# **Neutron diffraction**



#### **NEUTRON PROPERTIES**



### Discovered in 1932 by Chadwick

At least 50% of the Universe matter







Domicile : étoiles et noyau des atomes<<<<<<<<<<<<<< Profession : particule constitutive de la matière









#### **NEUTRON'S GENERATION: THE ILL REACTOR**





#### **HOW NEUTRONS ARE EXTRACTED AND GUIDED**



#### **WHY NEUTRONS?**



### **Light atoms, as hydrogen or oxygen, are scattered as much as heavy atoms**





- X-ray scattering is proportional to the number of electrons (*Z* 2 )
- Electron scattering depends on the electric potential
- Neutron scattering is similar for all atoms with a non-clear atomic (or isotopic) dependence.

#### **FORM FACTOR**

- The nuclear form factor is independent of *Q (neutrons)*
- The structural form factor is strongly dependent of *Q (X-ray)*
- High *Q*'s region is better explored using neutron diffraction



#### **PENETRATION ON THE MATTER**

**Neutrons interact with the tiny atomic nuclei and can localise the atoms more precisely**

They can penetrate deeper than X-rays





#### **MAGNETIC SCATTERING**

Neutrons have no charge, but they do have a magnetic moment.

The magnetic moment is given by the neutron's spin angular momentum: y do have a magnetic moment.<br>
y the neutron's spin angular mom<br>  $-\gamma \mu_N \hat{\sigma}$ 

$$
-\gamma\mu_{_N}\,\hat{\sigma}
$$

Where:

- $\gamma$  is a constant (=1.913)
- $\cdot$   $\mu_N$  is the nuclear magneton
- $\mu_N$  is the nuclear magneton<br>•  $\hat{\sigma}$  is the quatum mechanical Pauli spin operator

#### **Normally refer to neutron as a spin-1/2 particle**

### **MAGNETIC SCATTERING**

#### **Neutrons are strongly scattered by magnetic materials**



Ferromagnetic magnetite  $Fe<sub>3</sub>O<sub>4</sub>$ and antiferromagnetic manganese oxide MnO

- Neutrons are like small magnets
- The neutron's magnetic moment strongly interacts with the magnetic atomic moments
- Neutrons can determine magnetic structures and measure the magnetization with high precision

#### **Arrangement of spins in two different types of skyrmions**



I. Kezsmarki et al., Nature Materials, 2015, **14**, 1116; DOI: 10.1038/nmat4402ptions.

### **OVERVIEW**

### **X-rays**

- **Surface or small samples**
- Diffraction power correlated with  $Z$
- Difficult for determine light atoms
- **Low contrast for atoms close in the Periodic Table**
- $\blacksquare$  High flux

### Small samples

 $\blacksquare$  High resolution

#### **• Neutrons**

- **Bulk meausrements**
- **Diffraction power different for different isotopes**
- Can see light atoms (incoherent problem of H)
- Can distinguish neighbors in in the Periodic Table
- $\blacksquare$  Low flux
	- Big samples
- **Medium resolution**

**Magnetic Structures!!! « Magnetic studies »**

### **SCATTERING CONCEPTS**

$$
\hbar \mathbf{Q} = \hbar(\mathbf{k} - \mathbf{k}) = h\mathbf{s}
$$

The prime (') superscripts stand for final state of the particle

The differential scattering cross section can be defined based on the fraction of neutrons scattered into a solid angle  $d\Omega$ , with an angle  $\theta$ . The incident neutrons have an initial wave vector **k**, which change to **k**' after the scattering process. The final wave vector, **k**', lies within the solid angle  $d\Omega$ .  $G$ <br>  $SU$  al s<br>  $\alpha$  a ch e d<br>  $con$ <br>  $\alpha$  a prediction<br>  $\vdots$ <br>  $\vdots$  prediction by =  $\hbar(\mathbf{k'}-\mathbf{k}) = h\mathbf{s}$ <br>
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(') superscripts stand for final state of the particle<br>
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The prime (') superscripts stand for final state of the particle<br>
The differential scattering cross section can be defined based on the fraction of neutrons<br>
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Here we will consider only the magnetic interaction between the incident neutrons and the target.

$$
\left(\frac{d^2\sigma}{d\Omega dE}\right)_{s\lambda\to s'\lambda'} = \frac{k'}{k} \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \left|\left\langle \mathbf{k}'s'\lambda'\right| V_m\left|\mathbf{k}s\lambda\right\rangle\right|^2 \delta(E_\lambda - E_\lambda + \hbar\omega)
$$

Where  $E_i$  and  $E_{i'}$  denoting the initial and the final kinetic energy of the probe and and *s* and *s*' are spin states.



