Evolution of triaxiality in the mass $A \approx 110$ region

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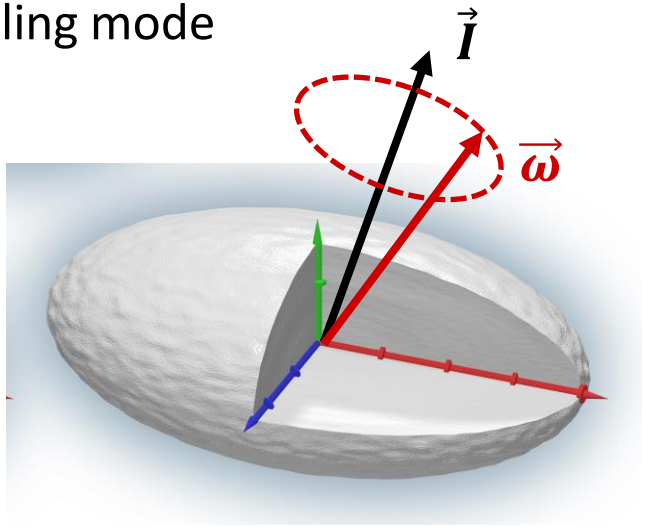
Triaxiality in atomic nuclei

High-spin phenomena that require stable triaxial shape:

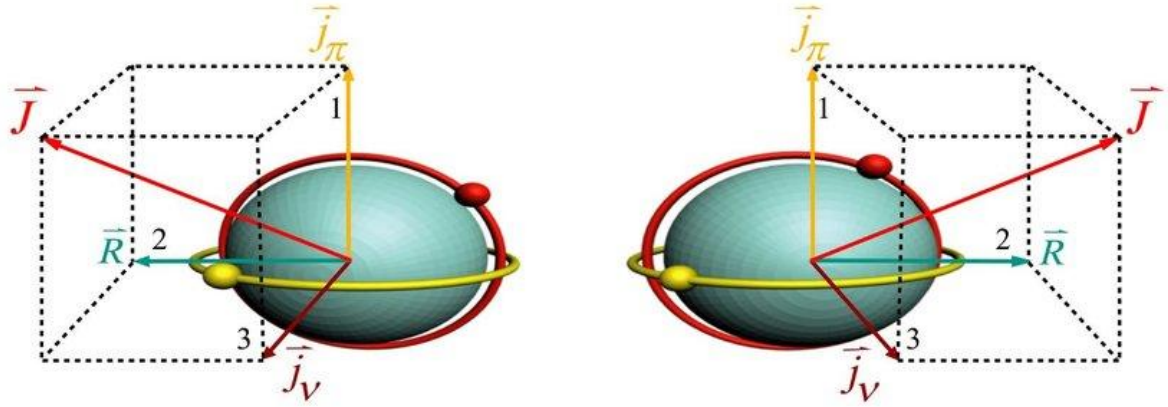
Wobbling mode



$$\hbar\omega_{\mathrm{wob}} = \hbar\omega_{\mathrm{rot}}\sqrt{\frac{(\mathcal{I}_x-\mathcal{I}_y)(\mathcal{I}_x-\mathcal{I}_z)}{\mathcal{I}_y\mathcal{I}_z}}$$

S.W. Ødegård et al, Phys. Rev. Lett. 86, 5866 (2001)
D.R. Jensen et al., Nucl. Phys. A 703, 3 (2002)
A. Görgen et al., Phys. Rev. C 69, 031301(R) (2004)

Chiral doublets



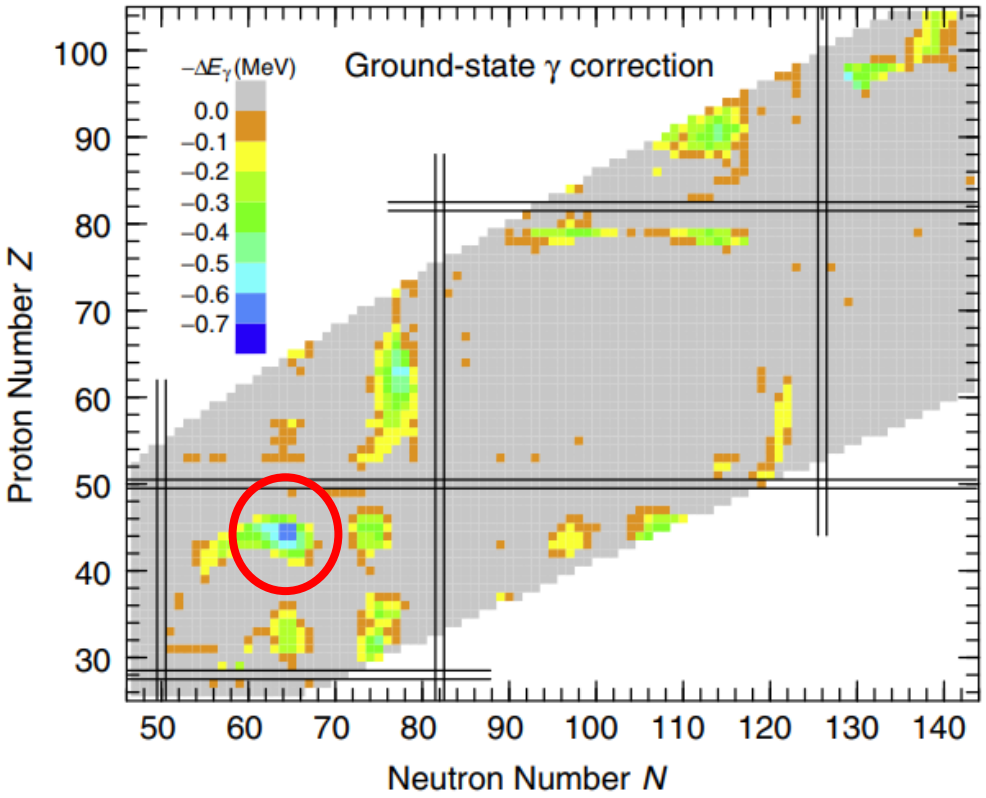
Triaxial odd-odd nuclei:
Angular momenta $\vec{J}$, $\vec{j}_\pi$, $\vec{j}_\nu$ form left- and right-handed systems.

⇒ pairs of near-degenerate rotational bands

K. Starosta et al., Phys. Rev. Lett 86, 971 (2001)

Triaxial ground-state deformation?

Theoretical predictions for ground-state triaxiality



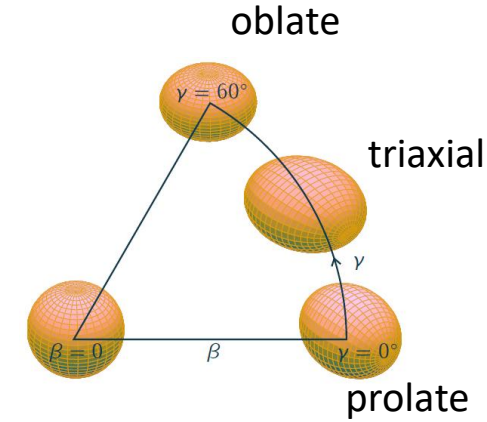
microscopic-macroscopic
finite-range liquid drop model (FRLDM)

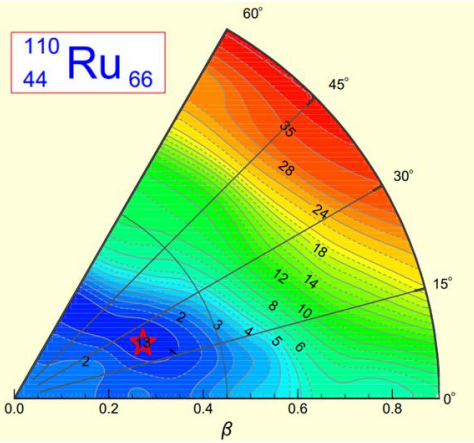
- ground-state masses
- with correction for axial asymmetry

strongest energy gain
due to axial asymmetry
for $Z \approx 44, N \approx 64$

γ – rigid ?
γ – soft ?

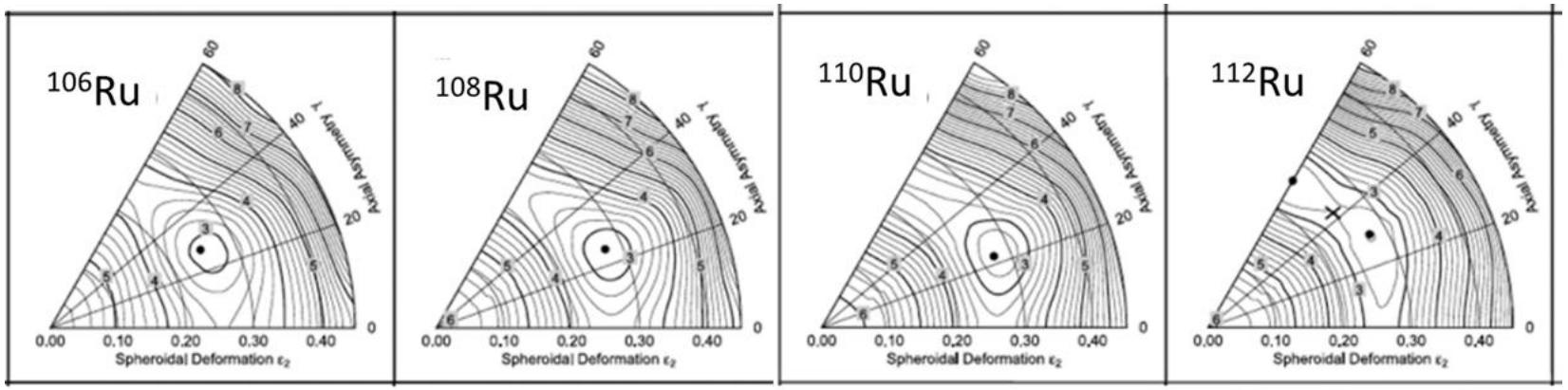
P. Möller et al.,
At. Data Nucl. Data Tables 94, 758 (2008)



