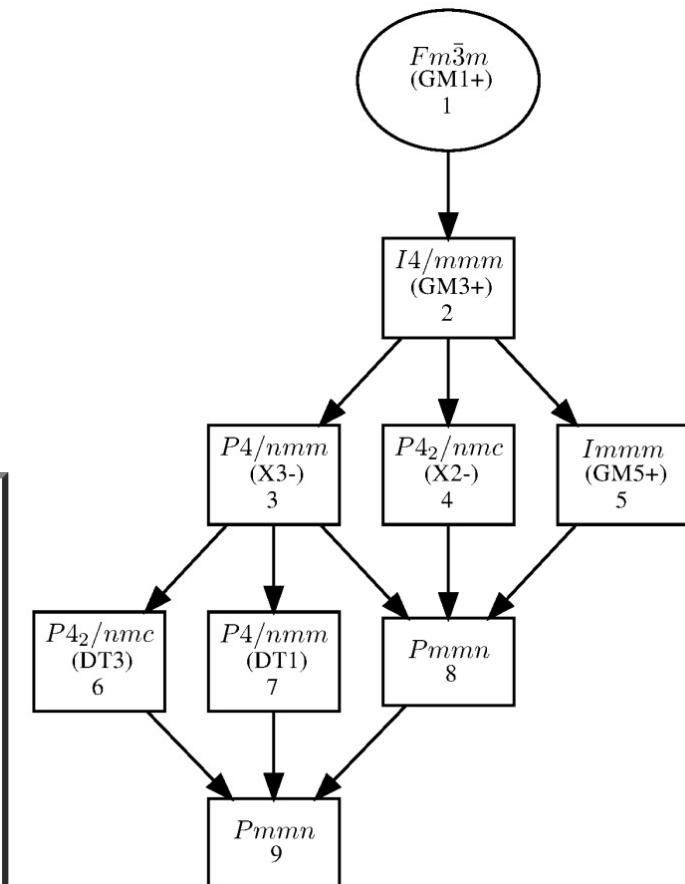
Input data

| Group→subgroup                        | Transformation matrix   |
|---------------------------------------|---|
| $Fm\bar{3}m$ (N. 225)→ $Pmmn$ (N. 59) | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$ |

Representations and order parameters

Show the graph of isotropy subgroups

| k-vectors                       | irreps and order parameters            | isotropy subgroup transformation matrix               | link to the irreps     |
|---------------------------------|--|---|------------------------|
| GM: (0,0,0)                     | GM <sub>1</sub> <sup>+</sup> : (a)     | $Fm\bar{3}m$ (No. 225)<br>a,b,c;0,0,0                 |                        |
|                                 | GM <sub>3</sub> <sup>+</sup> : (a,0)   | $I4/mmm$ (No. 139)<br>a/2-b/2,a/2+b/2,c;0,0,0         | matrices of the irreps |
|                                 | GM <sub>5</sub> <sup>+</sup> : (a,0,0) | $Immm$ (No. 71)<br>a/2+b/2,-1/2a+b/2,c;0,0,0          |                        |
| DT: (0,1/2,0)(1/2,0,0)(0,0,1/2) | DT <sub>1</sub> : (0,0,0,a,a)          | $P4/nmm$ (No. 129)<br>a/2-b/2,a/2+b/2,2c;1/4,0,-1/4   | matrices of the irreps |
|                                 | DT <sub>3</sub> : (0,0,0,a,-a)         | $P4_2/nmc$ (No. 137)<br>a/2-b/2,a/2+b/2,2c;1/4,0,-1/4 |                        |
| X: (0,1,0)(1,0,0)(0,0,1)        | X <sub>2</sub> <sup>-</sup> : (0,0,a)  | $P4_2/nmc$ (No. 137)<br>a/2-b/2,a/2+b/2,c;0,1/4,1/4   | matrices of the irreps |
|                                 | X <sub>3</sub> <sup>-</sup> : (0,0,a)  | $P4/nmm$ (No. 129)<br>a/2-b/2,a/2+b/2,c;1/4,0,1/4     |                        |



