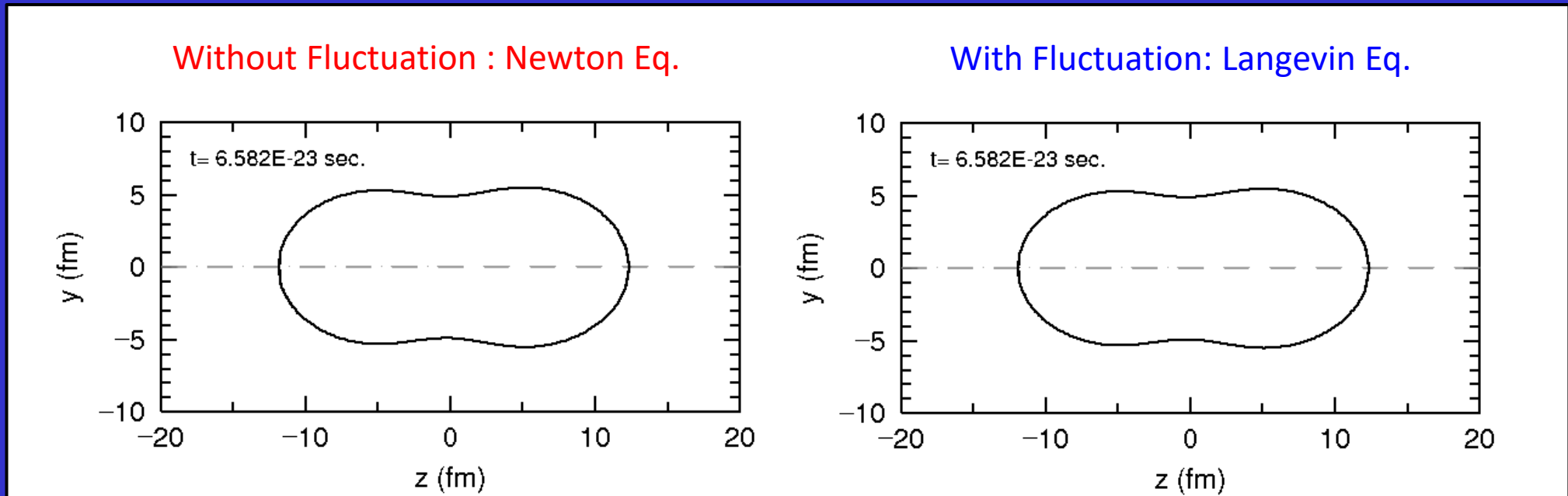


Role of **Macroscopic** Effects in Dynamics of Nuclear Fission



Yoshihiro Aritomo, Shinya Takagi, Kohta Nakajima, Kosuke Kawai
Faculty of Science and Engineering, Kindai University, Osaka, Japan