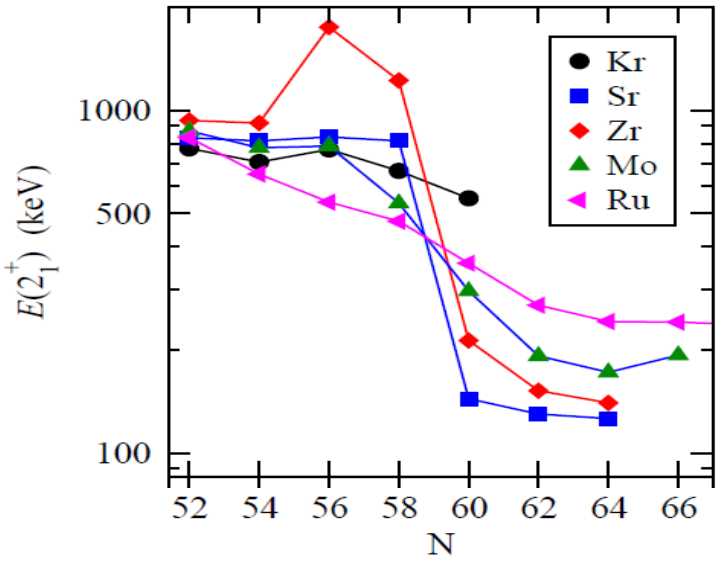


fully microscopic DFT
Gogny D1S

S. Hilaire, M. Girod
Eur. Phys. J. A 33, 237–241 (2007)

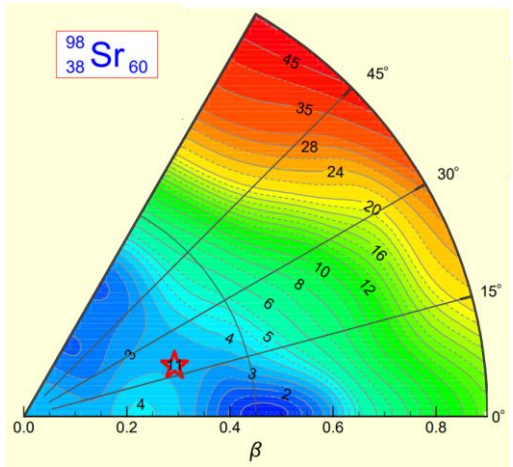


Shape transition and coexistence in at $N = 60$



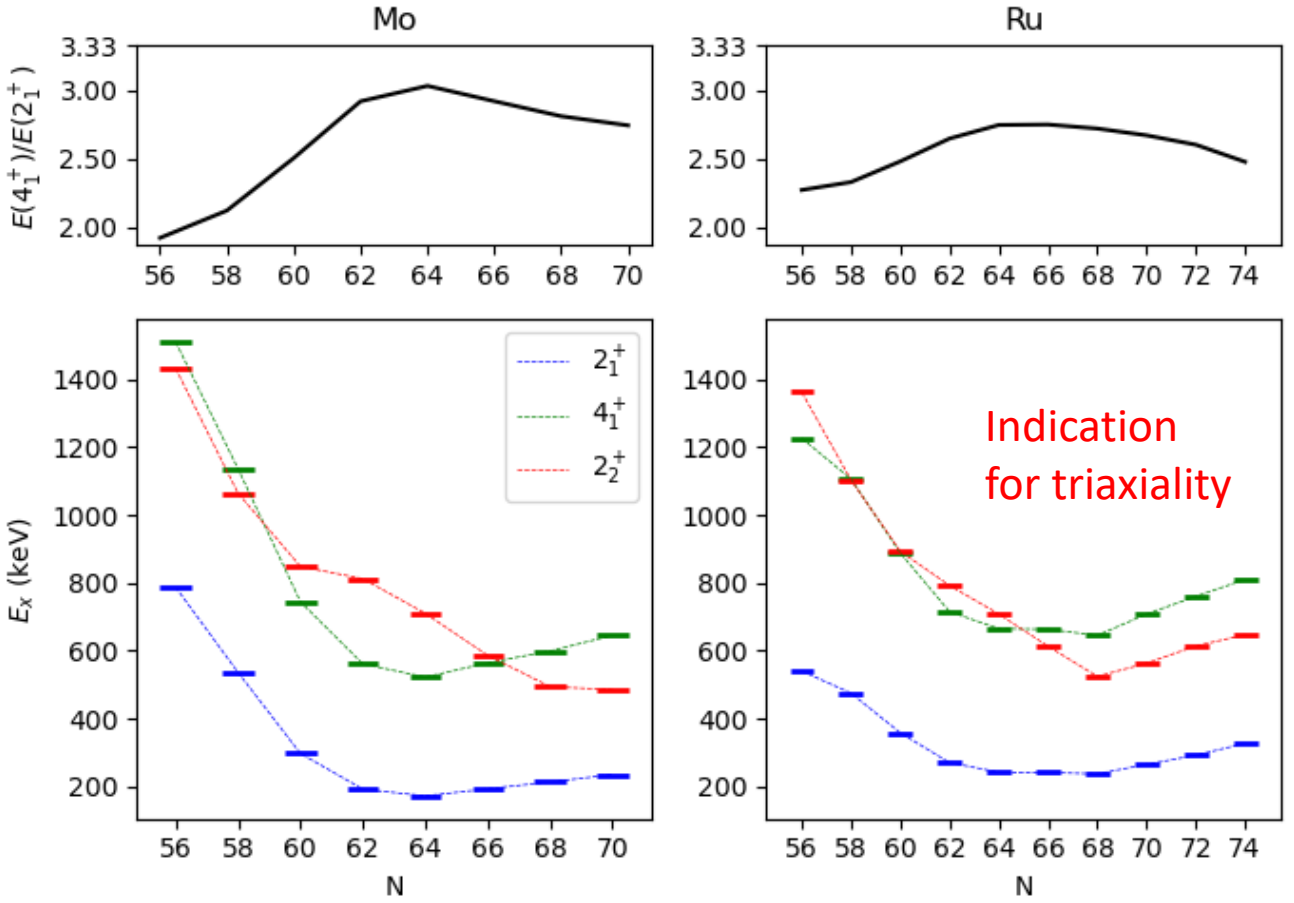
A.Görgen, W.Korten, J. Phys. G 43, 024002 (2016)

- rapid onset of deformation at $N = 60$ for Sr and Zr
- smooth transition for Mo and Ru
- extremely low-lying 2_2^+ states and γ-bands in Mo and Ru beyond $N \approx 66$



- shape coexistence in the transition region

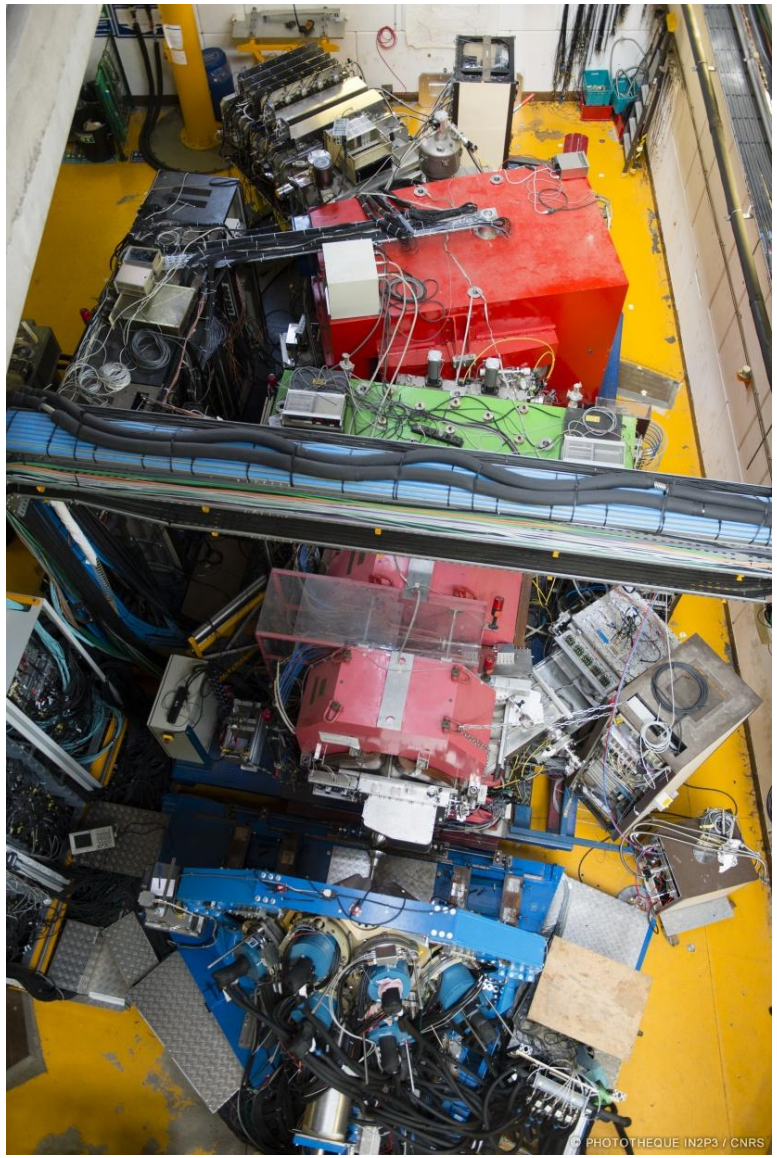
E. Clément et al.,
PRL 117, 099902 (2016)
PRC 94, 054326 (2016)
P. Singh et al.,
PRL 121, 192501 (2018)



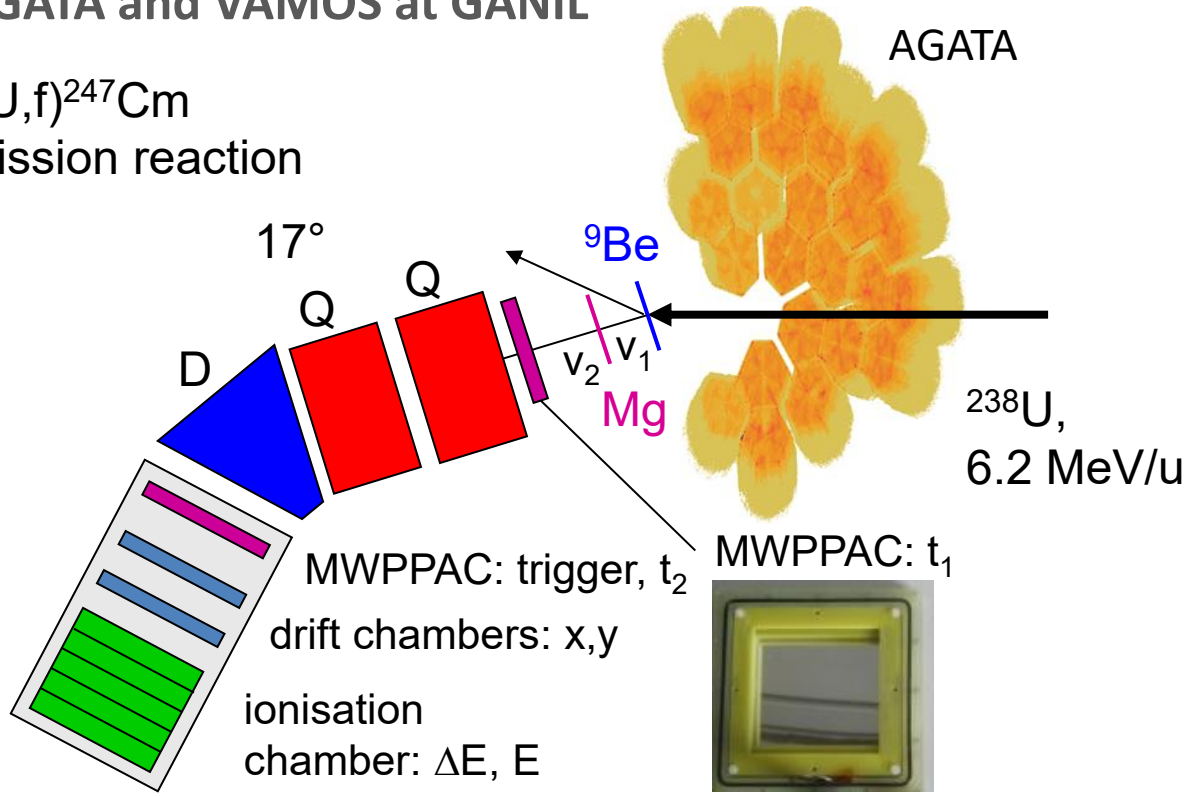
J. Ha et al.,
PRC 101, 044311 (2020)

