

Cy Jeffries EMBL Hamburg





Small angle scattering

Wave properties: Scattering amplitudes



When the incident beam is **scattered elastically** (no change in λ) and if preserved distance correlations exist between the scattering centres, a **coherent wavefront** develops that emanates from the sample where both constructive and destructive interference occurs in the wave amplitudes.

The magnitude of the **coherent scattering amplitudes** as a function of angle relates to spatial correlations between scattering centres.

Destructive interference





Of course, macromolecules have many, many atom pair distance correlations within extent of their volume boundary. The coherent wave front is derived from the *sum* of the scattered waves from all of these correlations as a function of angle.



More formally:

If the distances, *r*, between the atoms of a marcomolecule are preserved then the amplitudes of the *coherent* wave front through *s* are proportionate to the sum of the atomic scattering factors (i.e., probability to scatter) weighted by the *distribution of the distances* between scattering pairs.





The issue?

We cannot access the amplitudes experimentally. We measure the *intensity* of the scattered radiation, I(s).

For solution-based SAXS, the sample particles are tumbling in solution!

Sample



$$I(s) = \left\langle A(s)A(s)^* \right\rangle \xrightarrow{\text{All orientations}} \underset{\text{isotropic scattering}}{\text{All orientations}}$$

l(s) fundamentally boils down to the form factor of the particles, P(s), their volume and scattering power!



The scattering intensity l(s) – and thus the associated form factor in reciprocal space – relates to an atom-pair distance distribution function of the particle p(r) in real space by a Fourier transform:



How does the sample properties combined with the measurement impact our approach to modelling data?

Populations of macromolecules in the sample

Sum of spherically averaged scattering amplitudes

population state(s) distribution

 $I(s) = \left\langle A(s)A(s)^* \right\rangle$

Distance distributions are encoded in the scattering intensities – not x,y,z atomic coordinates





The Complication

For biological macromolecules in solution, we forgot the solution!

It is obvious that macromolecules of a sample will scatter. The amplitudes arising from preserved distance correlations will *sum* to produce coherent scattering intensities at low angle.

The lower the angle (lower *s*), the *longer* the correlated distances, *d*:

$$s = 2\pi/d$$

However, the solution, i.e., the solvent of the sample, also scatters! As the solvent (hopefully) does not have any time-preserved long-range distance correlations, its scattering contributions add as a 'flat incoherent background' in the SAS regime.



But you need contrast as well: 'excess scattering power'

l(s) in the small-angle region depends, and indeed only arises, if there is a difference between the average scattering length density of the **solvent** and the average scattering length density of the particles of interest. This difference is known as contrast and is represented as

$$\Delta \rho = \overline{\rho} - \rho_{\rm s},$$

where $\overline{\rho}$ and ρ_s are the mean scattering length densities of the particle and the solvent, respectively.





 $I(s) \propto \Delta \rho^2$



How do I calculate the Contrast

http://smb-research.smb.usyd.edu.au/NCVWeb/

MULCh: Modules for the analysis of small-angle neutron contrast variation data from bio-molecular assemblies.

For X-rays: Convert the SLD, ρ (10¹⁰ cm⁻²) to electron density by dividing by the Thomson electron radius: 2.8179 x 10⁻¹³ cm. The answer is in e/cm³, so divide again by 10^{24} to get e/Å³...or more quickly:

$$\frac{\rho}{28.179}$$
 e/Å³

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0.836

1.526

matchpoints: v/v 2H_O

3.451 2.228

2.893

2.335

1.778 0.588 0.042

0.468

1.682

1.135

-0.505 0.663 0.105

1.052

ii) Total

neutron

SANS

Whole complex SANS

matchpoint: v/v 2H2O

contrast for



After background subtraction...

I(*s*) will represent the time and rotationally averaged squared scattering amplitudes from the particle population expressed as the summed contribution from each individual particle, *i*, in the sample.





For a PURE, MONODISPERSE and IDEAL sample

The concentration.

$$V$$

$$I(s) = N(\Delta \rho V)^2 P(s)$$

If all particles are identical, and do not interact, the *l*(*s*) profile (*after* background scattering has been subtracted) will represent the time and rotationally averaged squared scattering amplitudes, i.e., the scattering intensity, from a *SINGLE PARTICLE*.



How do I <u>maybe</u> know I have an ideal system?

The molecular mass estimates through a concentration series.

The MM, the MM, the MM, the MM, the MM.