#### **SCATTERING CONCEPTS**

 $S =$ <br> $\frac{1}{2}$ <br>:  $\frac{1}{2}$  $\left|\frac{\mathbf{x} \cdot \lambda}{W_m} \right| \mathbf{k} s \lambda \Big|^2 \delta(E_{\lambda} - E_{\lambda} + \hbar \omega)$ <br>
ysics<br>
Conservation of the Energy CEPTS<br>  $\begin{aligned} \sum_{s,\lambda \to s',\lambda'} &= \frac{k'}{k} \left( \frac{m_n}{2\pi\hbar^2} \right)^2 \Big| \langle \cdot \rangle \end{aligned}$ <br>
which contain most of the probe-tain to the final state.  $\begin{aligned} \mathsf{P}\mathsf{T}\mathsf{S} \ \text{and} \ \mathsf{S} \ \text{in} \ \mathsf{S} \ \text{in} \ \mathsf{S} \ \text{in} \ \mathsf{S} \end{aligned}$ **CONCEPTS**<br>  $\frac{d^2\sigma}{\Omega dE'}$ <br>  $= \frac{k'}{k} \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \left|\left\langle \mathbf{k}'s'\right\rangle\right|$ <br>
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initial to the final state.  $\int s \lambda' |V_m| \mathbf{k} s \lambda \rangle|^2 \delta(E_{\lambda} - E_{\lambda'} + \hbar \omega)$ <br> **Since**<br> **Example 2**<br> **Example 2**<br> **Example 2**<br> **Example 2**<br> **Example 2**<br> **Example 2**<br> **Example 2 CONCEPTS**<br>  $\frac{d^2 \sigma}{d\Omega dE}$ <br>  $\frac{d}{dx} \left( \frac{m_n}{2\pi \hbar^2} \right)^2 \left| \left\langle \mathbf{k}^1 s^1 \lambda^1 |V_m| \mathbf{k} s \lambda \right\rangle \right|^2 \delta(E_z - E_x + \hbar \omega)$ <br>
ment, which contain most of the physics<br>
matrix element gives the transition<br>
initial to the final sta **PTS**<br> $=\frac{k'}{k} \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \left|\left\langle \mathbf{k}' s'\lambda'\right| V_m \left|\mathbf{k} s\lambda\right\rangle\right|^2 \delta(E_\lambda - E_\lambda + \hbar \omega)$ <br>ch contain most of the physics Conservation of the Energy  $\lambda$   $|V_m|$  ks  $\lambda$   $|^{2}$   $\delta(E_{\lambda} - E_{\lambda} + \hbar \omega)$  **k k** The matrix element, which contain most of the physics  $\Box$  Conservation of the Energy

 $2 \times 11^{1}$   $\frac{1}{2}$ 

'

 $\rightarrow$ s' $\lambda$ '  $K \setminus 2\pi h^{-}$  )

 $n_n$ 

2  $\alpha$ 

2  $\left| \int_{\mathbb{R}} \mathbb{E} \left| \int_{\mathbb{R}} \mathbb{E} \left| \int_{\mathbb{R}} f(x) \right| \mathbb{E} \left| \int_{\mathbb{R}} f(x) \right| \right| \right| \leq C \left| \mathcal{L} \right|$ 

 $m \mid \mathbf{D}$ <sup> $\mathcal{U}$ </sup>

The square of the matrix element gives the transition probability from the initial to the final state.

 $\sigma$  |

This transition probability is related to the probe-target interaction through the potential  $V_m$ , which is the potential felt by the neutron due to the magnetic field created by moving electrons of the target.



Where **B** is the magnetic induction

#### **ELASTIC SCATTERING**

If the incident neutron energy  $=$  the final neutron energy, the scattering is considered as "**elastic**".

gy = the final neutron energy, the scatté

\n
$$
\left(\frac{d^2\sigma}{d\Omega}\right)_{s \to s'} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \left|\left\langle \mathbf{k} \, s' | V_m | \mathbf{k} s \right\rangle\right|^2
$$
\nneutron scattering we can assume than s

\nume **(r)**

If we are using unpolarised neutron scattering we can assume than  $s = s'$ .