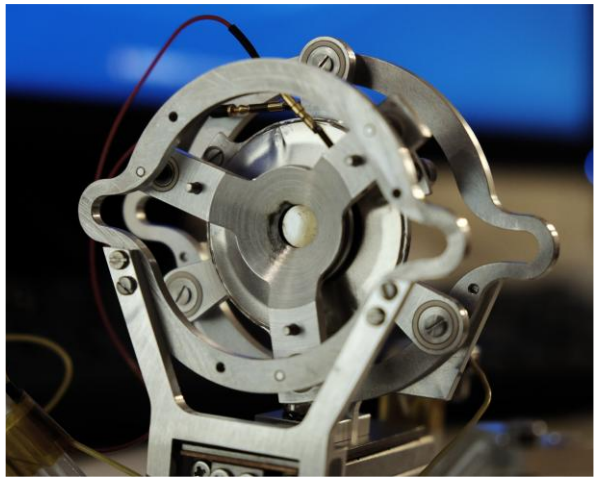
P.A. Söderström et al.,
PRC 88, 024301 (2013)

Lifetime measurement in fission fragments with AGATA and VAMOS at GANIL



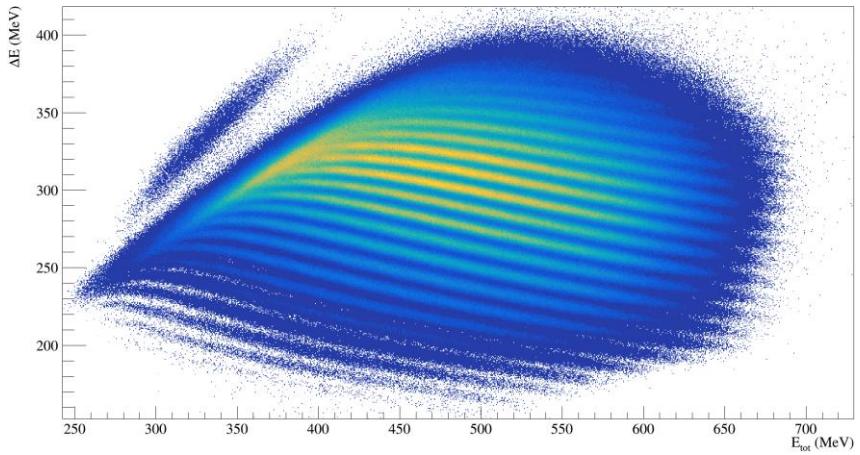
$^{9}Be(^{238}U,f)^{247}Cm$
fusion-fission reaction



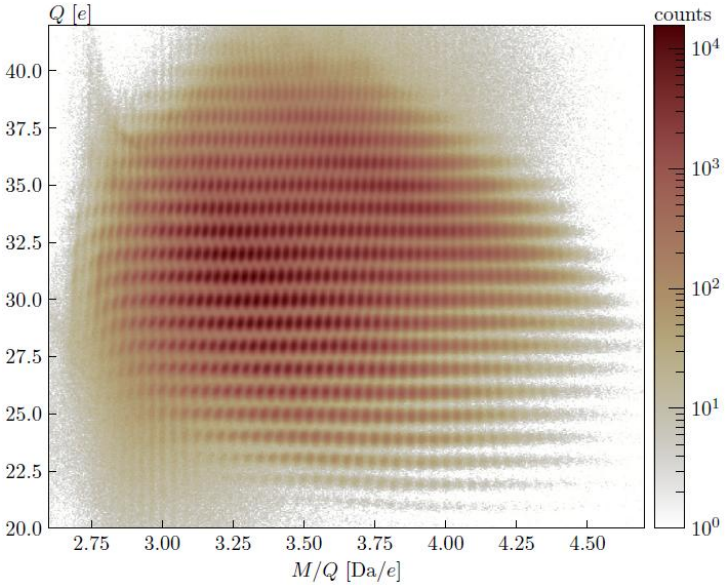


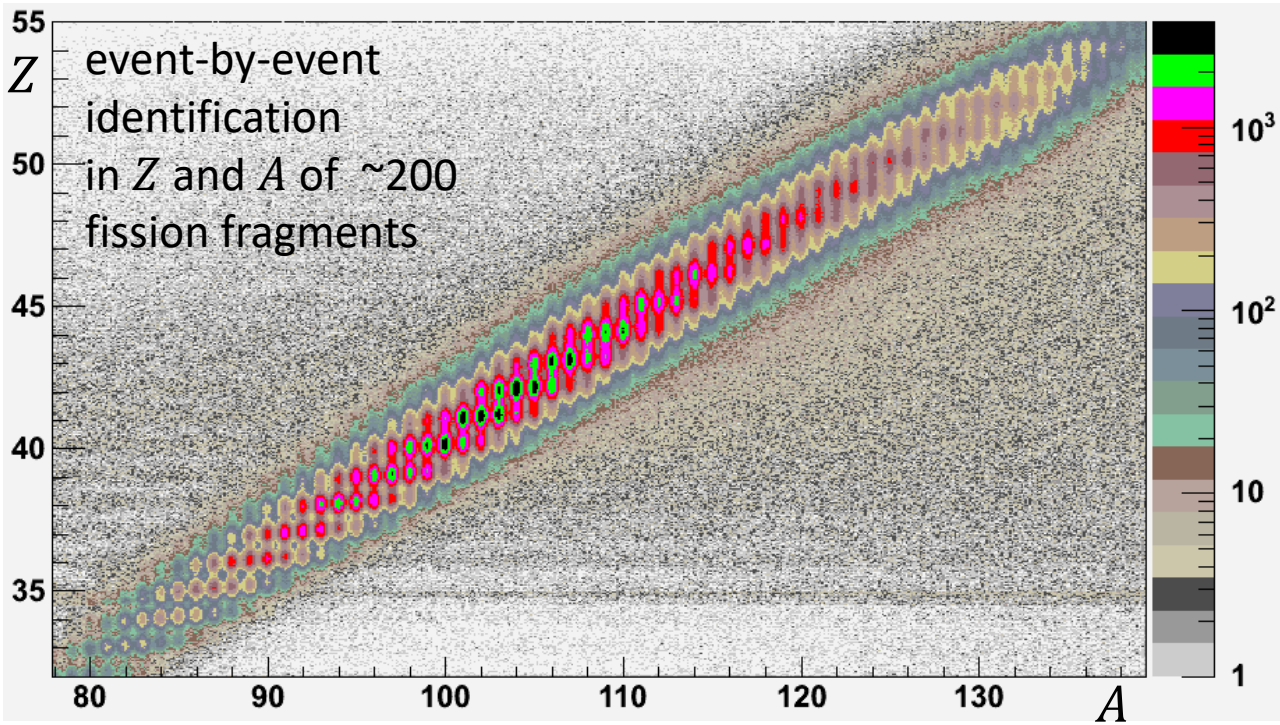
Fission fragment identification in VAMOS

energy loss and total energy $\Rightarrow Z$

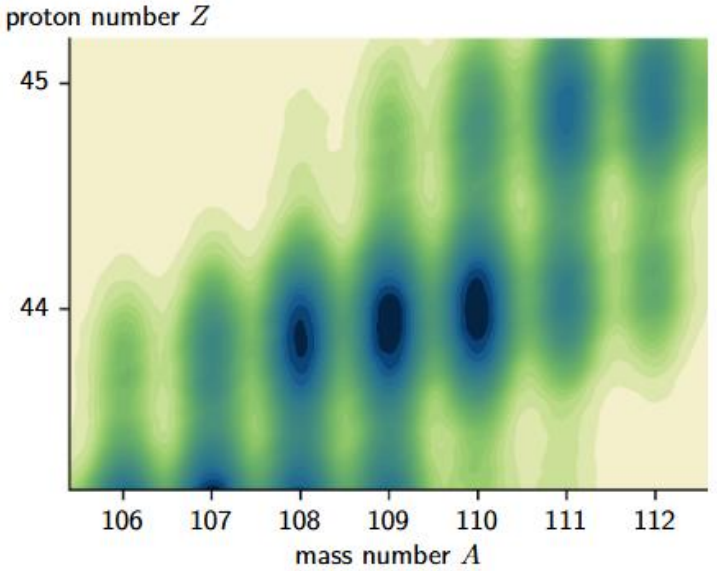


magnetic rigidity, time of flight $\Rightarrow$ mass M and M/Q





Ru



more results:

Shape evolution in even-mass ^{98}Zr – ^{104}Zr

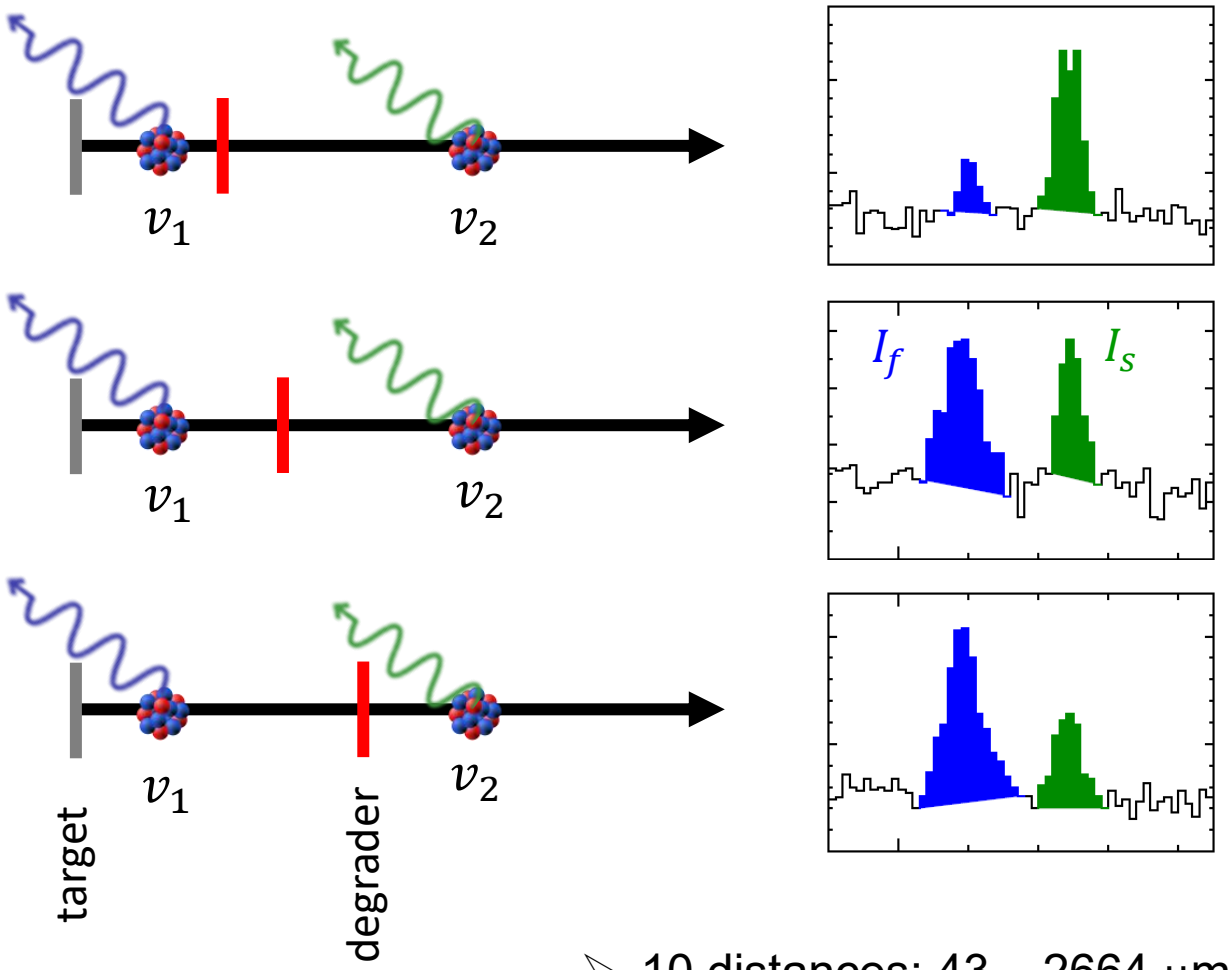
G. Pasqualato et al.,
Eur. Phys. J. A 59, 276 (2023)

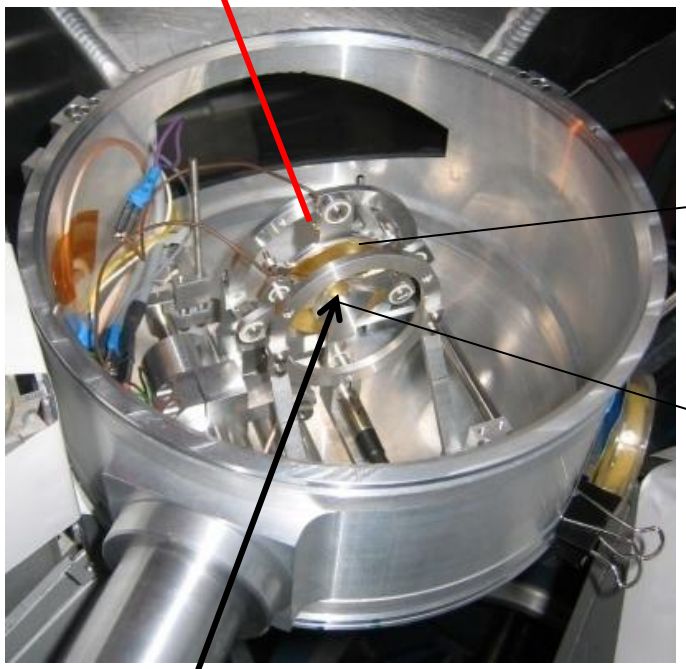
Triaxiality in ^{108}Mo

J.S. Heines et al., to be published

and more to come…

Recoil distance Doppler shift (RDDS) method

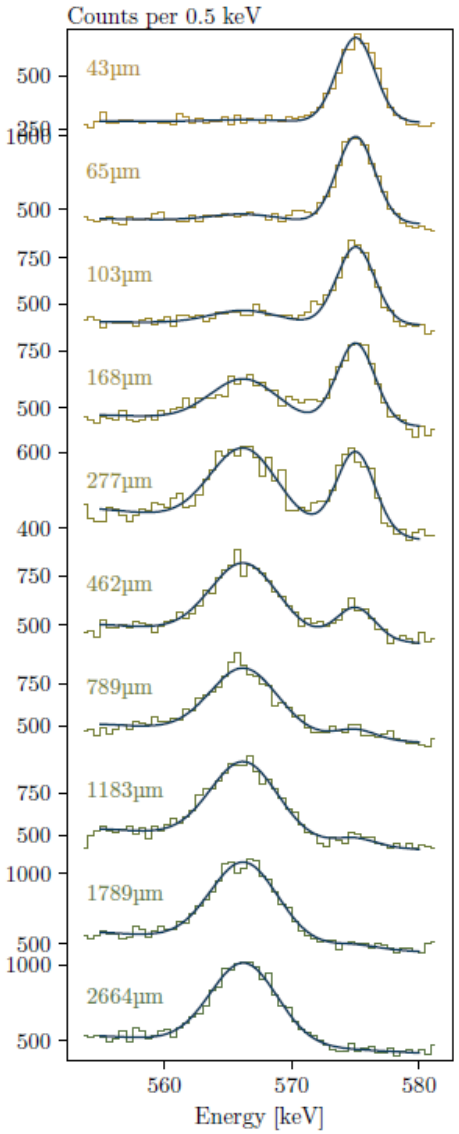




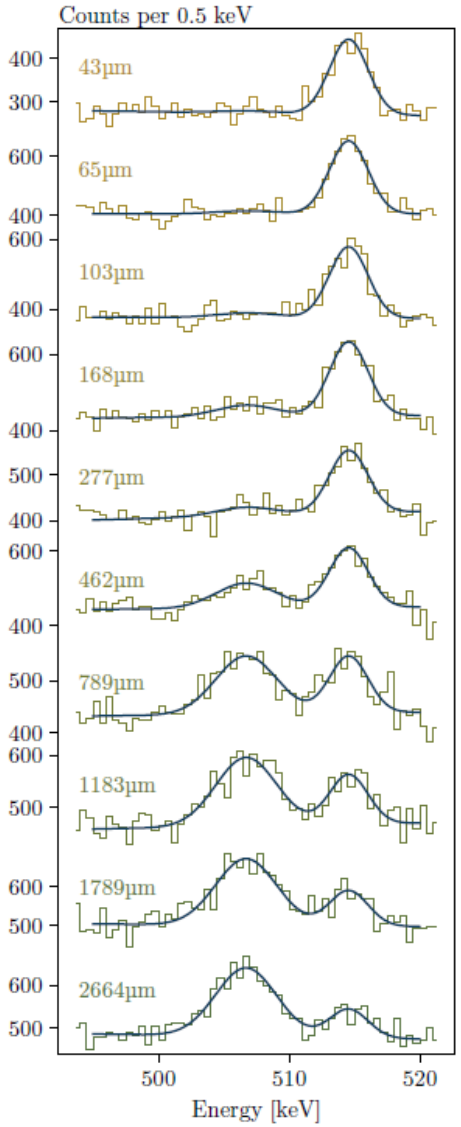
- 10 distances: 43 – 2664 μm
- ~18 h per distance
- sensitive to lifetimes in the range 1 – 100 ps