(+/-10 %)

Think about this – there is no point generating a single model to describe a 100 kDa protein if the experimental MW of the protein from SAS is 125 kDa (probably a mixture).

$$I(s) = \sum_{i}^{n} \left[\left(\Delta \rho_{i} V_{i} \right)^{2} P_{i}(s) \right]$$

If *i* are not identical, model as a mixture

multiple population states: mixtures

 $I(s) = N(\Delta \rho V)^2 P(s)$

If *i* are all identical, model as a single particle



The case of intrinsically disordered proteins or modular proteins connected by flexible linkers.

$$I(s) = \sum_{i}^{n} \left[\left(\Delta \rho_{i} V_{i} \right)^{2} P_{i}(s) \right]$$

Model as a structural ensemble!

population state distribution

MM is correct!

BUT

Still not ideal, i.e., cannot be modelled as a non-interacting single particle because the protein is structurally heterogeneous!



I(0)

At zero angle (s = 0) the magnitude of I(s) will primarily depend on the number of scattering centres within the bound squared-volume of a macromolecule – independent of the shape – weighted by the concentration and contrast squared:

$$I(0) \approx N(\Delta \rho V)^2$$

From this parameter, it is possible to obtain the *molecular weight*.

Data scaled to a standard protein with a KNOWN concentration and molecular weight

$$MW_{sample} = \frac{I(0)_{sample} N_{A}}{c_{sample} (\Delta \rho v_{sample})^{2}}$$

 $MW_{sample} = \frac{I(0)_{sample}}{I(0)_{standard}} \times \frac{c_{standard} \Delta \rho_{standard}^2}{c_{sample} \Delta \rho_{sample}^2} \times MW_{standard}$ An assumption that a target has a similar scattering length

density and partial specific volume as the secondary standard!

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Absolute scaling - requires partial specific volume and contrast.

Porod volumes and Kratky plot

The determination of MW from I(0) requires an accurate assessment of the concentration of a protein in solution that in and of itself can be difficult to determine!

An alternative concentration-independent estimate of MW is based on the volume of a protein in solution. Porod showed that for uniform particles with sharp boundaries the excluded volume *Vp* can be calculated as:

$$V_p = \frac{2\pi^2 I(0)}{Q}$$

where Q is the Porod invariant or the area under a plot of $I(s)s^2$ vs s calculated to $s = \infty$, or Kratky plot.

The Vp of a protein in nm^3 is typically 1.5–1.6 times the MW in kilodaltons (kDa).

However, caution must be applied when dealing with highly anisotropic or highly flexible/disordered proteins. In the case of flexible, or rod-like proteins, the decay in scattering intensities at high angle deviates sufficiently from Porod's law that the estimation of *Q* will incur errors in the volume estimation!

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Why is all this stuff important for SANS?





The dependency of I(0) with f_{D20} should be represented accurately by a parabolic curve that intersects the f_{D20} axis at a single point, corresponding to the matchpoint of the complex. The dependency of I(0) with f_{D20} should be reasonably well represented by a parabolic curve that intersects the f_{D20} axis at a single point. The parabola will typically miss data points at the

The dependency of I(0) with f_{D20} should be reasonably well represented by a parabolic curve that intersects the f_{D20} axis at a single point. The parabola will typically miss data points at the extremities of the plot, but the deviation of the estimated match-point from the real value is usually small. Of note, I(0)values from a dissociating complex can be accurately represented by general quadratic function, but that function will not intersect the f_{D20} axis.

The dependency of I(0) with f_{D20} will not be reasonably well represented by a parabolic curve that intersects the f_{D20} axis at a single point. The parabola will typically miss many data points, and the deviation of the estimated match-point from the real value is usually significant.

0.2 0.4 0.6 0.8 1.0 f_{D20}

c. Oligomerisation/

Aggregation

9.0

0,2 0,4



Modelling SAS data – before you leap into danger.

- Understand the data get the unit right, nm or Å, etc.
- Extract structural parameters and additional information *BEFORE* you begin modelling: if there is one thing you can trust it is the structural parameters from SAS data!





Modelling SAS data – before you leap into danger.

• Obtain as much information as possible about your system.



Get to know your data before you model!

Fundamental plots

- 1) Guinier
- 2) P(r)
- 3) Dimensionless Kratky
- 4) Porod-Debye



- Receveur-Bréchot & Durand (2012) Current Protein and Peptide Science, 13, 55-75.
- Durand D, Vivès C, Cannella D, Pérez J, Pebay-Peyroula E, Vachette P, Fieschi F. (2010) *J Struct Biol.* 169: 45-53.