We can integrate over all volume (**r**)

$$
argy = the final neutron energy, the scattering\n
$$
\left(\frac{d^2\sigma}{d\Omega}\right)_{s \to s'} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \left|\left\langle \mathbf{k} \, s' | V_m | \mathbf{k} s \right\rangle\right|^2
$$
\n
$$
d \text{ neutron scattering we can assume than } s = s'.
$$
\n
$$
d \text{volume (r)}
$$
\n
$$
\left|\left\langle \mathbf{k} \, v' | V_m | \mathbf{k} \right\rangle\right| = \left|\int_{0}^{s} V_m e^{i\mathbf{Q} \cdot \mathbf{r}} d\mathbf{r}\right| = \left|\mathbf{M}_{\perp}(\mathbf{Q})\right|
$$
\n
$$
d \text{with } \mathbf{Q} = \mathbf{k} \cdot \mathbf{k}
$$
\n
$$
d \text{the Fourier transform}
$$
\n
$$
d \text{is in Fourier space, otherwise called reciprocal space
$$
$$



The elastic cross-section is then directly proportional to the **Fourier transform squared** of the potential.

Neutron scattering thus works in Fourier space, otherwise called *reciprocal space*.

#### **MAGNETIC GROUND STATE**

The magnetic ground state of a spin system, should provide a magnetic structure compatible with the minimum energy of the classical spin Hamiltonian .m, should provide a<br>bin Hamiltonian<br> $\begin{array}{l} \frac{\alpha\beta}{j l ,im} {S}_{jl\alpha} {S}_{im\beta} \; + ... \end{array}$ nodels: ould provide a magnetic structure compatible<br>miltonian<br> ${}_{\alpha}S_{im\beta} + ...$ <br>: m system, should provide a magnetic structure<br>sical spin Hamiltonian<br> $\sum_{\alpha, \text{im}\beta} J^{\alpha\beta}_{jl, \text{im}} S_{jla} S_{im\beta} + ...$ <br>netic models: E<br>
spin system, should provide a magnetic structure compatible<br>
classical spin Hamiltonian<br>
=  $\sum_{jl\alpha,im\beta} J_{jl,im}^{\alpha\beta} S_{jn\beta} + ...$ <br>
magnetic models:<br>
1-1-1-1-1-1- Ferromagnet

**ATE**

\nof a spin system, should provide the classical spin Hamiltonian

\n
$$
H = \sum_{jl\alpha, im\beta} J_{jl,im}^{\alpha\beta} S_{jl\alpha} S_{im\beta} + \dots
$$

\n1D magnetic models:

\n**4.-1.-1.-1.-1.-1.-1.** Ferromannet

Here below we present some 1D magnetic models:



#### **MAGNETIC FIELD EXPERIENCED BY A TRAVELING NEUTRON:**

Magnetism is caused by unpaired electrons or by movement of charges.

The magnetic field felt by the neutron due to the spin (S) and orbital (L) moments of an electron can be expressed using the following equation:



Where the two term within braces correspond to the *magnetic vector potential of a dipolar field due to the electron spin moment* and the second correspond to the *Biot-Savart law for a single electron with linear momentum p*.

#### **MAGNETIC FIELD EXPERIENCED BY A TRAVELING NEUTRON:**

The evaluation of the **spatial part of the transition matrix** element for an electron *j* is described by the following expression: part of the transition matrix element for an elect<br>ssion:<br> $\pi \exp(i\mathbf{OR}.) \left\{\hat{\mathbf{O}} \times (\mathbf{s} \cdot \times \hat{\mathbf{O}}) + \frac{i}{\sqrt{2}} (\mathbf{p} \cdot \times \hat{\mathbf{O}}) \right\}$ 



to the component of the local magnetization that is **perpendicular** to the scattering vector !!

• The interaction between a body characterized by the magnetic density  $\rho_m(r)$  and a neutron beam provides:

$$
f(\mathbf{Q}) = \int_{S} \rho_{m}(\mathbf{r}) \exp(2\pi i \mathbf{Q} \cdot \mathbf{r}) d\mathbf{r}
$$

• Magnetic form factor, f(**Q**), is the **Fourier transform** of the magnetic density.



The above formulas are quite general: they can be applied to the cases in which  $\rho(r)$  represents the electron density of an electron, of an atom, of a molecule, of the unit cell, of the full crystal.

# **Diffraction Equations for crystals**

 $\mathbf{r}$  is a recipi<br> $\mathbf{s} = \mathbf{H}$ **Laue conditions**: the scattering vector is a reciprocal lattice vector of the crystal

**The Laue conditions have as a consequence the Bragg Law**



# **Ewald construction**



## **Ewald construction: single crystal case**

Ewald Sphere **z***D* **z***L* P **s***L*  $\boldsymbol{\mathsf{D}}_D$  $\mathbf{x}_D$ **h**  $\mathbf{s}_{0L}$  $2\theta$ O  $\mathbf{\overline{v}}$ Incident beam  $V_D$  $\overline{\gamma}$  $1/\lambda$  $\rightarrow$ <sup>y</sup>*L*</sup>  $\overline{\gamma_D}$ **crystal**  $\mathbf{x}_L$ Laboratory Frame and detector