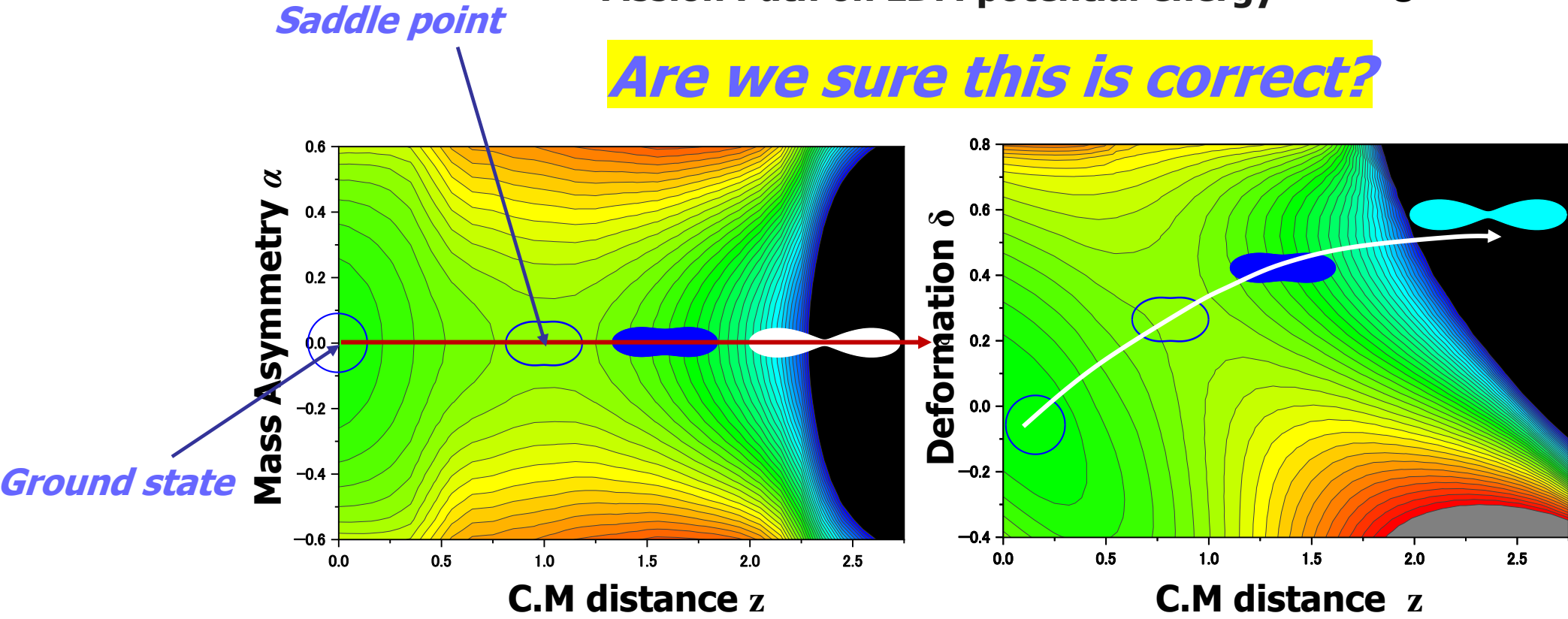


Fission 2026
Salle Totem, Le Bachat, Chamrousse, France
March 13, 2026

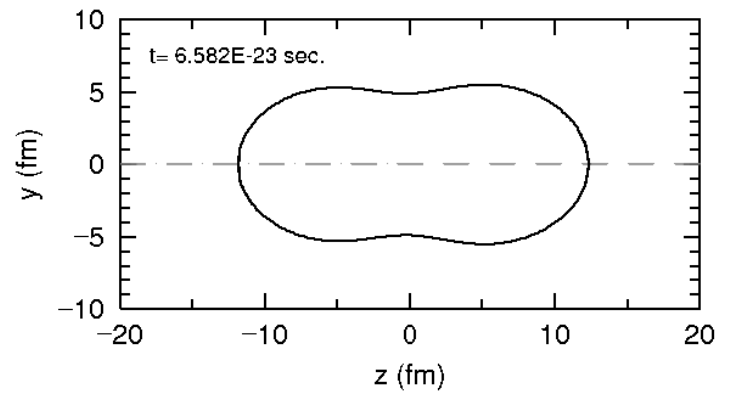
We would like to start this talk with a question

Fission Path on LDM potential energy ^{236}U

Are we sure this is correct?



*Draw fission path....
In general,*



Contents

1. Introduction

Microscopic and Macroscopic properties

2. Model

Dynamical model

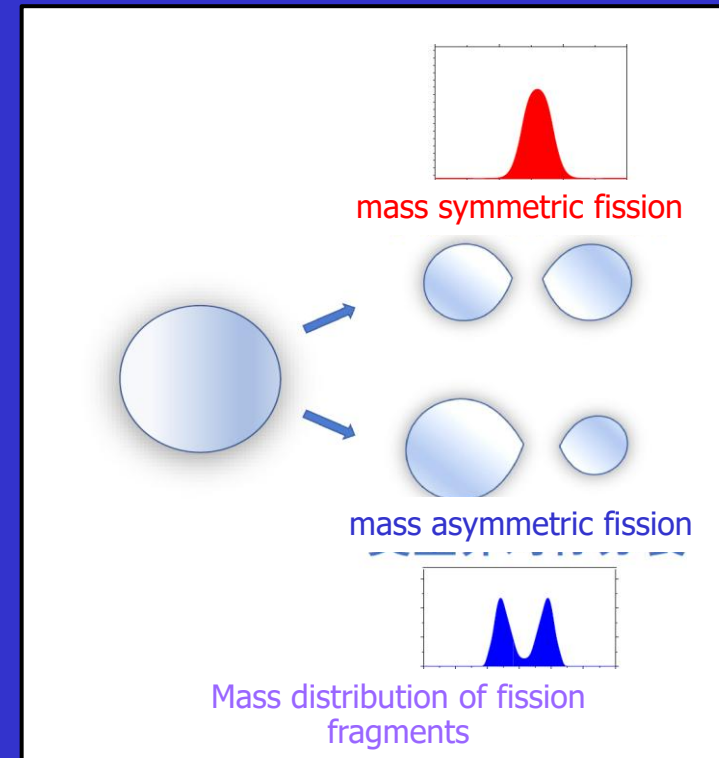
Two-center shell model + Langevin equation