RDDS spectra for ^{110}Ru and ^{112}Ru: ground-state band and γ band

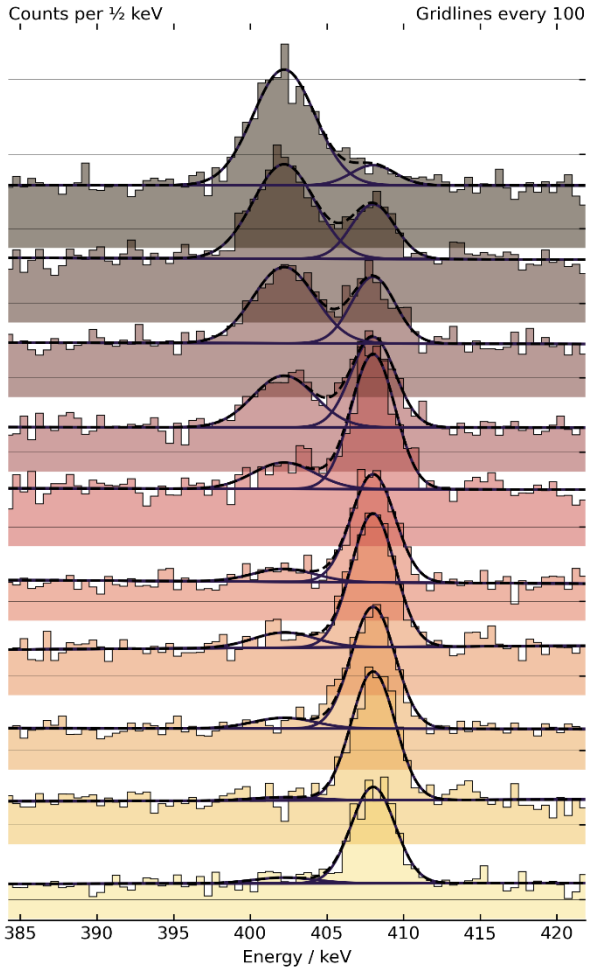
^{110}Ru: $6_1^+ \rightarrow 4_1^+$



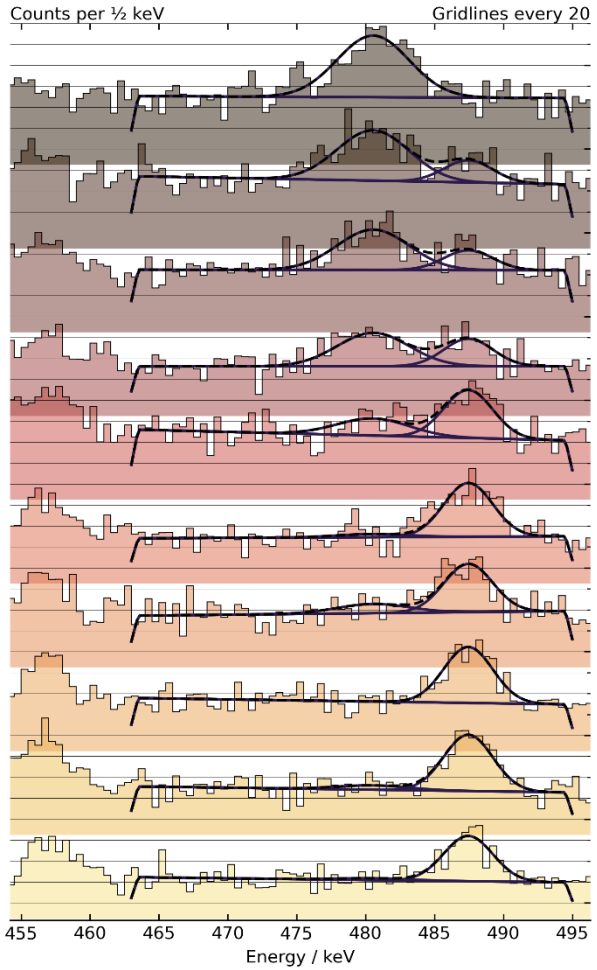
^{110}Ru: $5_1^+ \rightarrow 3_1^+$



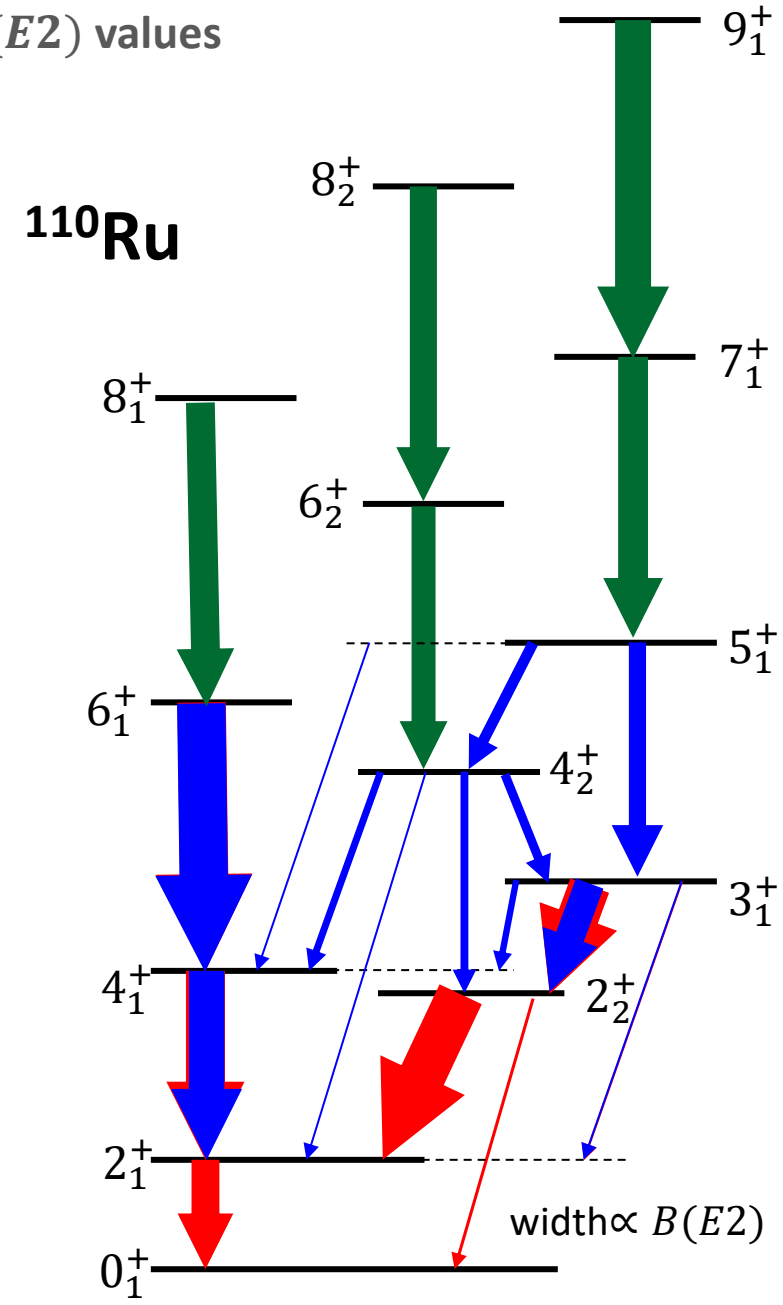
^{112}Ru: $4_1^+ \rightarrow 2_1^+$



^{112}Ru: $5_1^+ \rightarrow 3_1^+$

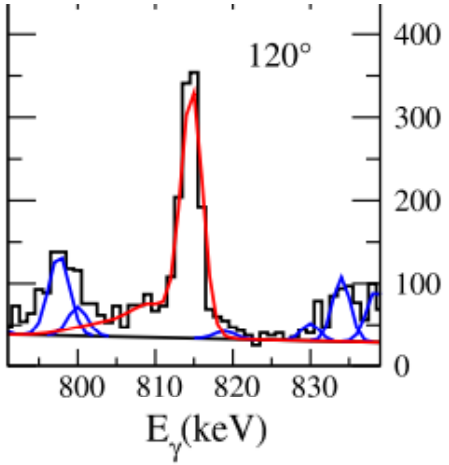


$B(E2)$ values



DSAM, ^{252}Cf
Hercules
Gammasphere

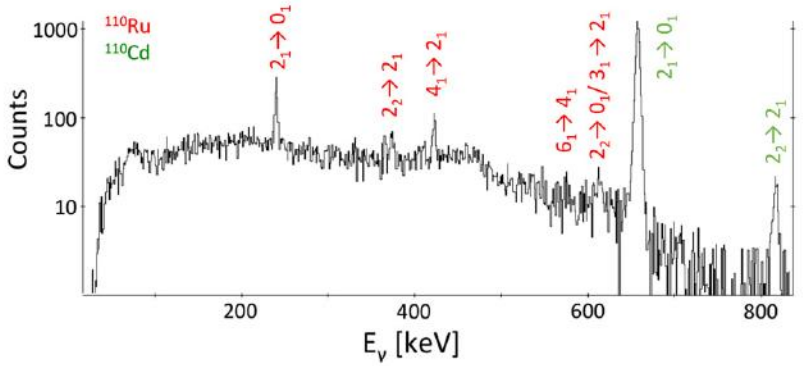
J.B. Snyder et al,
PLB 723, 61 (2013)



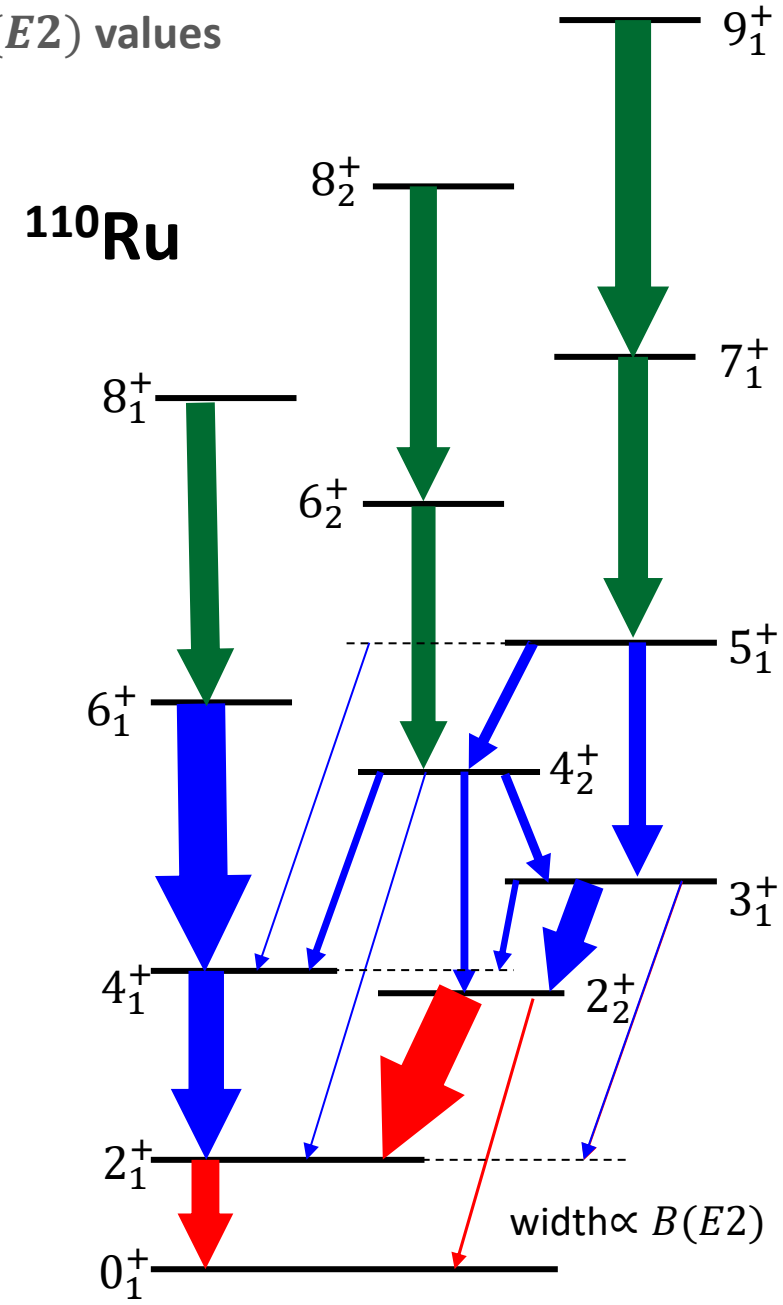
RDDS (this work): 5 lifetimes
Branching ratios ⇒ 12 $B(E2)$ values
(assuming pure E2 for $\Delta I = 1$)