# **Diffraction patterns**

**Single Xtal - 2D image + scan –> 3D Int vs 2θ Powder - 2D image –> 1D Int vs 2θ**



**Powder or polycrystalline solid**

## **Ewald construction Laue**



**Cyclops** 

Cooling down from RT up to 20K

- **I. Conmensurate structure**
- **II & III. Inconmensurate structures**

 $\left[\text{NH}_3\text{CH}_3\right]_{\text{N}}\left[\text{Co}^{\text{II}}(\text{HCOO})_6\right]_{\text{N}}$ 

**IV. Conmensurate structure ↔ Twinned crystal**



**I. Conmensurate structure II & III. Inconmensurate structures IV. Conmensurate structure ↔ Twinned crystal** 77K 128K

# **Single Crystal and Powder Diffraction**

**Single Crystal diffraction** allows to get with high precision subtle structural details: thermal parameters, anharmonic vibrations.

**Drawbacks:** big crystals for neutrons, extinction, twinning

**Data reduction:** Needs only the indexing and integration of Bragg reflection and obtain structure factors. List:  $h \, k \, l \, F^2 \, \sigma(F^2)$ 

**Data Treatment:** SHELX, FullProf, JANA, GSAS, …

**Powder diffraction** no problem with extinction or twinning. **Data reduction:** minimalistic, needs only the profile intensities and their standard deviations

**Data Treatment:** FullProf, JANA, GSAS, TOPAS, …

#### **POWDER DIFFRACTOMETER**



$$
2d = n\lambda sen(\theta)
$$

#### **MAGNETIC POWDER DIFFRACTION**

Based on the Rietveld Method

$$
y_i = \sum_{\mathbf{H}} I_{\mathbf{H}} \Omega(T_i - T_{\mathbf{H}}) + D_i + B_i
$$

Where  $y_i$  is the number of counts, the subscript "i" represents a discrete observation at the scattering variable T<sup>i</sup>

The T variable to describe either, the scattering angle 2 $\theta$ , the time of flight t or the scattering vector modulus **Q** or s.

H corresponds to Bragg peaks contributing to the channel "*i*"

In is the integrated intensity of the reflection  $H$ 

 $\Omega(T_i - T_H)$  is the value of the normalised profile function of the Bragg reflection at the position  $T_i$  due to the reflection  $H$  at the position  $T_H$ 

D<sup>i</sup> is the diffuse scattering due to defects

B<sup>i</sup> is the background

#### **MAGNETIC POWDER DIFFRACTION**

The intensity of each magnetic reflection is affected by other parameters than should be taken into account in the previous equation:

 $I_{M,H}(\mathbf{h}) \approx \left\{ j \cdot L \cdot A \cdot O \cdot E \cdot \mathbf{M}_{\perp}(\mathbf{h}) \cdot \mathbf{M}_{\perp}^*(\mathbf{h}) \right\}_{H}$ 

**DIFFRACTION**<br>
etic reflection is affected by other parameters<br>
revious equation:<br>  $M_{M,H}(\mathbf{h}) \approx \{ j \cdot L A \cdot O \cdot E \cdot \mathbf{M}_{\perp}(\mathbf{h}) \cdot \mathbf{M}_{\perp}^*(\mathbf{h}) \}_{\mathbf{H}}$ <br>
eflection H. *L* is the Lorentz factor. *A* is the alvith pref **DIFFRACTION**<br>
netic reflection is affected by other parameters than should be<br>
previous equation:<br>  $I_{M,H}(\mathbf{h}) \approx \{ j \cdot LAO \cdot E \cdot \mathbf{M}_{\perp}(\mathbf{h}) \cdot \mathbf{M}_{\perp}^*(\mathbf{h}) \}_H$ <br>
reflection H. *L* is the Lorentz factor. *A* is the a **)IFFRACTION**<br>tic reflection is affected by other parameters<br>vious equation:<br> $\mathbf{H}(\mathbf{h}) \approx \{ j \cdot L \cdot A \cdot O \cdot E \cdot \mathbf{M}_{\perp}(\mathbf{h}) \cdot \mathbf{M}_{\perp}^*(\mathbf{h}) \}_\mathbf{H}$ <br>election **H**. *L* is the Lorentz factor. *A* is the abth preferred *j* is the multiplicity of the reflection H. *L* is the Lorentz factor. *A* is the absorption correction. *O* is to handle with preferred orientations and *E* is the primary extinction correction. multiplicity<br>( ion. *O* is to h<br>( )<br>( ) sho known the shourned in the shourned in the should be a should be a should be a should be a should be a<br>matter of the should be a sh

The  $\;I_{M,H}^{}(\mathbf{h})\;$  <u>should be determined experimentally </u>and <u>fitted using theoretical models</u>.