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Research Article

Characterizing flexible and intrinsically unstructured biological macromolecules by SAS using the Porod-Debye law

Robert P. Rambo, John A. Tainer 🔀

First published: 20 April 2011 | https://doi.org/10.1002/bip.21638 | Citations: 412

Get to *know your data* before you model!

2) Five programs:

- AutoRg for first assessments of s_{min}
- SHANUM define the useful s_{max} .
- DATCLASS machine-learning method for the rapid geometric classification of SAXS data (from proteins).
- DARA kd-tree searching of the PDB + Alphafold DB for similar scattering profiles.
- AMBIMETER assess the ambiguity of the scattering data.





After cleaning up! Is there sufficient data at the very lowest s to encompass the particle size?

At minimum, s_{min} should = π/D_{max} Better rule of thumb, s_{min} = $1/D_{max}$ EMBL

Shanum and s_{max}

Shanum takes into account the statistical variance in the data to assess the useable s_{max} .



Shanum will also estimate D_{max} for you (or you can enter it yourself)



Datclass

 Classification of a protein shape using machine learning methods based on the scattering profiles calculated from a continuum of 488 000 geometric objects including intrinsically disordered polymers



99.98% of the PDB maps into the classifier space.





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Ambimeter

Based on a set of (several thousand) shape topologies with pre calculated scattering profiles.



- Provides a sense of how ambiguous a dataset is with respect to fitting models.
- An ambimeter score of 0 to 1.5(ish) are 'potentially unique' shapes.
- An ambimeter score of 2.9, for example, is very highly ambiguous.

What to do?

- Always run modelling routines several times!
- Use information from other techniques.
- Perform parallel modelling against several SAS datasets.



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More information = less ambiguity! From other methods.



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https://doi.org/10.1107/S2059798324005497





Summary: Know your data.

- *R_g* and *I*(0) from Guinier and *p*(*r*) check for consistency through a concentration series. Identify concentration independent interparticle interactions: coulombic-repulsive or aggregation. Deal with it.
- *Prepare your data for modelling:* s_{min} and s_{max} from AUTORG and SHANUM. Make sure s_{min} is at least = π/D_{max} , or better yet, $1/D_{max}$!
- Molecular mass estimates very important for guiding the modelling approach!
- Ambiguity.



Any questions so far?


Lets do some atomistic model fitting!

https://www.sasbdb.org/data/SASDFP8/



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Approach	Modeling of the hydration layer	Representation of the molecule	References	
CRYSOL	Implicit layer using an envelope function	All-atom	Svergun et al. J. Appl. Cryst. (1995)	CRYSON – for SANS
AXES	Explicit water molecules using equilibrated water boxes	All-atom	Grishaev <i>et al.</i> <i>JACS</i> (2010)	
FoXS	Implicit layer based on surface accessibility	All-atom or coarse-grained	Schneidman- Duhovny <i>et al.</i> <i>NAR</i> (2010)	FoXS – Debye formula $\sin qr_{ii}$
HyPred	Explicit water molecules based on MD simulations	All-atom	Virtanen <i>et al.</i> <i>Biophys. J.</i> (2011)	$I(q) = \sum_{i} \sum_{j} w_i w_j \frac{1}{q r_{ij}}$
AquaSAXS	Solvent-density map using the dipolar PB- Langevin approach	All-atom	Poitevin <i>et al.</i> Di NAR (2011)	na Schneidman-Duhovny…is here!



Ар	proach	Modeling of the hydration lay	Representation	References	de		
CR	YSOL	Implicit layer us envelope funct	WAXS	S	WAXS in Solvent (WAXSIS) computes small- and wide-angle X-ray scattering curves based on explicit-solvent all-atom molecular dynamics simulations.		
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Pepsi	-SAXS			Pepsi-SANS		
PEPSI-SAXS and PEPSI-SANS					EMBL 🌒	

research papers

IUCRJ ISSN 2052-2525 BIOLOGY MEDICINE Benchmarking predictive methods for small-angle X-ray scattering from atomic coordinates of proteins using maximum likelihood consensus data

Jill Trewhella,^a* Patrice Vachette^b* and Andreas Haahr Larsen^c

Received 5 March 2024 Accepted 23 May 2024 ^aSchool of Life and Environmental Sciences, University of Sydney, NSW 2006, Australia, ^bInstitute for Integrative Biology of the Cell (12BC), Université Paris-Saclay, CEA, CNRS, Gif-sur-Yvette, Paris 91198, France, and ^cDepartment of Neuroscience, University of Copenhagen, Blegdamsvej 3, 2200 Copenhagen, Denmark. *Correspondence e-mail: jill.trewhella@sydney.edu.au, patrice.vachette@i2bc.paris-saclay.fr

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Workhorse in ATSAS: CRYSOL (for SAXS)

SLD of the solvent

Convert the atomic coordinates of a model into a convenient mathematical expression for fitting or modelling.