Analysis by trajectory calculations

3. Fission processes

Role of transport coefficients

4. Summary



Nuclear Physics Theoretical research interests

Microscopic feature

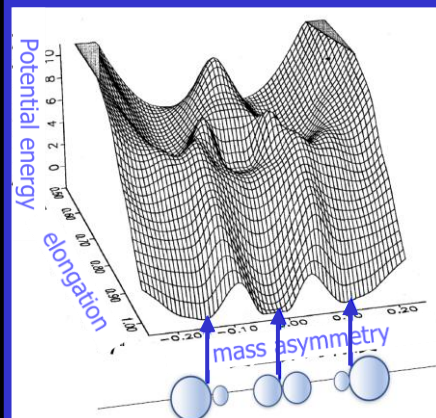
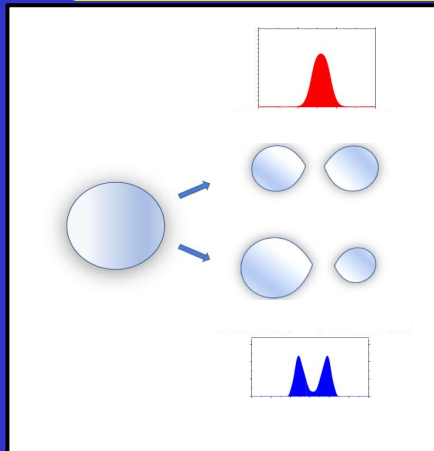
As discussed at this workshop until yesterday