In order to achieve to measure adequately the intensity of magnetic reflections some consideration are needed.

### **CAGLIOTI'S EQUATION**

The Caglioti's relations is generally used to calculate an approximate resolution function of the two axis diffractometer.

$$
FWHM = (U \tan^2 \theta + V \tan \theta + W)^{1/2}
$$

The full width at half maximum (FWHM) of Bragg reflections varies with the scattering angle.

U, V and W are parameter that the user should be provide to the refinement program (FullProf, Jana, Topas, etc…).

Why does on powder diffractometers the detector is placed on the positives values of  $\theta$  (right side from sample to detector)?



### **CAGLIOTI'S EQUATION**

```
FWHM = (U \tan^2 \theta + V \tan \theta + W)^{1/2}
```
The resolution on powder diffraction is defined as the ability to distinguish between two adjacent reflections.

This is correlated with the value of the full width at half maximum (FWHM) at each scattered position.

U, V and W are parameters can be correlated with angular divergence of the incoming neutrons to the monochromator,  $\alpha$ , the angular aperture of a monochromator-to-sample collimator,  $\alpha$ , the collimation between sample and detector,  $\alpha$ , the take-off angle of the monochromator,  $2\theta_m$  and the mosaicity of the monochromator,  $\beta$ .



#### **CAGLIOTI-PAOLETTI-RICCI EQUATIONS**

The Caglioti-Paoletti-Ricci equations are able to determine U, V and W through angular

divergences 
$$
\alpha_l
$$
,  $\alpha_2$  and  $\alpha_3$ , the take-off angle and the mosaicity of the monochromator,  $\beta$ .  
\n
$$
U = 4(\alpha_1^2 \alpha_2^2 + \alpha_1^2 \beta^2 + \alpha_2^2 \beta^2)/[\tan^2 \theta_m (\alpha_1^2 + \alpha_2^2 + 4\beta^2)]
$$
\n
$$
V = -4\alpha_2^2 (\alpha_1^2 + 2\beta^2)/[\tan \theta_m (\alpha_1^2 + \alpha_2^2 + 4\beta^2)]
$$
\n
$$
W = [\alpha_1^2 \alpha_2^2 + \alpha_1^2 \alpha_3^2 + \alpha_2^2 \alpha_3^2 + 4\beta^2 (\alpha_2^2 + \alpha_3^2)]/(\alpha_1^2 + \alpha_2^2 + 4\beta^2)
$$

For a powder diffractometer used to determine magnetic structures, typically we look for:

- long wavelengths (to separate nuclear and magnetic reflections),
- maximum of resolution on the low angle region (magnetic form factor)
- High flux

#### **CAGLIOTI-PAOLETTI-RICCI EQUATIONS**

**AGLIOTI-PAOLETTI-RICCI EQUATIONS**  
\n
$$
U = 4(\alpha_1^2 \alpha_2^2 + \alpha_1^2 \beta^2 + \alpha_2^2 \beta^2)/[\tan^2 \theta_m (\alpha_1^2 + \alpha_2^2 + 4\beta^2)]
$$
\n
$$
V = 4\alpha_2^2 (\alpha_1^2 + 2\beta^2)/[\tan \theta_m (\alpha_1^2 + \alpha_2^2 + 4\beta^2)]
$$
\n
$$
W = [\alpha_1^2 \alpha_2^2 + \alpha_1^2 \alpha_3^2 + \alpha_2^2 \alpha_3^2 + 4\beta^2 (\alpha_2^2 + \alpha_3^2)]/(\alpha_1^2 + \alpha_2^2 + 4\beta^2)
$$

Here a comparison of two instrument configuration the **blue** one correspond with a instrument configuration dedicated to **magnetism** while the **green** one is dedicated to **structural crystallography**.

**There is always a price to paid!!**



#### **MAGNETIC DIFFRACTION**

The integrated intensity of a magnetic Bragg reflections is proportional to the square of the so called magnetic interaction vector.

$$
I_M(\mathbf{h}) \approx |\mathbf{M}_{\perp}(\mathbf{h})|^2 = |\mathbf{M}(\mathbf{h}) - (\mathbf{M}(\mathbf{h}) \cdot \mathbf{e}) \mathbf{e}|^2
$$

Magnetic Interaction Vector

$$
\begin{aligned} \n\text{[for]}\\ \n\mathbf{M}_{\perp \mathbf{h}} &= \mathbf{e} \times \mathbf{M}(\mathbf{h}) \times \mathbf{e} = \mathbf{M}(\mathbf{h}) - \mathbf{e} \ (\mathbf{e} \cdot \mathbf{M}(\mathbf{h})) \n\end{aligned}
$$

Magnetic Structure Factor

**FRACTION**

\nensity of a magnetic Bragg reflections is proportional to the square of  
\netic interaction vector.