Calculate the envelope function from the centre of the macromolecule from a common/coincident grid origin.

Take into account the atomic scattering, the excluded volume and hydration shell scattering.





Workhorse in ATSAS: CRYSOL

$$\mathbf{I}(\mathbf{s}) = \left\langle \left| \mathbf{A}(\mathbf{s}) \right|^2 \right\rangle_{\Omega} = \left\langle \left| \mathbf{A}_{a}(\mathbf{s}) - \rho_{s} \mathbf{A}_{s}(\mathbf{s}) + \delta \rho_{b} \mathbf{A}_{b}(\mathbf{s}) \right|^2 \right\rangle_{\Omega}$$

- Either fit the experimental data by varying the density of the hydration layer δρ (affects the third term) and the total excluded volume (affects the second term).
- Or predict the scattering from the atomic structure using default parameters (theoretical excluded volume and bound solvent density of 1.1g/cm³).
- Provide output files (scattering amplitudes) for rigid body refinement routines.
- Compute particle envelope function F(ω)





atom in the sample.

The 'spherical wave bit' can be mathematically expressed in terms of a summed set of independent *spherical harmonics* (as a multipole expansion):

$$I(s) = 2\pi^{2} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} |A_{lm}(s)|^{2}$$

In 1970, Stuhrmann showed that the information content of a SAXS profile can be conveniently described in terms of a sum of spherical harmonic functions.



Essentially given a set of atomic coordinates in 3-dimensions (i.e., x, y, z coordinates), and knowing the identity of each atom at that coordinate (i.e., the atomic form factor), as well as the atomic volumes and scattering length densities, we can calculate the scattering amplitudes from the entire structure. As a result we can calculate the scattering intensities (i.e., the square of the scattering amplitudes.)



How many spherical harmonics to use in CRYSOL?

If you use the first harmonic only, i.e., zeroth-order, then the calculated intensities from the model will be a sphere. This is okay only if you want to describe the overall SIZE of the object, i.e., at the very lowest of angles in the Guinier region of the scattering profile. The zeroth-order harmonic dominates the very lowest angles of a calculated scattering profile!

If you use two harmonics, you will introduce an additional 'shape feature' into the calculated scattering intensities across *s*...but the resulting shape will probably still look like a sphere...with a couple of very low humps.

If you continue to increase the number of harmonics, you introduce additional shape features across **s**. However, the more harmonics you introduce the less impact on the overall calculated scattering is observed at the low angles (i.e., in the SAXS regeime).

Typically 15-30 harmonics are used to describe size and the shape of the object. However, this depends on the <u>CLASSIFICATION</u> of an object. Clearly, if the object is an extended rod, you probably need additional spherical harmonics terms.



I(s) from a globular structure using different numbers of harmonics





...yes this protein is real. It is called SASG – SASBDB search it. Lower order capture Guinier Higher-orders required to describe the anisotropic structure! Computationally expensive! 30 and 50 10 5 0.1 0.2 0.3 0.4

I(s) from an extended structure using different numbers of harmonics



Centre your atomic models!

 THE MODEL SCATTEING AMPLITUDES (and therefore the resulting intensities) MUST BE CALCULATED FROM THE ORIGIN, i.e., the models must be centred, otherwise you loose low-order harmonic contributions.

ATSAS tool: alpraxin



For macromolecules with cavities and holes – explicit hydration water using CRYSOL3

- Hydration shell representation as envelope function (CRYSOL implicit solvent layer) or dummy solvent beads, i.e., explicit solvent layer (CRYSOL 3).
- Explicit solvent modelling is important for internal cavities!



Outer shell

- Internal shell
- Small cavities (excluded volume adjustment)

Especially important for ring-shaped, hollow sphere, very small (less than 10 kDa) or very extended particles. Otherwise CRYSOL is fine.



Assessing data-model fits – χ^2

...knowing what model does NOT fit the data can be as valuable as knowing what model(s) do fit the data!





A note on
$$\chi^2$$



The errors on the scattering intensities need to be correctly specified, otherwise the test is, by default, INVALID. Errors follow Poisson counting statistics that limit to a gaussian distribution after many repetitions (for photon counting detectors).

If the errors are correctly specified and no significant (systematic) deviations are present between the experimental and modeled intensities, the value should lie in the range of approximately 0.9-1.1 depending on the number of points in the dataset (0.9-1.1 is typical for over-sampled SAXS data on modern detectors).