# Example: Fullerene-cubane crystal

Use the option *Get irreps* for the other possible symmetries

|    |                      |  |        |                 |            |
|----|----------------------|--|--------|-----------------|------------|
| 15 | <i>Pnma</i> (No. 62) | $\begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 1/4 \\ -2 & 0 & 0 & -1/4 \end{pmatrix}$ | 24=4x6 | Conjugacy Class | Get irreps |
| 16 | <i>Pmmn</i> (No. 59) | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 24=4x6 | Conjugacy Class | Get irreps |
| 17 | <i>Pccn</i> (No. 56) | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$  | 24=4x6 | Conjugacy Class | Get irreps |
| 18 | <i>Pmma</i> (No. 51) | $\begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 0 \\ -2 & 0 & 0 & 1/2 \end{pmatrix}$    | 24=4x6 | Conjugacy Class | Get irreps |
| 19 | <i>Pccm</i> (No. 49) | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 24=4x6 | Conjugacy Class | Get irreps |
| 20 | <i>Pmmm</i> (No. 47) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$     | 24=4x6 | Conjugacy Class | Get irreps |

Only two of the symmetries can be result of a single irrep order parameter (fulfill the Landau condition)

**DT5**

Fm-3m → *Pnma* ( $-2c, -1/2a + 1/2b, 1/2a - 1/2b; 0, 1/4, -1/4$ )

**DT5**

Fm-3m → *Pmma* ( $-2c, 1/2a + 1/2b, 1/2a - 1/2b; 0, 0, 1/2$ )

# Example: Fullerene-cubane crystal

Go back to the input page

Enter the serial number of the space group:  225

Introduce the supercell  
Alternatively give the modulation wave-vectors

|  |  |  |
|--|--|--|
| $a_s =$<br><input type="text" value="1/2"/><br>+ | $b_s =$<br><input type="text" value="1/2"/><br>+ | $c_s =$<br><input type="text" value="0"/><br>+ |
| <input type="text" value="a"/>                   | <input type="text" value="a"/>                   | <input type="text" value="a"/>                 |
| <input type="text" value="-1/2"/><br>+           | <input type="text" value="1/2"/><br>+            | <input type="text" value="0"/><br>+            |
| <input type="text" value="b"/>                   | <input type="text" value="b"/>                   | <input type="text" value="b"/>                 |
| <input type="text" value="0"/><br>+              | <input type="text" value="0"/><br>+              | <input type="text" value="2"/><br>+            |
| <input type="text" value="c"/>                   | <input type="text" value="c"/>                   | <input type="text" value="c"/>                 |

The supercell is centred:

**Optional:** Show only subgroups that can be the result of a Landau-type transition (single irrep order parameter).

## Possible limitations of the subgroup list.

(Check only one option on the left and the specific value on the right)

(Check only one option on the left and the specific value on the right)