Mass-asymmetric fission

Shell structure

One of the features that appear in finite quantum many-body systems

Quantum Mechanics



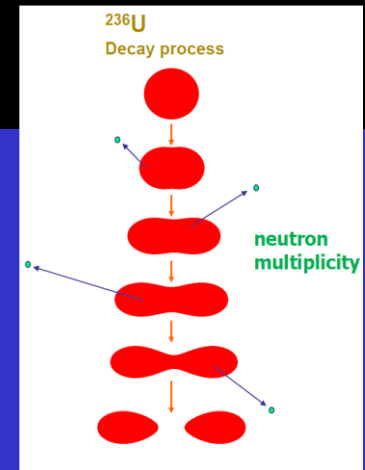
Macroscopic feature

Fission process

time scale

viscosity → friction coefficient

Easiness of deformation → inertia mass



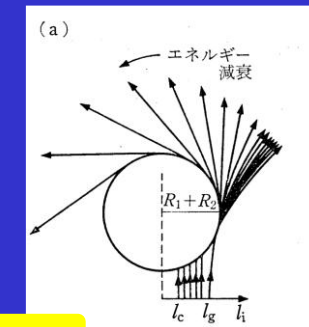
Fusion process

Scattering angle

of grazing collision

Dissipation

of kinetic energy



Liquid Drop Model Classical Mechanics

Phenomenological model

Advantage of using Macroscopic model

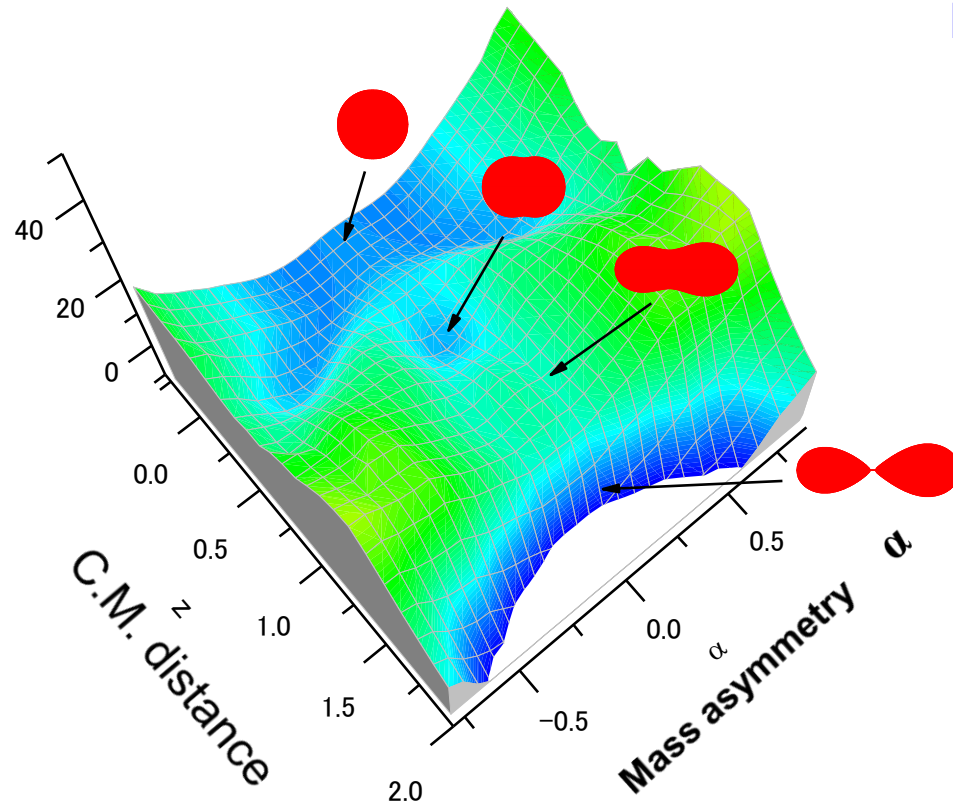


Model

Dynamical Model

Two-center shell model

Langevin equation



Dynamical calculation

Time-evolution of nuclear shape
in fission process

Two Items

1. Potential energy surface
2. Trajectory \leftarrow described by
Equation of Motion

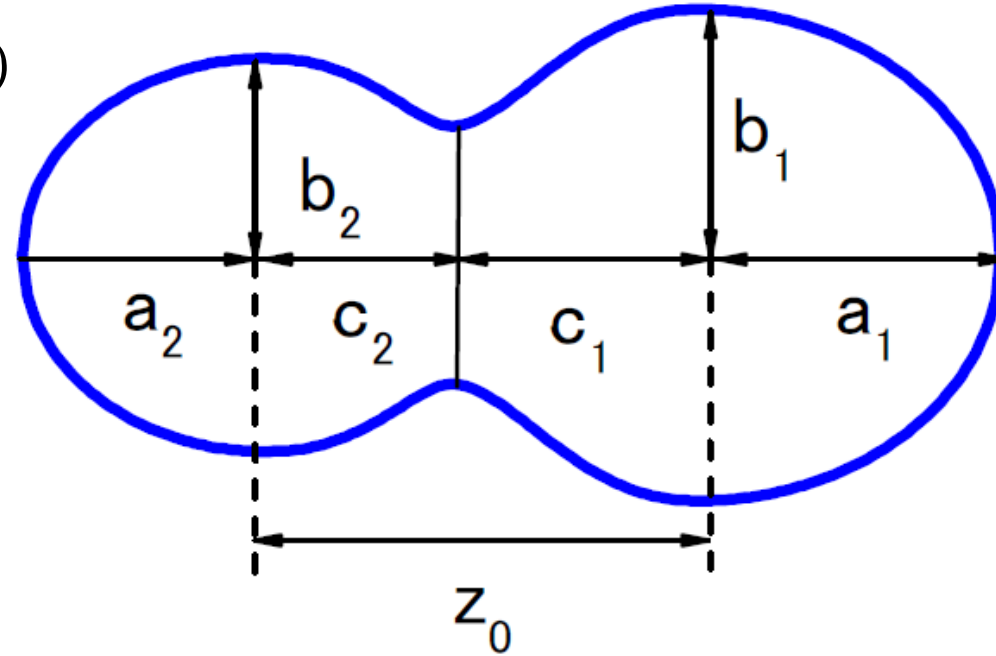
***Trajectory on potential
energy surface***

Nuclear Shape

two-center parametrization (z, δ, α)

(Maruhn and Greiner,
Z. Phys. 251(1972) 431)

$$q(z, \delta, \alpha)$$



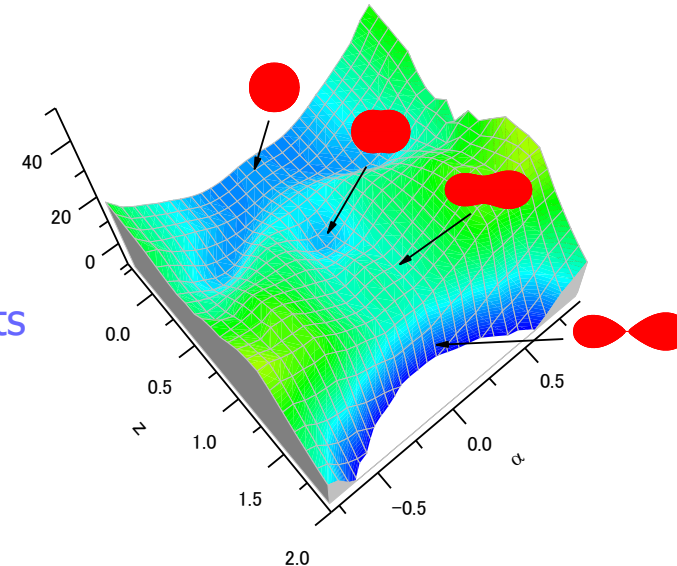
$$z = \frac{z_0}{BR} \quad \text{Distance between center of mass}$$

$$B = \frac{3 + \delta}{3 - 2\delta}$$

R : Radius of the spherical compound nucleus

$$\delta = \frac{3(a-b)}{2a+b} \quad (\delta_1 = \delta_2) \quad \text{Deformation of fragments}$$