Same intensities, same model, but different error estimates



Correlation Map: CorMap P

$$J = \begin{pmatrix} \vdots \\ I(q_k) \\ \vdots \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \ddots & \\ & \sigma(I(q_k))^2 & \cdots & \sigma(I(q_k), I(q_l)) \\ & \vdots & \ddots & \vdots \\ & \sigma(I(q_l), I(q_k)) & \cdots & \sigma(I(q_l))^2 \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & &$$

$$\sigma(I_{\exp}(q_k))^2 = \frac{1}{m-1} \sum_{i=1}^m (I_{\exp}(q_k)_i - \overline{I}_{\exp}(q_k))^2$$

On-diagonal variance.

$$\begin{split} \sigma(I_{\exp}(q_k), I_{\exp}(q_l)) &= \\ \frac{1}{m-1} \sum_{i=1}^m (I_{\exp}(q_k)_i - \overline{I}_{\exp}(q_k)) (I_{\exp}(q_l)_i - \overline{I}_{\exp}(q_l)) \end{split}$$

Off-diagonal co-variance between all pointto-point q_k and q_l .



Error normalized residual plots

+3

 $\Delta \sigma$

-3

Model fits to the data are also evaluated using • normalized residual plots to help assess systematic model-fit deviations from the data in addition to over or under-estimation of the errors.



resudual =

(I(s)experiment - cI(s)model)

Lets do some atomistic model building!

SREFLEX OLIGOMER SASREF BUNCH CORAL Ensemble Optimization Method (EOM)



Modelling 3D-structures that fit SAXS data is perhaps the fundamental 'art' of small-angle scattering!

The major considerations to keep in mind when modelling SAS data are:

There is often more than one model that fits the data equally well.

SAS data is inherently noisy. SAS data is inherently ambiguous.



Lets do the easy bit first: get the right sequence and the right PDB (or .cif) file(s).

- You should know the amino acid sequence of the protein (or polynucleotide, any other macromolecule, etc.) used for the SAS experiment. You should also know if the macromolecule binds metals, ligands, lipids, detergents, is glycosylated, etc.
- For proteins, use UNIPROT as a a fundamental resource to obtain the correct canonical sequence: <u>www.uniport.org</u>
- You should know what rigid-body (or bodies) you want to use for the modelling, i.e., the atomic coordinate PDB or .cif files.

• Extract the amino acid sequence from the PDB file.

- Align the atomic coordinate (.pdb/cif) amino acid sequence with the amino acid sequence of the
 <u>EXACT</u> protein used for the SAS measurement.
- Deal with missing side-chains in the atomic coordinate file (account for ALL OF THE MASS).









Clustal Omega is a new multiple sequence alignment program that uses seeded guide trees and HMM profile-profile techniques to generate alignments between three or more sequences. For the alignment of two sequences please instead use our pairwise sequence alignment tools.

Important note: This tool can align up to 4000 sequences or a maximum file size of 4 MB.

STEP 1 - Enter your input sequences	Input form	Web services	s
Enter or paste a set of			
PROTEIN *	Tools > Multip	le Sequence A	ligr
sequences in any supported format:	Results f	or job clus	sta
> <u>SAS_</u> Protein	rioountori		510
HMHHHHHTTRGSNNEEAICSLCDKKIRDRFVSKVNGRCYHSSCLRCSTCKDELGATCFLREDSMYCRAHFYKKFGTKCSSCNEGIVPDHV/RKASN HVYHVECFQCFICKRSLETGEEFYLIADDARLVCKDDYEQARDGGSGGHMGSGGGIGPLMVQPATPHIDNTLGGPIDIQHF	Alignments	Result Summ	ary
>PDB_sequence GSNNEEAICSLCDKKIRDRFVSKVNGRCYHSSCLRCSTCKDELGATCFLREDSMYCRAHFYKKFGTKCSSCNEGIVPDHVVRKASNHVYHVECFQCF ICKRSLETGEEFYLIADDARLVCKDDYEQARDGGSGGHMGSGGGGPLMVQPATPHIDNTLGGPIDICHF	Download A	lignment File	Sh
Or, upload a file: Browse No file selected.	CLUSTAL O(1.2	.4) multiple se	que

Oops! Part of the sequence missing in the PDB file! This missing fragment will have to be built. Do not worry...ATSAS rigid-body modelling programs can deal with this!



PLEASE NOTE: Showing colors on large alignments is slow.