\n
$$
I_M(\mathbf{h}) \approx |\mathbf{M}_{\perp}(\mathbf{h})|^2 = |\mathbf{M}(\mathbf{h}) - (\mathbf{M}(\mathbf{h}) \cdot \mathbf{e}) \mathbf{e}|^2
$$
\non Vector

\n
$$
\mathbf{M}_{\perp \mathbf{h}} = \mathbf{e} \times \mathbf{M}(\mathbf{h}) \times \mathbf{e} = \mathbf{M}(\mathbf{h}) - \mathbf{e} \cdot (\mathbf{e} \cdot \mathbf{M}(\mathbf{h}))
$$
\nFactor

\n
$$
\mathbf{M}(\mathbf{h}) = p \sum_{j=1}^{n} O_j f_j(\mathbf{h}) T_j \sum_{s} \mathbf{S}_{\mathbf{k} s} \exp \{2\pi i \left[ (\mathbf{H} + \mathbf{k}) \{ S | \mathbf{t} \}, \mathbf{r}_j \right] \} \qquad 1/2 \cdot r_e \cdot \gamma = p = 0.2696 \cdot 10^{-12} \text{ cm}/\mu_{\text{B}}
$$

*j* : index running for all magnetic atom sites in the magnetic asymmetric unit  $(j = 1,...n)$ 

*s* : index running for all atoms of the orbit corresponding to the magnetic site *j* (*s*=1,… *n<sup>j</sup>* ). Total number of atoms:  $N = \sum n_j$ 

 $\{S | \mathbf{t}\}_s$  Symmetry operators of the propagation vector group

# **Diffraction Patterns of magnetic structures**

#### Portion of reciprocal space



- Magnetic reflections
- Nuclear reflections

**Magnetic reflections**: indexed by a set of propagation vectors {**k**}

- **h** is the scattering vector indexing a magnetic reflection
- **H** is a reciprocal vector of the crystallographic structure
- **k** is one of the propagation vectors of the magnetic structure
	- ( **k** is reduced to the Brillouin zone)

Representation in the Real Space Diffraction pattern Reciprocal Space



# **Commensurate Magnetic structures can be described by the periodic repetition of a magnetic unit cell**, just as crystal structures are described by translation of a nuclear unit cell

We are going to use a description based on the **nuclear unit cell** and a **'propagation vector**', **k**, that describes the relation between moment orientations of equivalent magnetic atoms in different nuclear unit cells

### **Diffraction Patterns of magnetic structures**



The magnetic moment " $m_{li}$ " associated with the atom " $j$ " at the unit cell with origin in **R***<sup>l</sup>* , is determined by the Fourier series:

$$
\mathbf{m}_{lj} = \sum_{\mathbf{k}} \mathbf{S}_{\mathbf{k}j} \exp\{-2\pi i \mathbf{k} \mathbf{R}_{l}\}
$$

Where **Sk***<sup>j</sup>* are the Fourier components (linear combination of Basis vectors of irreps) with propagation vector **k** corresponding to the atom *j* in the zero<sup>th</sup> unit cell.

For getting  $\mathbf{m}_{li}$  as real vectors  $\mathbf{S}^*_{ki} = \mathbf{S}_{-ki}$  should be satisfied

The summation is taken over a discrete set of wavevectors that are confined to the first Brillouin zone of the Bravais lattice of the nuclear cell

The magnetic moment can also be written using the next formulation:  
\n
$$
\mathbf{m}_{ij} = \sum_{\mathbf{k}} \mathbf{S}_{\mathbf{k}j} \exp\{-2\pi i \mathbf{k} \mathbf{R}_{l}\} = \sum_{\mathbf{k}} \mathbf{T}_{\mathbf{k}j} \exp\{-2\pi i \mathbf{k} \mathbf{R}_{lj}\}
$$

Where:

$$
\mathbf{R}_{lj} = \mathbf{R}_l + \mathbf{r}_j = l_1 \mathbf{a} + l_2 \mathbf{b} + l_3 \mathbf{c} + x_j \mathbf{a} + y_j \mathbf{b} + z_j \mathbf{c}
$$



With  $\mathbf{R}_{\mathsf{I}} = (I_{1}, I_{2}, I_{3})$  being a translation vector and  $(x_{j}, y_{j}, z_{j})$  being the fractional coordinates of atom *j* within the unit cell.