$$\alpha = \frac{A_1 - A_2}{A_{CN}} \quad \text{Mass asymmetry}$$

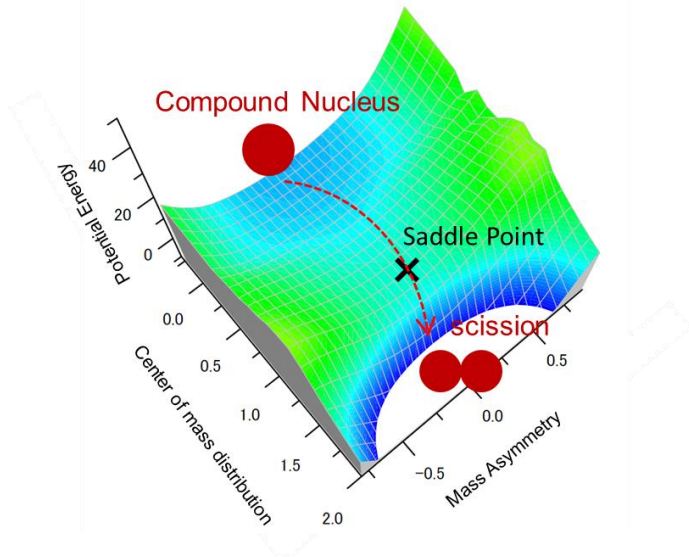


Potential energy

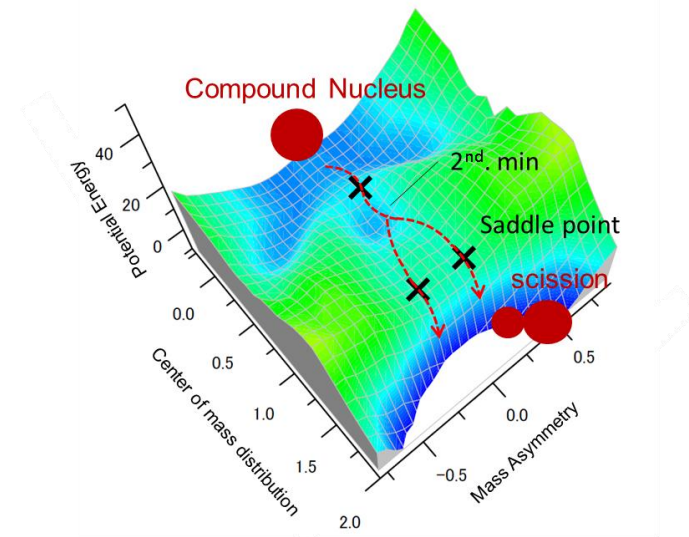
$$V(q, T) = V_{LD}(q) + V_{SH}(q, T)$$

Microscopic model part

Liquid Drop Model



Liquid Drop Model
+ Shell correction energy



Multi-dimensional Langevin Equation

$$\frac{dq_i}{dt} = (m^{-1})_{ij} p_j$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2} \frac{\partial}{\partial q_i} (m^{-1})_{jk} p_j p_k - \gamma_{ij} (m^{-1})_{jk} p_k + g_{ij} R_j(t)$$

Friction
dissipation

Random force
fluctuation

Newton equation
ordinary differential equation

$\langle R_i(t) \rangle = 0$, $\langle R_i(t_1) R_j(t_2) \rangle = 2\delta_{ij} \delta(t_1 - t_2)$: white noise (Markovian process)

$$\sum_k g_{ik} g_{jk} = T \gamma_{ij}$$

Einstein relation

Fluctuation-dissipation theorem

q_i : deformation coordinate (z, δ, α)
two-center parametrization

(nuclear shape)

(Maruhn and Greiner, Z. Phys. 251(1972) 431)

p_i : momentum

m_{ij} : Hydrodynamical mass

(inertia mass)