For proteins RNA, DNA, etc, just use AlphaFold3

https://golgi.sandbox.google.com/





But my structure *almost* fits the data, can I just wiggle it a bit? - SREFLEX

Employs normal modes pattern of motion on domain-partitioned structures.

Automated or manual domain partitioning possible.

Works with proteins!



Deciphering conformational transitions of proteins by small angle X-ray scattering and normal mode analysis

A. Panjkovich, D.I. Svergun (2016) *Phys Chem Chem Phys.* 18, 5707-19

Used for spatial refinement of models using small structural adjustments.

Great for assessing whether slight conformational movements are required to fit SAXS data (e.g., from crystal or AF-predicted structures).





ATSAS online version applied additional CONCORD refinement





ATSAS online version applied additional CONCORD refinement



Combine SREFLEX with Multi-FoXS



Scattering from mixtures

 Possible to obtain the volume fraction contribution to the total scattering profile of individual components of mixtures.

$$I(s) = \sum_{k} v_{k} I_{k}(s)$$



ATSAS program: OLIGOMER





foXS combined with Multi-foXS!

roxs	
Fast SAXS Profile Computation with Debye Formula	
・ <u>Sali Lab Home</u> ・ <u>ModWeb</u> ・ <u>ModBase</u> ・ <u>ModEval</u> ・ <u>PCSS</u> ・ <u>FoXS</u> ・ <u>IMP</u> ・ <u>MultiFit</u> ・ <u>ModPipe</u> ・ X	< · · · · · · · · · · · · · · · · · · ·
Login About FoXS Home Queue Help FAQ Download Links Type PDB code of input molecule or upload files in PDB or mmCIF format (zip file with several PI uploaded): (PDB:chainId e.g. 6lyz:A) or upload file: Choose File no file see Experimental profile: Choose File no file selected (optional) sample input e-mail address: (optional, the results are sent to this address) (optional) Job name: (optional) (optional)	DB/mmCIFs can be Can upload a zip file with multiple structures
Submit Form Clear Advanced Options NEW! MultiPoXS Now with conformational sampling and multi-state modeling, try here	Assess the individual model fits, then also pass the models to Multi-foXS for oligomeric analysis

Schneidman-Duhovny D, Hammel M, Tainer JA, and Sali A. Accurate SAXS profile computation and its assessment by contrast variation experiments. Biophysical

Journal 2013. 105 (4), 962-974 Schneidman-Duhovny D, Hammel M, Tainer JA, and Sali A. FoXS, FoXSDock and MultiFoXS: Single-state and multi-state structural modeling of proteins and their complexes based on SAXS profiles NAR 2016 [FREE Full Text]

Contact: foxs@salilab.org



Structure still does not fit - try some rigid body modelling

- The structures of two (or more) subunits in reference positions are known.
- Arbitrary complex can be constructed by moving and rotating the subunits.
- This operation depends on three Euler rotation angles and three Cartesian shifts.





Structure does not fit - try some rigid body modelling





The target function:

$$E(\lbrace X\rbrace) = \chi^2[(I(s), I_{\exp}(s)] + \sum_i \alpha_i P_i$$

is minimized...basically χ^2 plus penalties!

Penalties describe model-based restraints and/or introduce the available additional information from other methods: MX, NMR, EM, Alphafold etc).

A brute force (grid) search is applied if the number of free parameters is small.

Otherwise a Monte-Carlo based technique (e.g. simulated annealing) is employed to perform the minimization of $E({X})$.







Incorporate information from EM, crystallography, NMR, biochemistry (e.g., cross linking, Mass-spec), FRET and bioinformatics...and of course for proteins...Alphafold!



Default 'sensible' modelling restraints like:

- Minimise clashes.
- Maintain contacts.

+

- Don't shift too far from the origin!
 - For dummy residues, make dihedral angles and Ramachandran geometry sensible.
 - Do not inter-penetrate subunits (interconnectivity).


SASREF (for SAXS), SASREFcv (for SAXS and SANS)

- Each subunit is treated as an individual rigid body. Protein, DNA, RNA, etc.
- Assumes the atomistic models are COMPLETE i.e., no missing fragments or mass!
- Options to perform *MIXTURE* modelling (e.g., monomer-dimer; SASREFmx) or *CONTRAST VARIATION* (SAXS and SANS; SASREFcv).
- Start from arbitrary initial orientations of the subunits at the grid origin.
- Simulated annealing is employed.
- Search of interconnected spatial arrangement of the subunits without clashes.
- Random movement/rotation at one SA step.
- Fitting the scattering data by minimizing the target function.
- · Additional restraints may be applied.