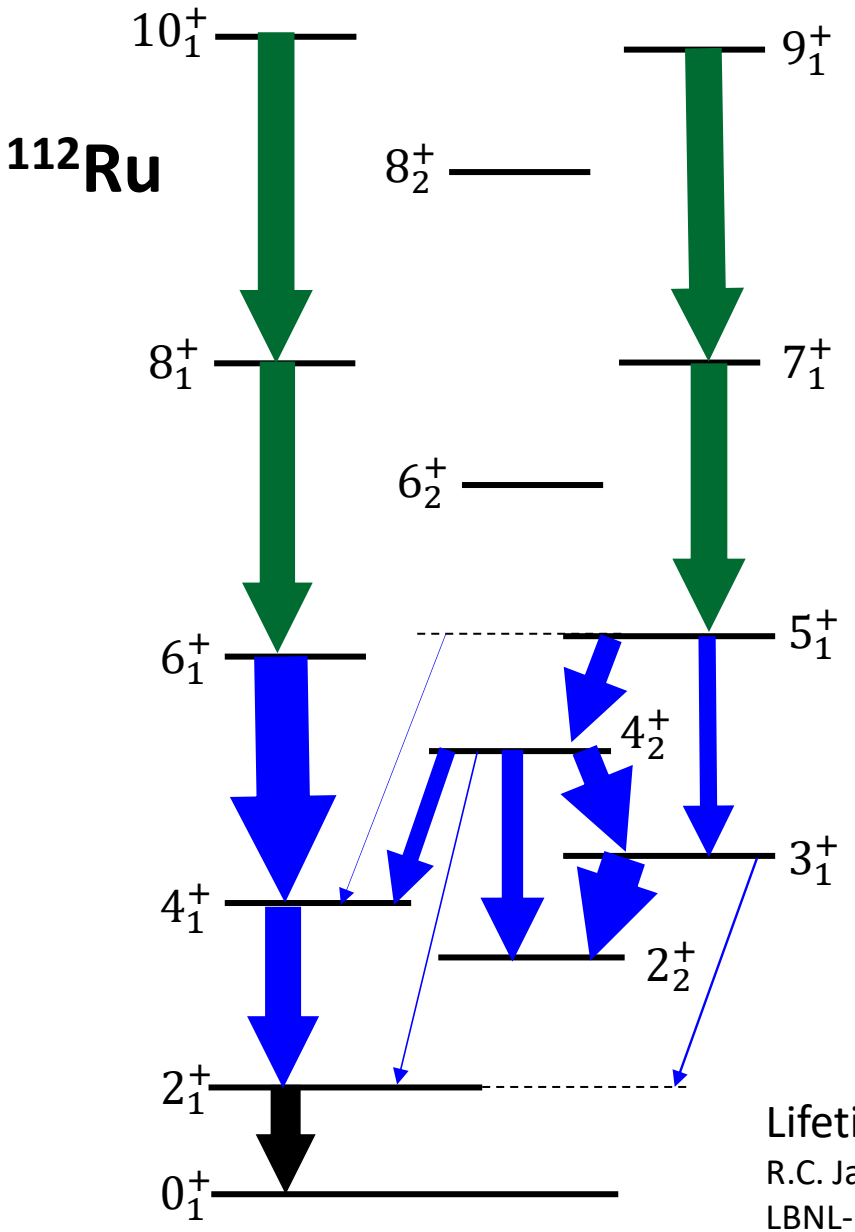
Coulomb excitation
CARIBU+GRETINA

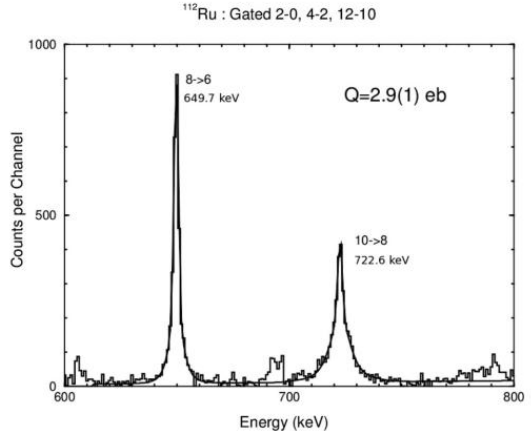
D.T. Doherty et al.,
PLB 766, 334 (2017)



$B(E2)$ values





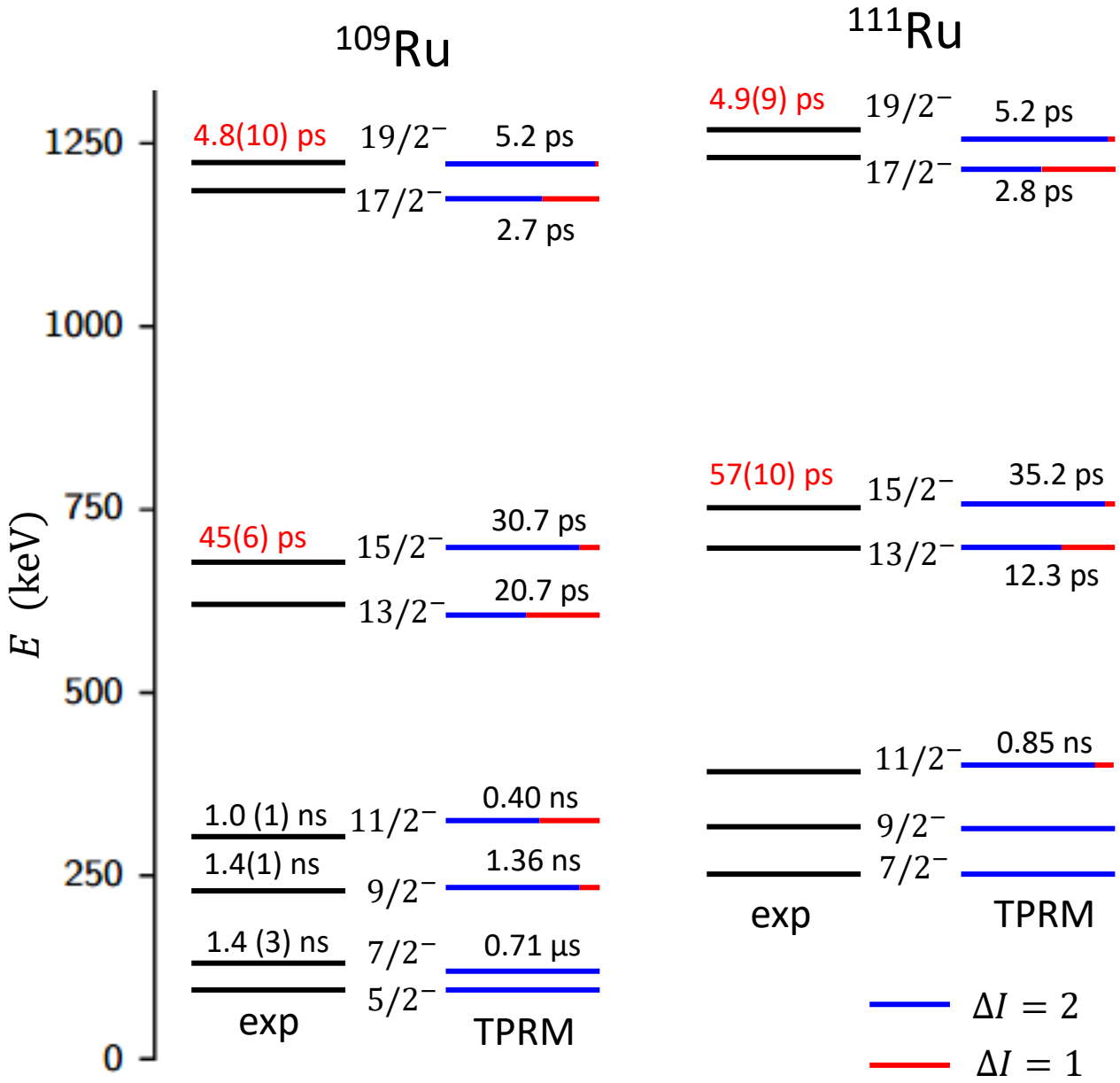


Doppler profile method, ^{248}Cf
A.G. Smith et al.,
Phys. Rev. C 86, 014321 (2012)

RDDS (this work): 5 lifetimes
Branching ratios
$\Rightarrow$ 11 $B(E2)$ values
(assuming pure E2 for $\Delta I = 1$)

Lifetime measurement ^{252}Cf
R.C. Jared et al.
LBNL-2366, p.38 (1984)

Odd-mass isotopes: $\nu[532]5/2^-$ bands



Triaxial particle-rotor calculations

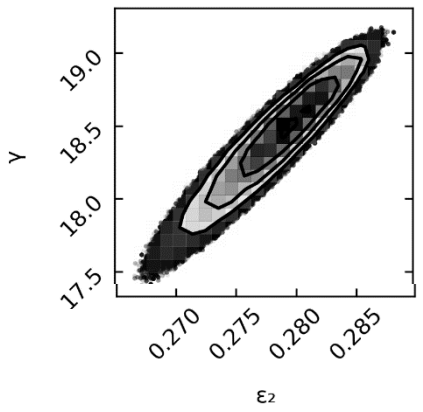
odd neutron coupled to triaxial even-even core

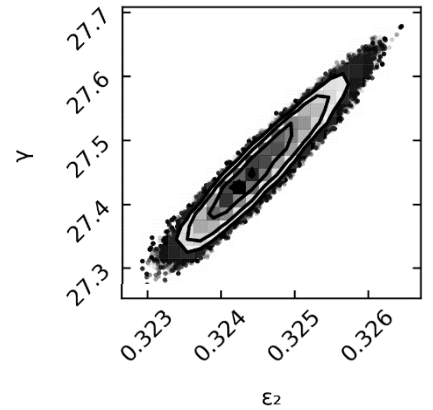
parameters:

- (ε_2, γ) deformation of the core
- scaling parameter for moment of inertia
- Coriolis attenuation factor

I. Ragnarsson, P. B. Semmes,
Hyperfine Interact. 43, 423 (1988)

	^{109}Ru	^{111}Ru
ε_2	0.279(4)	0.324(1)
γ	18.4(4)°	27.5(1)°





Even-even isotopes: Generalized triaxial rotor model

Hamiltonian: $H = A_1\hat{I}_1^2 + A_2\hat{I}_2^2 + A_3\hat{I}_3^2 = A\hat{I}^2 + F\hat{I}_3^2 + G(\hat{I}_+^2 + \hat{I}_-^2)$

$$F = \frac{E(2_2^+) - E(2_1^+)}{4\sqrt{1 + \tan^2(2\Gamma)}}$$

Parameters:

- Quadrupole moment Q_0
- triaxiality γ
- Mixing angle Γ between ground-state and γ band

The inertia parameters are related to the mixing angle Γ and 2^+ energies:

$$A = \frac{E(2_1^+) + E(2_2^+) - 4F}{12}$$

$$G = \frac{F}{2\sqrt{3}}\tan(2\Gamma)$$

analytic expressions for E2 matrix elements:

$$\langle 0_1^+ || \mathcal{M}(E2) || 2_1^+ \rangle = \sqrt{\frac{5}{16\pi}} Q_0 \cos(\gamma + \Gamma)$$

$$\langle 2_1^+ || \mathcal{M}(E2) || 3_1^+ \rangle = \sqrt{\frac{25}{32\pi}} Q_0 \sin(\gamma + \Gamma)$$