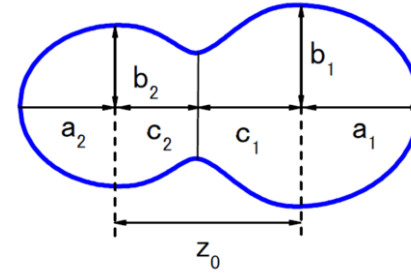
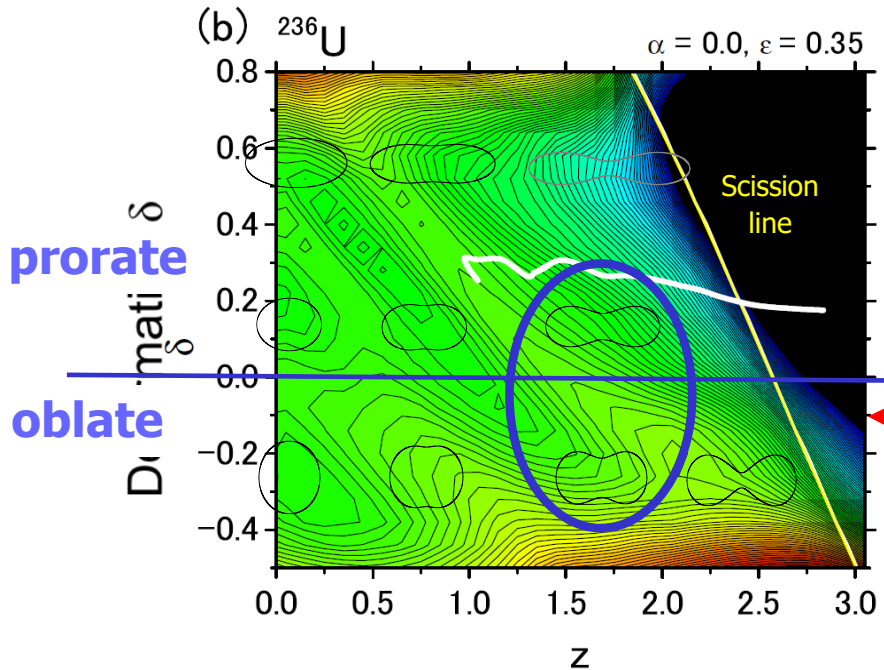
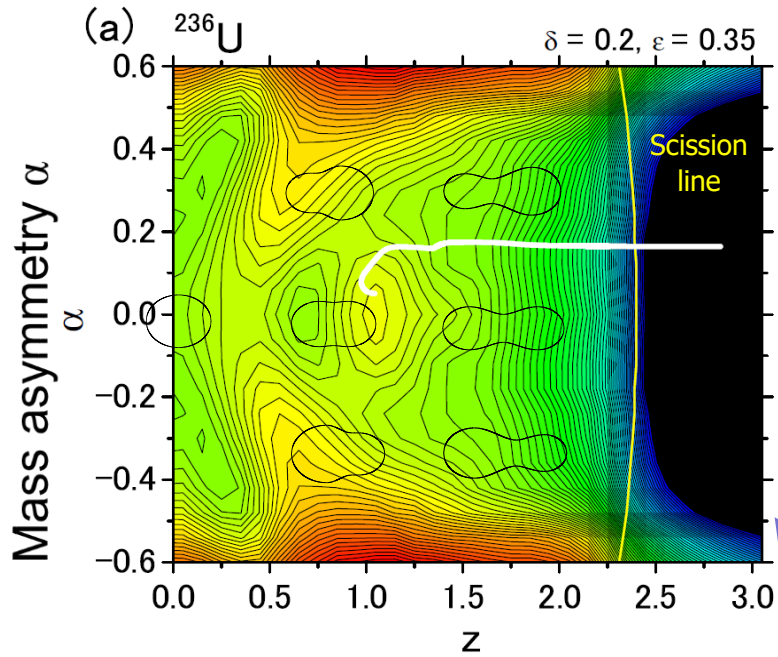
γ_{ij} : Wall and Window (one-body) dissipation

(friction)