Lowest space group to consider

1

Lowest point group to consider

-----

Lowest crystal system to consider

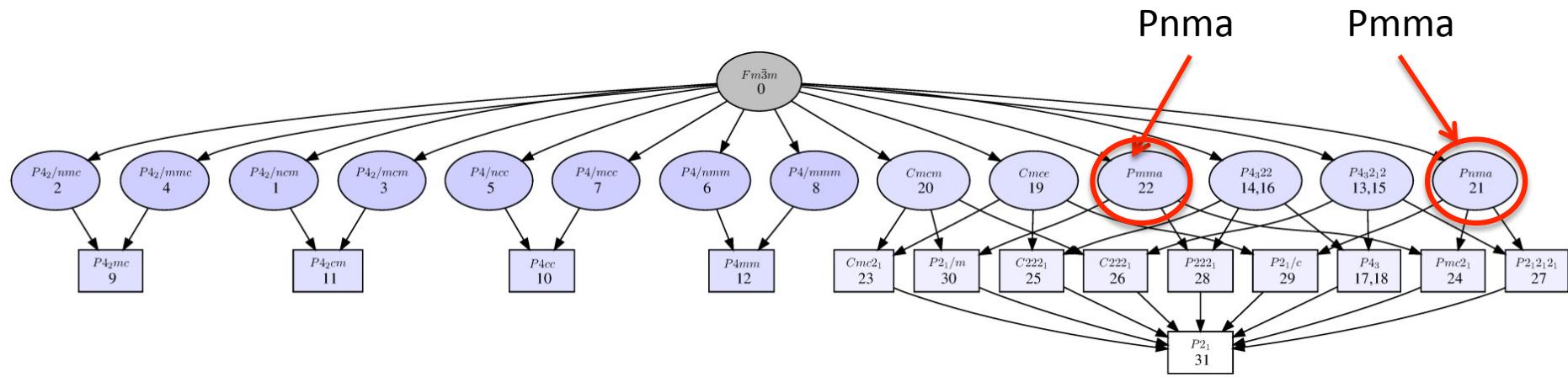
-----

Only maximal subgroups

This option allows to discard all possible symmetries that cannot be reached by the action of a **single irrep distortion (a single order parameter)**

# Example: Fullerene-cubane crystal

Landau-type possible symmetry breaks (one order parameter according to an irrep)



This condition reduces the number of possible distinct symmetries from 99 to 31

From the six-non-centered subgroups with point group mmm, only the subroups of type  $Pnma$  and  $Pmma$  appear

# Structure transformation to a lower symmetry

## TRANSTRU

### Transform Structure

**Transform Structure**

TRANSTRU transforms the structure to the low symmetry space group basis, taking care of all possible splittings of the Wyckoff positions.

Structure

|     |        |        |          |          |          |          |
|-----|--------|--------|----------|----------|----------|----------|
| 221 | 3.9064 | 3.9064 | 3.9064   | 90.      | 90.      | 90.      |
| 3   |        |        |          |          |          |          |
| La  | 1      | 1a     | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| Co  | 1      | 1b     | 0.500000 | 0.500000 | 0.500000 | 0.500000 |
| 0   | 1      | 3c     | 0.500000 | 0.500000 | 0.500000 | 0.000000 |

Low symmetry Space Group ITA number

Transformation Matrix:

In matrix form:

|             |    |    |              |
|-------------|----|----|--------------|
| Linear part |    |    | Origin Shift |
| 1           | -1 | -1 | 0            |
| 2           | 0  | 0  | 1/2          |
| -1          | -1 | 1  | 1/2          |

Show

Transformation of the set of operations forming the subgroup to standard setting of the space group

# TRANSTRU is now linked directly with SUBGROUPS!

## Input data

Subgroups of the space group :

$Fm\bar{3}m$  (N. 225)

Lowest space group to consider:

$P1$  (N. 1)

Supercell given by:

( $1/2, -1/2, 0$ ), ( $1/2, 1/2, 0$ ), ( $0, 0, 2$ )

No

Centred supercell:

No

Only subgroups compatible with a Landau-type transition

## List of subgroups that can be the result of a Landau-type transition

| N | Group Symbol         | Transformation matrix   | Group-Subgroup index | Other members of the Conjugacy Class           | irreps                                    | TRANSTRU                 |
|---|----------------------|---|----------------------|--|---|--------------------------|
| 1 | $P4_2/ncm$ (No. 138) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$ | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input type="checkbox"/> |
| 2 | $P4_2/nmc$ (No. 137) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/4 \end{pmatrix}$  | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input type="checkbox"/> |
| 3 | $P4_2/mcm$ (No. 132) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$    | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |                          |
| 4 | $P4_2/mmc$ (No. 131) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix}$      | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |                          |
| 5 | $P4/ncc$ (No. 130)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/4 \end{pmatrix}$  | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |                          |
| 6 | $P4/nmm$ (No. 129)   | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/4 \\ -1/2 & 1/2 & 0 & 0 \end{pmatrix}$                     | 12=4x3               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> |                          |