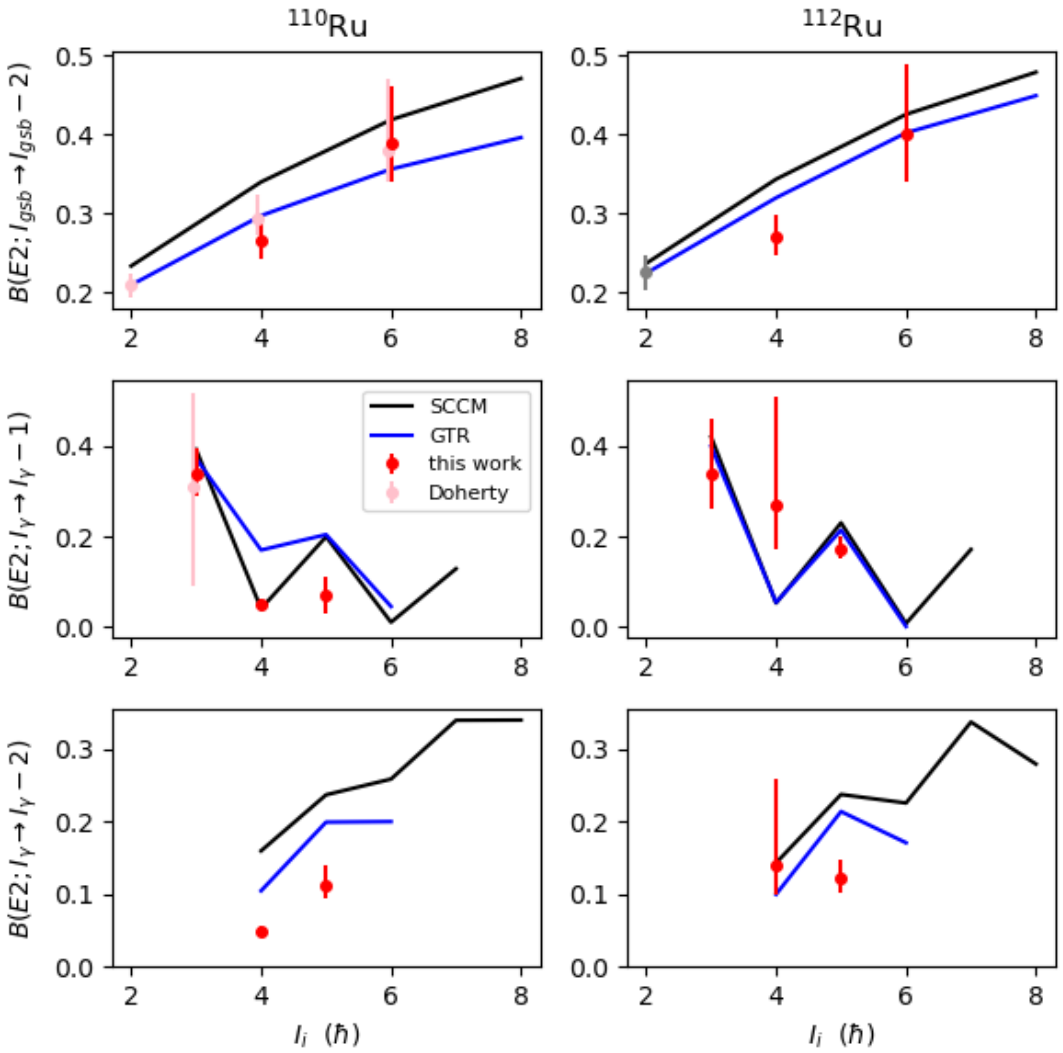
$$\frac{\langle 0_1^+ || \mathcal{M}(E2) || 2_2^+ \rangle}{\langle 2_1^+ || \mathcal{M}(E2) || 2_1^+ \rangle} = \sqrt{\frac{7}{10}} \frac{\sin(\gamma + \Gamma)}{\sin(\gamma - 2\Gamma)}$$

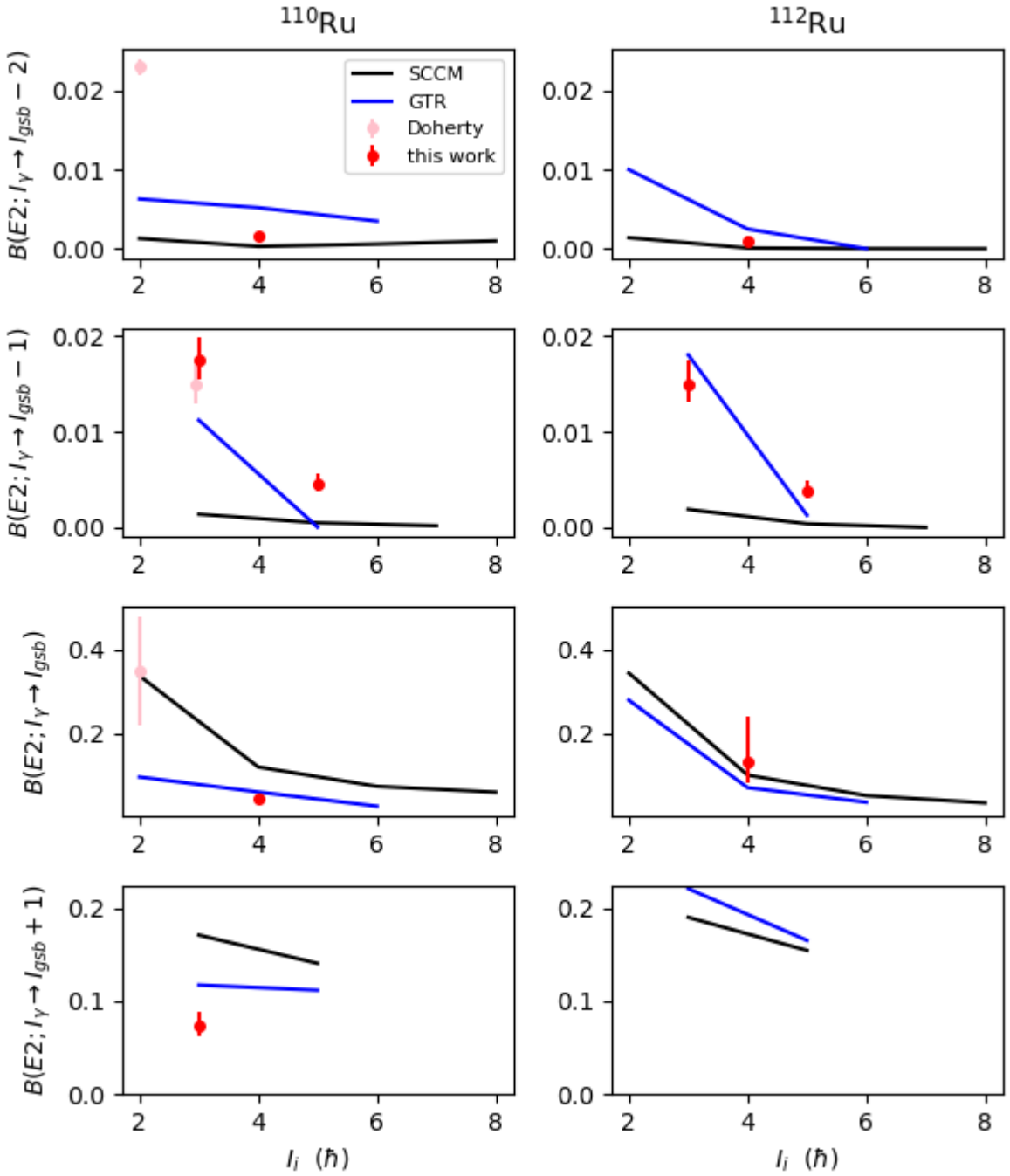
Fitting experimental E2 matrix elements yields:

	^{110}Ru	^{112}Ru
Q_0	3.32(11) eb	3.42(16) eb
γ	22.6(9)°	26(4)°
Γ	– 10.4(7)°	– 15(4)°

J.L.Wood et al., PRC 70, 024308 (2004)
W.D.Kulp et al., PRC73, 014308 (2006)
J.M.Allmond et al., PRC78, 014302 (2008)

Comparison with experimental B(E2) values



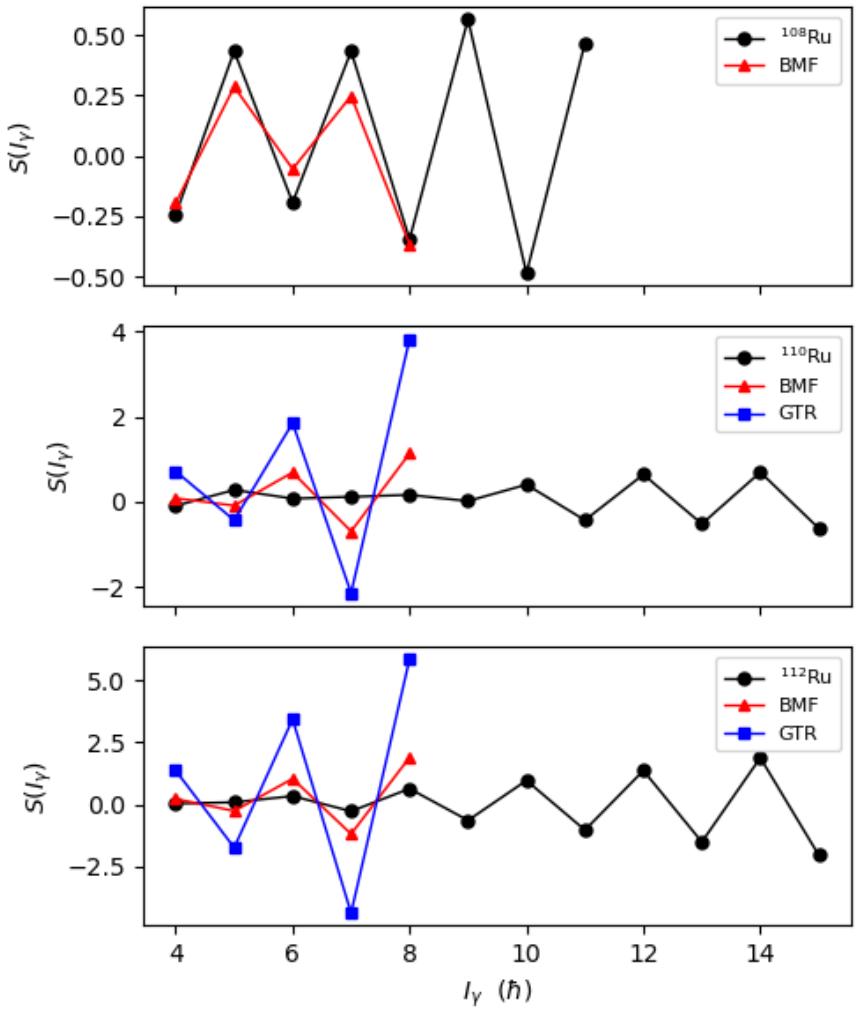


SCCM: symmetry-conserving configuration mixing (GCM) variation after particle-number projection, Gogny D1S EDF

J.S. Heines et al., Eur. Phys. J. A 62, 38 (2026)

Energy staggering of the γ band

$$S(I_\gamma) = \frac{\left([E(I_\gamma) - E(I_\gamma - 1)] - [E(I_\gamma - 1) - E(I_\gamma - 2)]\right)}{E(2_1^+)}$$