$$E_{\text{int}} = E^* - \frac{1}{2} (m^{-1})_{ij} p_i p_j - V(q)$$

E_{int} : intrinsic energy, E^* : excitation energy

Without fluctuation



$$z = \frac{z_0}{BR}$$

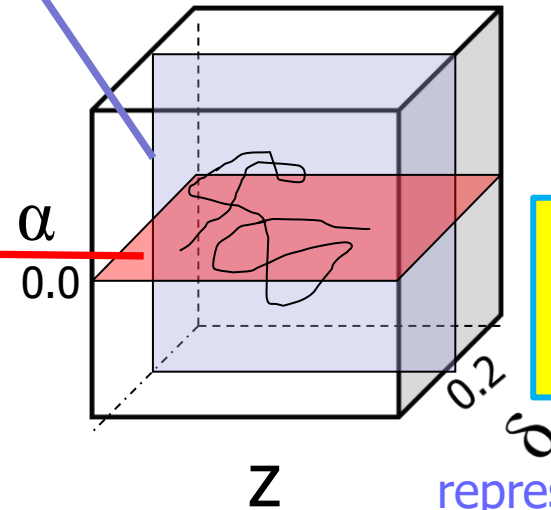
$$B = \frac{3 + \delta}{3 - 2\delta}$$

R : Radius of the spherical compound nucleus

$$\delta = \frac{3(a - b)}{2a + b}$$

$$\alpha = \frac{A_1 - A_2}{A_{CN}}$$

3D coordinate space

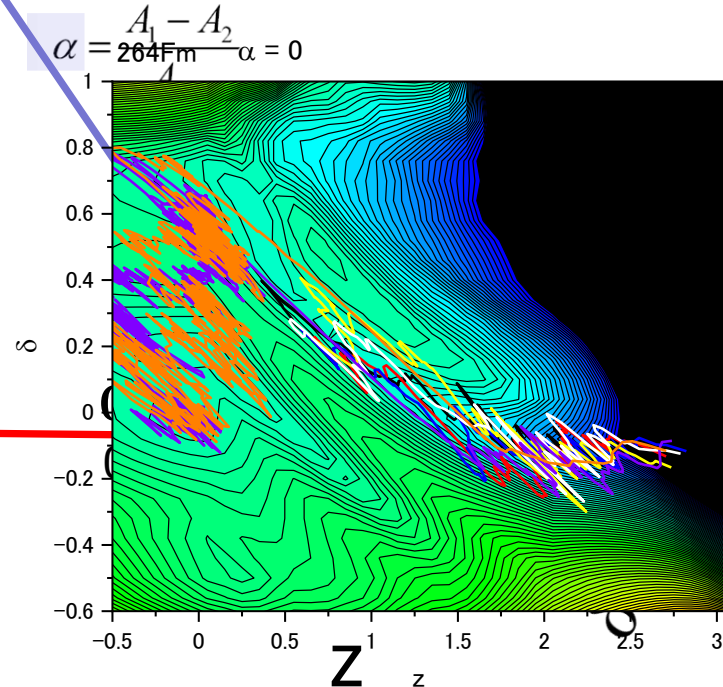
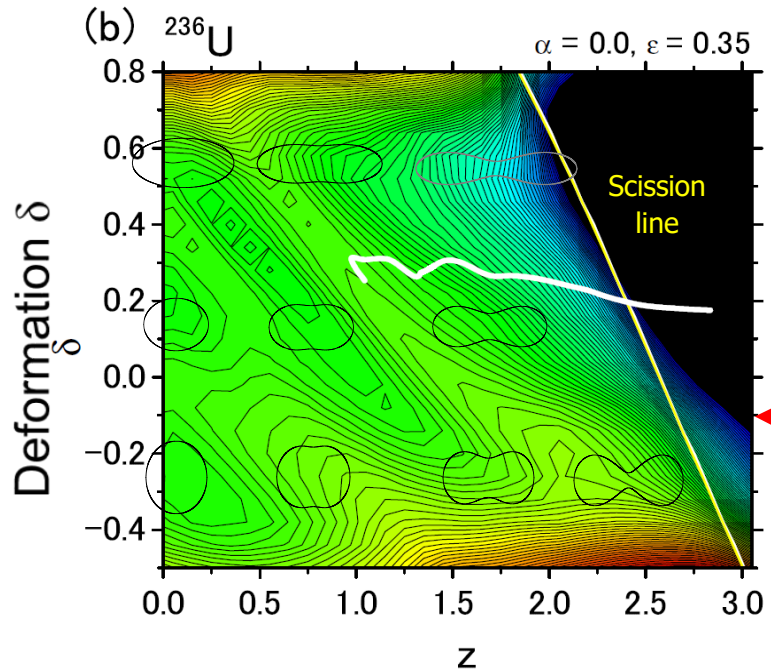
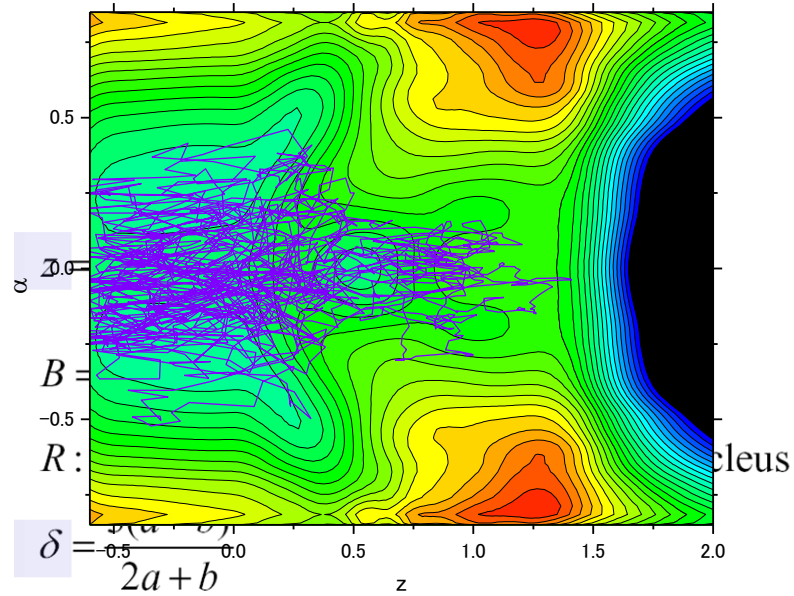
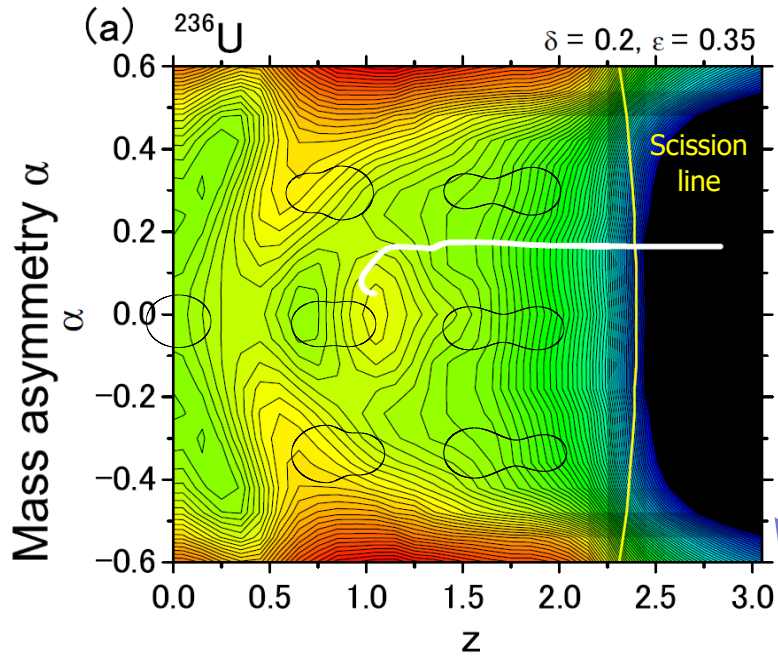