One can use whatever of the two formulae keeping in mind the relation:  $\mathbf{S_{kj}} = \mathbf{T_{kj}} \exp\{-2\pi\mathbf{i}\ \mathbf{r}_j\}$ 

The **Sk***<sup>j</sup>* (or **Tk***<sup>j</sup>* ) correspond to the *Fourier components* of the magnetic moment of atom *j.* **Sk***<sup>j</sup>* is a complex vector so *6 components are needed.*

A magnetic structure could be fully described by:

- Wave-vector(s) or propagation vector(s) {**k**}.
- Fourier components **Sk**<sup>j</sup> for each magnetic atom *j* and **k**-vector **Sk**<sup>j</sup> is a complex vector.

General expression of the Fourier coefficients (complex vectors) for an arbitrary site when **k** and – **k** are not equivalent can be written as:

$$
\mathbf{S}_{\mathbf{k}} = \frac{1}{2} (\mathbf{R}_{\mathbf{k}} + i \mathbf{I}_{\mathbf{k}}) \exp\{-2\pi i \phi_{\mathbf{k}}\}
$$

Only six parameters are independent. The writing above is convenient when relations between the vectors **R** and **I** are established (e.g. when  $|\mathbf{R}|=|\mathbf{I}|$ , or **R** . **I** =0). A phase  $\phi$  is also added to handle the possible shift among different magnetic sites.

**SIMPLE PROPAGATION VECTOR**  $K = (0, 0, 0)$ 







ingle magnetic site

\n
$$
\mathbf{m}_{ij} = \sum_{\{\mathbf{k}\}} \mathbf{S}_{\mathbf{k}j} \exp\left\{-2\pi i \mathbf{k} \mathbf{R}_{i}\right\} = \mathbf{S}_{\mathbf{k}j}
$$
\nwhere  $\mathbf{k} = (0, 0, 0)$ .

- **Fourier coefficients are real and equal to magnetic moments**
- **The magnetic cell is identical to the crystallographic unit cell**
- **Magnetic symmetry: conventional crystallography plus spin reversal operator crystallographic magnetic groups**

#### **SINGLE PROPAGATION VECTOR K=1/2 H**

The propagation vector is ½ a vector of the reciprocal space vector

$$
\mathbf{m}_{ij} = \sum_{\{\mathbf{k}\}} \mathbf{S}_{\mathbf{k}j} \exp \{-2\pi i \, \mathbf{k} \, \mathbf{R}_{i}\} = \mathbf{S}_{\mathbf{k}j} \, (-1)^{n(l)}
$$

 $S_{kj} = m_{0j}$  corresponds, in this case, to the magnetic moment of the atom *j* within the zero<sup>th</sup> (nuclear) cell.<br>
• REAL Fourier coefficients = magnetic moments<br>
• The magnetic symmetry may also be described using conve

- REAL Fourier coefficients  $\equiv$  magnetic moments
- 

#### **FOURIER COEFFICIENTS OF SINUSOIDAL STRUCTURES**

- **k** interior of the Brillouin zone (pair **k**, -**k**). In other words **k**-(-**k**) is not a reciprocal space vector

- Real **S<sup>k</sup>** , or imaginary component in the same direction as the real one

**IRIER COEFFICIENTS OF SINUSOIDAL STRUCTURES**

\nterior of the Brillouin zone (pair **k**, -**k**). In other words **k**-(-**k**)

\nat a reciprocal space vector

\n**all** 
$$
S_{\mathbf{k}}
$$
, or imaginary component in the same direction as the

\n**on**

\n
$$
\mathbf{m}_{ij} = \mathbf{S}_{kj} \exp(-2\pi i \mathbf{k} \mathbf{R}_{l} + \mathbf{S}_{-kj} \exp(2\pi i \mathbf{k} \mathbf{R}_{l})
$$

\n
$$
\mathbf{S}_{\mathbf{k}j} = \frac{1}{2} m_{j} \mathbf{u}_{j} exp(-2\pi i \phi_{kj}) exp(-2\pi i \mathbf{k} \mathbf{R}_{l} + \frac{1}{2} m_{j} \mathbf{u}_{j} exp(2\pi i \phi_{kj}) \cdot exp(2\pi i \mathbf{k} \mathbf{R}_{l})
$$

\n
$$
\mathbf{m}_{ij} = \frac{1}{2} m_{j} \mathbf{u}_{j} exp(-2\pi i (\mathbf{k} \mathbf{R}_{l} + \phi_{kj})) + \frac{1}{2} m_{j} \mathbf{u}_{j} exp(2\pi i (\mathbf{k} \mathbf{R}_{l} + \phi_{kj}))
$$