| N  | Group Symbol             | Transformation matrix  | Group-Subgroup index | Other members of the Conjugacy Class           | irreps                                    | TRANSTRU                            |
|----|--------------------------|--|----------------------|--|---|-------------------------------------|
| 21 | $Pnma$ (No. 62)          | $\begin{pmatrix} 0 & -1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 & 1/4 \\ -2 & 0 & 0 & -1/4 \end{pmatrix}$   | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input checked="" type="checkbox"/> |
| 22 | $Pmma$ (No. 51)          | $\begin{pmatrix} 0 & -1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ -2 & 0 & 0 & 1/2 \end{pmatrix}$      | 24=4x6               | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input checked="" type="checkbox"/> |
| 23 | $Cmc2_1$ (No. 36)        | $\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix}$                 | 48=4x12              | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input type="checkbox"/>            |
| 24 | $Pmc2_1$ (No. 26)        | $\begin{pmatrix} 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix}$         | 48=4x12              | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input type="checkbox"/>            |
| 25 | $C222_1$ (No. 20)        | $\begin{pmatrix} 1 & 0 & 0 & 1/4 \\ 0 & 1 & 0 & 1/4 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$           | 48=4x12              | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input type="checkbox"/>            |
| 26 | $C222_1$ (No. 20)        | $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$                | 48=4x12              | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input type="checkbox"/>            |
| 27 | $P2_{12}1_{21}$ (No. 19) | $\begin{pmatrix} 1/2 & 1/2 & 0 & 1/8 \\ -1/2 & 1/2 & 0 & -1/8 \\ 0 & 0 & 2 & -1/4 \end{pmatrix}$ | 48=4x12              | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input type="checkbox"/>            |
| 28 | $P222_1$ (No. 17)        | $\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 2 & 1/2 \end{pmatrix}$       | 48=4x12              | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input type="checkbox"/>            |
| 29 | $P2_1/c$ (No. 14)        | $\begin{pmatrix} 1/2 & 0 & -1 & 0 \\ -1/2 & 0 & 0 & 1/4 \\ 0 & 2 & 0 & -1/4 \end{pmatrix}$       | 48=4x12              | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input type="checkbox"/>            |
| 30 | $P2_1/m$ (No. 11)        | $\begin{pmatrix} 1/2 & 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & -2 & 0 & 1/2 \end{pmatrix}$      | 48=4x12              | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input type="checkbox"/>            |
| 31 | $P2_1$ (No. 4)           | $\begin{pmatrix} 1/2 & 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$        | 96=4x24              | <input type="button" value="Conjugacy Class"/> | <input type="button" value="Get irreps"/> | <input type="checkbox"/>            |

\* Clases with the same i superscript are conjugated through an improper symmetry operation.

Submit selected subgroups to TRANSTRU

This example was already in one of our tutorials from 2010:



Tutorial\_SPT\_abr2010.pdf (can be downloaded from the BCS but obsolete)

2011

DOI: 10.1021/cg101522t

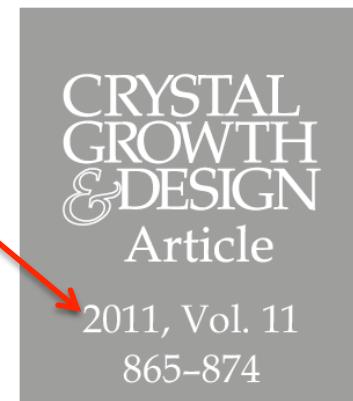
## Low Temperature Structure and Supramolecular Interactions of the C<sub>60</sub>–Cubane Cocrystal

Gábor Bortel,\* Sándor Pekker, and Éva Kováts

Research Institute for Solid State Physics and Optics, Hungarian Academy of Sciences, P.O. Box 49,  
Budapest H-1525, Hungary

Received November 16, 2010; Revised Manuscript Received December 21, 2010

**ABSTRACT:** The rotor–stator phase of the fullerene–cubane cocrystal, C<sub>60</sub>·C<sub>8</sub>H<sub>8</sub>, has an orientational ordering phase transition at around 140 K. On the basis of a simulated annealing analysis of the powder X-ray diffraction data, we determined the previously unknown crystal structure of the orthorhombic oriented phase: the space group is *Pnma*, the positions and the orientations of the constituent molecules significantly changed relative to the rock salt structure of the ambient temperature phase. The experimental structure is confirmed by cohesive energy calculations based on a pair potential method. Related structures, allowed by symmetry considerations, can be excluded by both the experiments and the calculations. The stability of the oriented phase is predominantly controlled by the nearest neighbor fullerene–fullerene and fullerene–cubane interactions.