Petoukhov, M. V., and Svergun, D. I. (2006). Eur Biophys J., 35, 567-576



SASREF restraints

Subunit clashes or disconnected models are penalised!



Inter penetrating subunits are penalised.



X Disconnected models are penalised.





SASREF inputs

For SAXS:

- Rigid body starting models centred to an origin. Protein, DNA, RNA, etc.
- Scattering amplitude files of each rigid-body model calculated using **CRYSOL**.
- Contacts file (optional).
- Symmetry information.

Contact information can be exceptionally useful!



No contact information



Single contact

Alphafold3 of course be used if no contact information is available.



Other docking methods

acromolecular Docking with SAXS Profile								
	• Sali Lab Hom	e • <u>ModWeb</u> • <u>ModBase</u> • <u>N</u>	lodEval • PCSS • FoXS • IMP • MultiFit • ModPipe • X					
	L	ogin Web Server About Fe	DXSDock Queue Help Download Links					
Type PDB codes of recepto	or and ligand mo	lecules or upload files in PDB	format sample input files					
Receptor		(PDB:chainId e.g. 2kai:A,B)	or upload file: Choose File no file selected					
Ligand		(PDB:chainId e.g. 2kai:I)	or upload file: Choose File no file selected					
Complex SAXS profile	Choose File	no file selected						
e-mail address			(the results are sent to this address, optional)					
Complex type	Default	0	Please specify receptor and ligand in the corresponding order					
Submit Clear								
Advanced Parameters								
Job name								
Weighted SAXS score			Weighted SAXS scoring that accounts for monomers contributed					
Distance constraints	Choose File	no file selected						
Submit Clear								

If you use FoXSDock (version), please cite:

Schneidman-Duhovny D, Hammel M, Sali A. Macromolecular docking restrained by a small angle X-ray scattering profile. J Struct Biol. 2010 [Abstract] Schneidman-Duhovny D, Hammel M, Tainer JA, and Sali A. FoXS, FoXSDock and MultiFoXS: Single-state and multi-state structural modeling of proteins complexes based on SAXS profiles. NAX SUG16 [FREE Full Text]

Contact: dina@salilab.org



pyDockSAXS Protein-protein interactions using SAXS and computational docking

lob

About

pyDockSAXS server is a web server for rigid-body protein-protein docking that combines computational and experimental information. On one side the server uses the new version of **pyDock** (pyDock 3.0) to energetically evaluate the interaction surface of multiple docking poses. On the other side, the server evaluates the capacity of each pose to describe the experimental curve of SAXS with the program CRYSOL. The appropriate combination of both scoring functions yields predictions of the 3D arrangement of the complex in solution. Please find the details of the method here. **Reference**

Please cite these references if you use

pyDockSAXS server in your research:

 Receptor PDB (?):
 Choose File no file selected

 Ligand PDB (?):
 Choose File no file selected

 SAXS experimental curve (?):
 Choose File no file selected

 Contact email (?): (optional)
 contact@research.edu

For advanced users only (optional):

If you have a rigid-body docking set from previous pyDock runs, you can upload it here (?): Choose File no file selected

For test purposes only:

pyDockSAXS: protein-protein complex structure by SAXS and computational docking, *B Jiménez-Garcia, C Pons, DI. Svergun, P Bernadó and J Fernández-Recio.* Nucleic Acids Research 2015; doi: 10.1093/nar/gkv368

Load example data

I accept that results from pvDockSAXS are offered without warranty



More complicated examples – Dealing with missing stuff, linkers, etc.





Missing stuff, linkers, etc





BUNCH – will optimize domain and dummy-amino acid positions

- For SAXS only!
- Single residue **polypeptide chain only, i.e., 'protein domains'!**
- With or without symmetry **multiple curves allowed**, e.g., domain truncation mutants.
 - Models missing linkers and mass as a set of dummy residues.
- A two-step procedure.
- pre_bunch
 - bunch
- Requires the domain PDB files and the EXACT protein sequence along with the SAXS data and scattering amplitudes calculated by CRYSOL.



BUNCH – will optimize domain and dummy-amino acid positions





ATSAS - CORAL

- SASREF is good for modelling whole/complete complexes against SAXS data.
- BUNCH is good form modelling single polypeptide chains with missing fragments, or linkers connecting modules/domains against SAXS data
 - **CORAL** combines both concepts into one!
- CORAL Protein, DNA, RNA, glycosylated systems and complexes...all are possible!
- Known subunit interfaces can be preserved by grouping subunits together.
- CORAL is also a great deal faster than BUNCH (CORAL can be used to model single polypeptide chains as well, and it is much faster!).