\n
$$
\mathbf{m}_{ij} = m_{j} \mathbf{u}_{j} \cos 2\pi (\mathbf{k} \mathbf{R}_{l} + \phi_{kj})
$$



#### **FOURIER COEFFICIENTS OF HELICAL STRUCTURES**

- **k** interior of the Brillouin zone - Real component of **S<sup>k</sup>** perpendicular to the imaginary component



$$
\mathbf{S}_{\mathbf{k}j} = \frac{1}{2} \Big[ m_{\mathbf{k}j} \mathbf{u}_{j} + i m_{\mathbf{k}j} \mathbf{v}_{j} \Big] exp(-2\pi i \phi_{\mathbf{k}j})
$$

$$
\mathbf{B}_{kj} - \frac{1}{2} \left[ m_{uj} \mathbf{u}_j + \iota m_{vj} \mathbf{v}_j \right] \exp(-2\pi i \varphi_{kj})
$$
  

$$
\mathbf{m}_{lj} = m_{uj} \mathbf{u}_j \cos 2\pi (\mathbf{k} \mathbf{R}_l + \phi_{kj}) + m_{vj} \mathbf{v}_j \sin 2\pi (\mathbf{k} \mathbf{R}_l + \phi_{kj})
$$

#### **REPRESENTATION ANALYSIS**

A reducible representation of the propagation vector group can be constructed by selecting the atoms of a Wyckoff position and applying the symmetry operators to both positions and axial vectors (spins).

This gives rise to the so called Magnetic Representation of dimension: 3n<sub>a</sub> (being n<sub>a</sub> the number of atoms in the primitive cell)

The basis functions, for each Irrep and each sublattice of a Wyckoff site, can be calculated by using the projection operator formula. The basis functions are constant vectors of the form (1,0,0), (0.5, 1,0) … with components referred to the crystallographic unitary frame: {**a**/a, **b**/b, **c**/c} attached to each sublattice.

**SARAh** (Spin and Representation Analysis How-To) – SARAh is widely used for magnetic symmetry analysis.

I**SODISTORT** – Part of the ISOTROPY software suite. ISODISTORT is widely used for symmetry analysis in structural refinements.

**BasIreps** – BasIreps is part of the FullProf suite and allows for the calculation of irreducible representations of space groups for structural and magnetic systems.

**Bilbao Crystallographic Server**. It provides tools for representation analysis specifically focused on coupling magnetic symmetry with crystallographic data, useful for magnetic structure analysis.

#### **SUITE OF DIFFRACTION INSTRUMENTS AT ILL**

Powder Diffractometers:

- **D1B** 2-axis diffractometer (CRG)
- **D2B** high-resolution 2-axis diffractometer
- **D4** diffractometer for liquids and amorphous substances
- **D7** diffuse scattering spectrometer (polarized neutrons)
- **D20** high-flux 2-axis diffractometer
- **SALSA** the strain imager

**XtremD** diffractometer for extreme conditions experiments (CRG)

#### **D1B** 2-axis diffractometer

PHYSICAL REVIEW B 109, 104411 (2024)

#### Exploring magnetism and magnetoelectric properties in the green phase of  $R_2BaCuO<sub>5</sub>$  $(R = Er, Eu, Y, Tm, and Lu)$ : The role of 4f-3d exchange coupling

Premakumar Yanda,<sup>1</sup> N. Boudjada,<sup>2</sup> Juan Rodríguez-Carvajal,<sup>3</sup> and A. Sundaresan<sup>1</sup>



FIG. 1. (a) Crystal structure of  $R_2BaCuO_5$  where R is rare-earth metal ion. (b) Lattice parameters with Vegard's law fitting and (c) Unit cell volume with respect to rare earth ionic radius.

#### **D20** 2-axis diffractometer





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#### Anionic and Magnetic Ordering in Rare Earth Tantalum Oxynitrides with an  $n = 1$  Ruddlesden-Popper Structure

Published as part of the Chemistry of Materials virtual special issue "C. N. R. Rao at 90".

Jhonatan R. Guarín, Carlos Frontera, Judith Oró-Solé, Bastian Colombel, Clemens Ritter, François Fauth, Josep Fontcuberta,\* and Amparo Fuertes\*



#### **SUITE OF DIFFRACTION INSTRUMENTS AT ILL**

Single Crystal Diffractometers:

- **D3** polarised hot-neutron diffractometer
- **D9** hot neutrons
- **D10** 4-circle & 3-axis
- **D23** 2-axis spectrometer (CRG)

**Orient Express** Laue diffractometer for align and check samples

#### **D9** hot neutron





#### **D10** thermal neutron diffractometer



# Thank you for your kind attention



71, rue des Martyrs - 38000 Grenoble - www.ill.eu