$S(I_\gamma) < 0$ for I_γ even

γ soft (Wilets-Jean)

Inversion of $S(I_\gamma)$ at ^{110}Ru

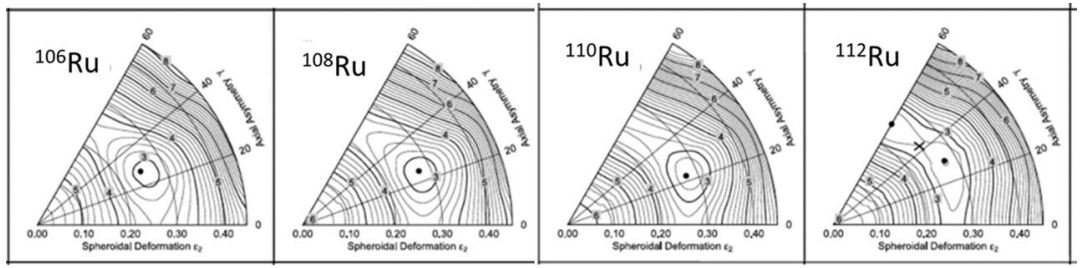
$S(I_\gamma) > 0$ for I_γ even

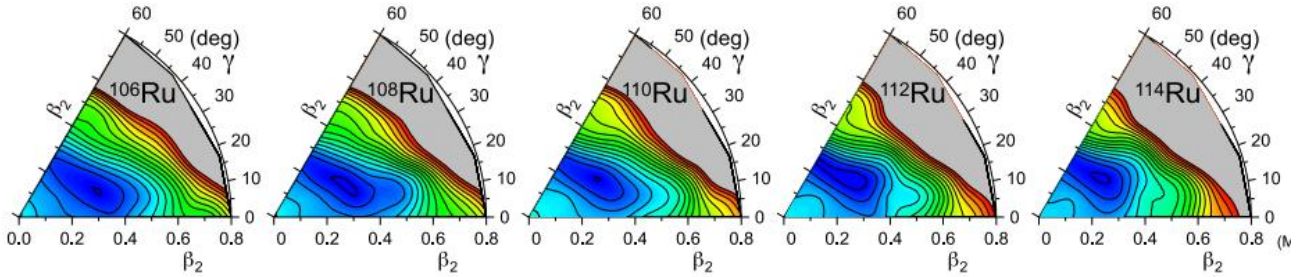
γ rigid (Davydov-Filippov)

SCCM calculations reproduce inversion of staggering (although the absolute energies are too high)

GTR model reproduces correct phase for ^{112}Ru, but magnitude of staggering is too large

FRLDM

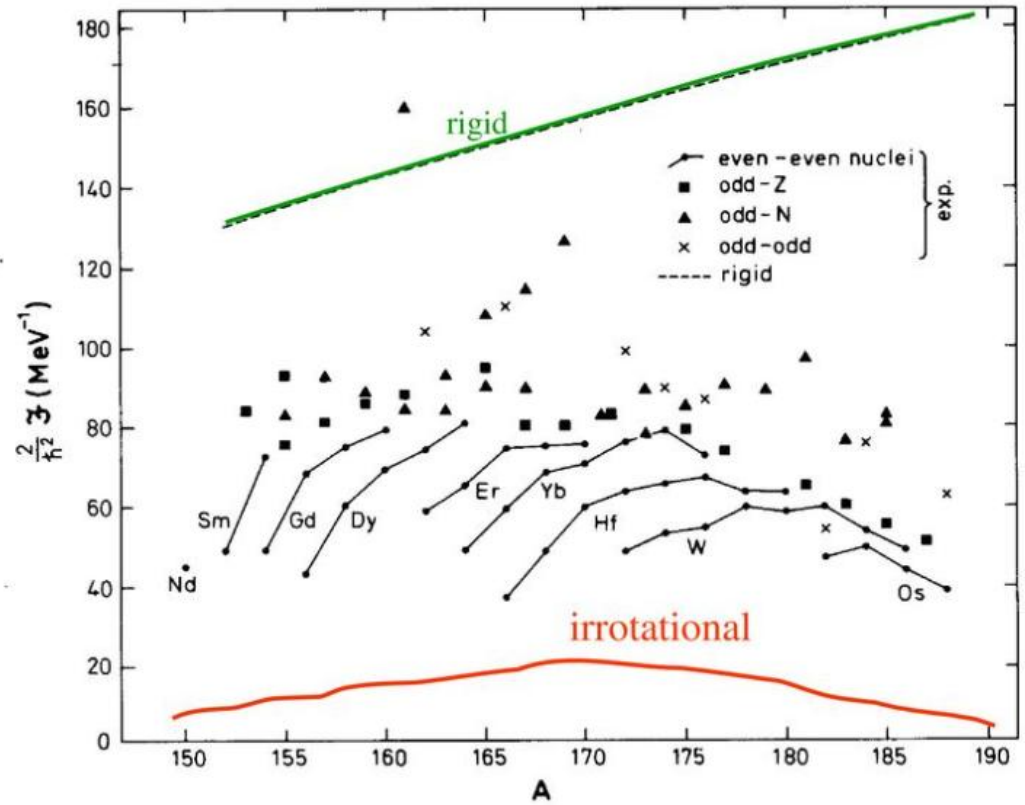




Gogny D1S

Moments of inertia

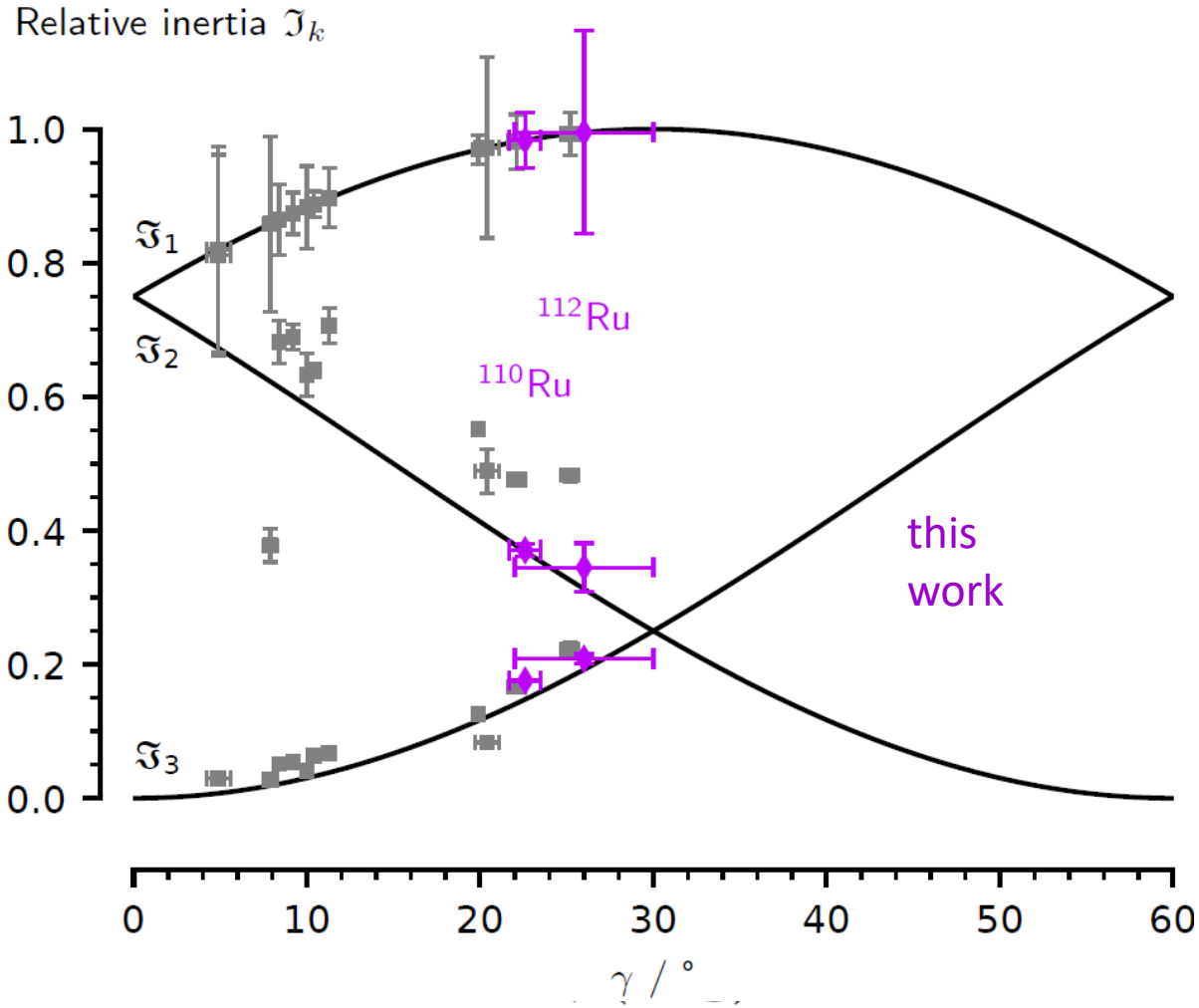
experimental moments of inertia:
in between values for rigid body
and irrotational flow



adapted from Bohr & Mottelson Vol.II

MoI from generalized triaxial rotor model

- fitted to experimental E2 matrix elements
- scaled to irrotational $\mathfrak{I}_1$



J.M. Allmond, J.L. Wood,
Phys. Lett. B 767, 226 (2017)

J.S. Heines et al.,
Eur. Phys. J. A. 62, 38 (2026)

Summary and conclusions

- RDDS lifetime measurements in neutron-rich fission fragments
- event-by-event identification of fission fragments in VAMOS

- 16 new lifetimes in Ru isotopes between $A = 108$ and $A = 112$

- odd mass: lifetimes in $\nu[532]5/2^-$ band in ^{109}Ru and ^{111}Ru
  - triaxial particle-rotor model suggests increase in γ deformation from ^{109}Ru and ^{111}Ru

- even mass: lifetimes both in ground-state band and γ band
  - B(E2) values for in-band and inter-band transitions
  - good agreement with Gogny-SCCM and triaxial rotor calculations
  - SCCM calculations reproduce staggering and transition from γ-soft to γ-rigid
  - GTR model suggests increase in γ-deformation from ^{110}Ru to ^{112}Ru
  - relative moments of inertia in GTR model consistent with irrotational flow
  - ^{112}Ru best candidate for triaxial rotor with $\gamma \approx 30^o$ and $\mathcal{J}_1 > \mathcal{J}_2 \approx \mathcal{J}_3$

Eur. Phys. J. A (2026) 62:38
https://doi.org/10.1140/epja/s10050-025-01782-4

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article - Experimental Physics

Triaxiality of neutron-rich ruthenium nuclei studied by lifetime measurements

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