...<u>SAXS only!!</u>



- CORAL requires the SAXS data, domain/subunit atomic coordinate files along with the scattering amplitudes calculated by CRYSOL. A contact file is also possible!
- CORAL requires and additional .con file telling the program where to generate the linkers for each subunit:



Always check the final model fits using CRYSOL

Approach	Modeling of the hydration layer	Representation of the molecule	References	
CRYSOL	Implicit layer using an envelope function	All-atom	Svergun et al. J. Appl. Cryst. (1995)	
AXES	Explicit water molecules using equilibrated water boxes	All-atom	Grishaev <i>et al.</i> <i>JACS</i> (2010)	WAXSI S I Save (MUSE) conjusts and and selected and and an analysis of the selected and an an analysis of the selected and an an analysis of the selected and and an an an analysis of the sel
FoXS	Implicit layer based on surface accessibility	All-atom or coarse-grained	Schneidman- Duhovny <i>et al.</i> <i>NAR</i> (2010)	B a variate NANO-D Agorithms for Modeling and Simulating Nanosystems Cortica @ So
HyPred	Explicit water molecules based on MD simulations	All-atom	Virtanen <i>et al</i> . <i>Biophys. J</i> . (2011)	NOM NOM NAME PARLOCIONS Software (statement) Contact Software P > Software > Software
AquaSAXS	Solvent-density map using the dipolar PB- Langevin approach	All-atom	Poitevin <i>et al.</i> NAR (2011)	NANO-D Agostithms for Modeling and Simulating Nanosystems <u>Contract</u> Work Now National Contract Noted Now National Contract National Contra

Pepsi-SANS

PEPSI-SAXS and PEPSI-SANS /IBL



Always check the final model fits using CRYSOL

CRYSOL Implicit la envelope	ayer using an function	All-atom	Svergun et al. J.	
			Appl. Cryst. (1995)	
AXES You can use of careful, not all dummy-residu				
HyPred Explicit w based on simulation	vater molecules MD ms	All-atom	Virtanen <i>et al.</i> <i>Biophys. J.</i> (2011)	Summer > Pagestess Pepsi-SAXS (8 g team.resp.* c)
AquaSAXS Solvent-o using the Langevin	lensity map dipolar PB- approach	All-atom	Poitevin <i>et al.</i> NAR (2011)	NANO-D Algorithms for Modeling and Simulating Nanosystems

Pepsi-SANS

PEPSI-SAXS and PEPSI-SANS /BL



If there is one message, I want to get across today – always consider ambiguity!

- You <u>must</u> run your selected rigid body modelling routines at least 10 times and check for the spatial consistency of the models (spatial alignment using supcomb).
- At the end of a BUNCH, SASREF or CORAL run check the fits with CRYSOL!
- Use Correlation Map to assess fits if you are unsure about your experimental errors!
- Error normalized residual plots are a great tool to visually assess systematic differences between modelled and experimental scattering intensities.
- ...also apply common sense.
- I usually do 20 modelling runs, check the individual model fits with CRYSOL (using 30 harmonics, minimum), then order the CRYSOL fits in terms of χ^2 and CorMap *P*, then spatially align all models that fit the data to assess consistency.







https://golgi.sandbox.google.com/





Combine with Alphafold!

emre.brookes@umontana.edu, jill.trewhella@sydney.edu.au

research papers



ISSN 1600-5767

AlphaFold-predicted protein structures and smallangle X-ray scattering: insights from an extended examination of selected data in the Small-Angle Scattering Biological Data Bank

^aDepartment of Chemistry and Biochemistry, University of Montana, 32 Campus Drive, Missoula, MT 59812, USA, ^bProteomica e Spettrometria di Massa, IRCCS Ospedale Policlinico San Martino, Largo R. Benzi 10, Genova 16132, Italy, ^cUniversité Paris-Saclay, CEA, CNRS, Institute for Integrative Biology of the Cell (I2BC), Gif-sur-Yvette 91198, France, and ^dSchool of Life and Environmental Sciences, The University of Sydney, NSW 2006, Australia. *Correspondence e-mail:

Emre Brookes,^a* Mattia Rocco,^b Patrice Vachette^c and Jill Trewhella^d*

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J. Appl. Cryst. (2023). 56, 910-926







Is it only a matter of time before AlphaFold can build ensemble models by itself? ...yes



Thank you and goodbye!

SAXS Team@EMBL

Dmytro Soloviov Melissa Gräwert Clement Blanchet Aleksi Sutinen



Everyone involved in ATSAS over the years!