This example was already in one of our tutorials from 2010:



Tutorial\_SPT\_abr2010.pdf (can be downloaded from the BCS but obsolete)

2011

DOI: 10.1021/cg101522t

## Low Temperature Structure and Supramolecular Interactions of the C<sub>60</sub>–Cubane Cocrystal

Gábor Bortel,\* Sándor P

Research Institute for Solid State Physics and Optics  
Budapest H-1525, Hungary

Received November 16, 2010

**ABSTRACT:** The rotor–rotor interaction between the C<sub>60</sub> molecules in the C<sub>60</sub>–cubane cocrystal at low temperature was studied by X-ray diffraction. The C<sub>60</sub>–cubane cocrystal has an orientational order–disorder phase transition at around 140 K. The space group of the low-temperature phase was determined to be Pnma, and the orientations of the C<sub>60</sub> molecules were found to be randomly oriented in the c axis direction. The experimental results are compared with the calculated structures, allowed by symmetry, and the oriented phase is predicted to be the rock salt structure of the ambient phase. The calculations based on a pair potential model are in good agreement with the experiments and the calculations.

CRYSTAL  
GROWTH  
& DESIGN  
Article

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865–874

ystal, C<sub>60</sub>·C<sub>8</sub>H<sub>8</sub>, has an orientational order–disorder phase transition of the powder X-ray diffraction data, and phase: the space group is *Pnma*, the powder pattern corresponds to the rock salt structure of the ambient phase. The calculations based on a pair potential model are in good agreement with the experiments and the calculations. The interactions between the C<sub>60</sub>–fullerene and fullerene–cubane

yystal,  $C_{60} \cdot C_8H_8$ , has an orientational cysis of the powder X-ray diffraction data, d phase: the space group is *Pnma*, the poe to the rock salt structure of the ambient calculations based on a pair potential model the experiments and the calculations. The fullerene–fullerene and fullerene–cubai

# More Examples:

More examples in the tutorial of SUBGROUPS that can be downloaded from the webpage of the program ...

**Subgroups: Subgroups compatible with a given supercell or some propagation vector(s).**

The program *Subgroups* provides the possible subgroups of a space group which are possible for a given **supercell**. The program provides a list of the set of space groups or a graph showing the group-subgroup hierarchy, grouped into conjugacy classes. More optional information about the classes or subgroups is also given.

Other alternatives for the input of the program:

- Instead of the whole set of subgroups, the output can be limited to subgroups having a chosen common subgroup of lowest symmetry, common point group of lowest symmetry, or groups which belong to a specific crystal class.
- Instead of a supercell, a set of modulation wave vectors can be given, including complete or partial wave-vectors stars.
- The subgroups compatible with intermediate unit cells between the unit cell of the parent space group and the given supercell (or the supercell determined by the given wave vector(s) when the previous option is used) can be included.
- When a set of wave-vectors is used as input, the output can be further refined introducing the Wyckoff positions of the atoms and/or a set of irreducible representations.

Tutorial\_SUBGROUPS: [download](#)

See the [Help](#) for details.

Enter the serial number of the space group:  choose it

Introduce the supercell

Alternatively give the modulation wave-vectors

|  |  |  |
|--|--|--|
| $a_s =$  | $b_s =$  | $c_s =$  |
| <input type="text"/> 1 <input type="text"/> a      | <input type="text"/> 0 <input type="text"/> a      | <input type="text"/> 0 <input type="text"/> a      |
| +<br><input type="text"/> 0 <input type="text"/> b | +<br><input type="text"/> 1 <input type="text"/> b | +<br><input type="text"/> 0 <input type="text"/> b |
| +<br><input type="text"/> 0 <input type="text"/> c | +<br><input type="text"/> 0 <input type="text"/> c | +<br><input type="text"/> 1 <input type="text"/> c |

The supercell is centred:

Include the subgroups compatible with intermediate cells.  
(It is not applied when only the maximal subgroups are calculated)

Optional: refine further the subgroups of the output giving the Wyckoff positions of the atoms

Give the Wyckoff positions

Optional: Show only subgroups that can be the result of a Landau-type transition (single irrep order parameter).