To perform trajectory analysis

Projection on two-dim. plane

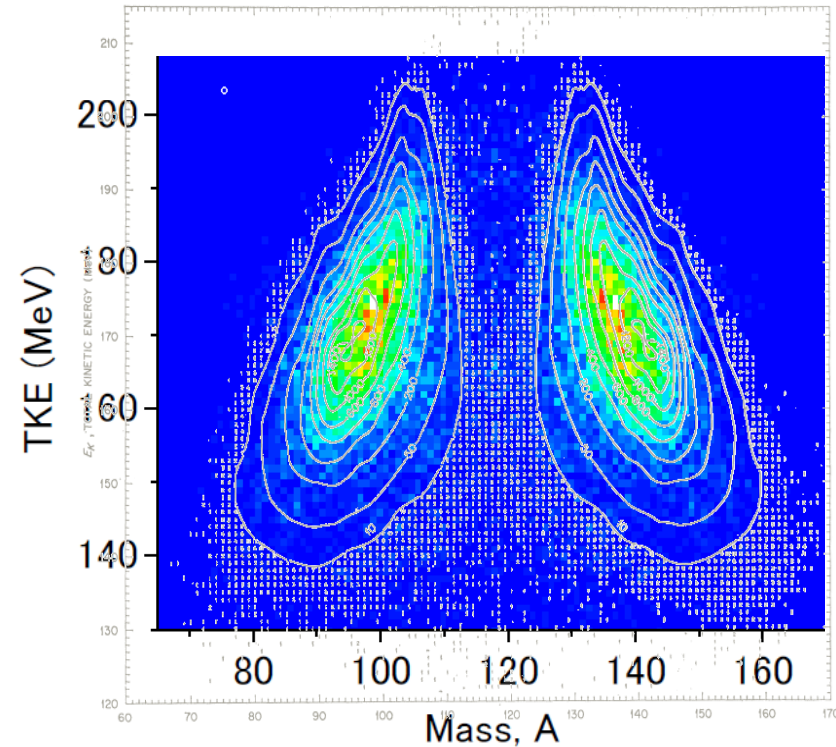
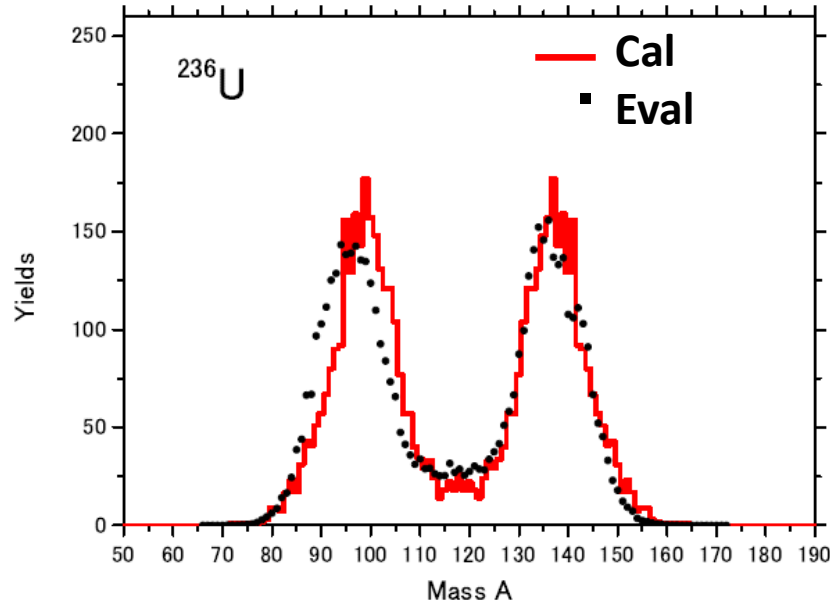
representative point

Without fluctuation



Projection on two-dim. plane

$^{236}\text{U}^*$ ($E^* = 20\text{MeV}$)



evaluation value J.Katakura, JENDL FP Decay Data File 2011
and Fission Yields

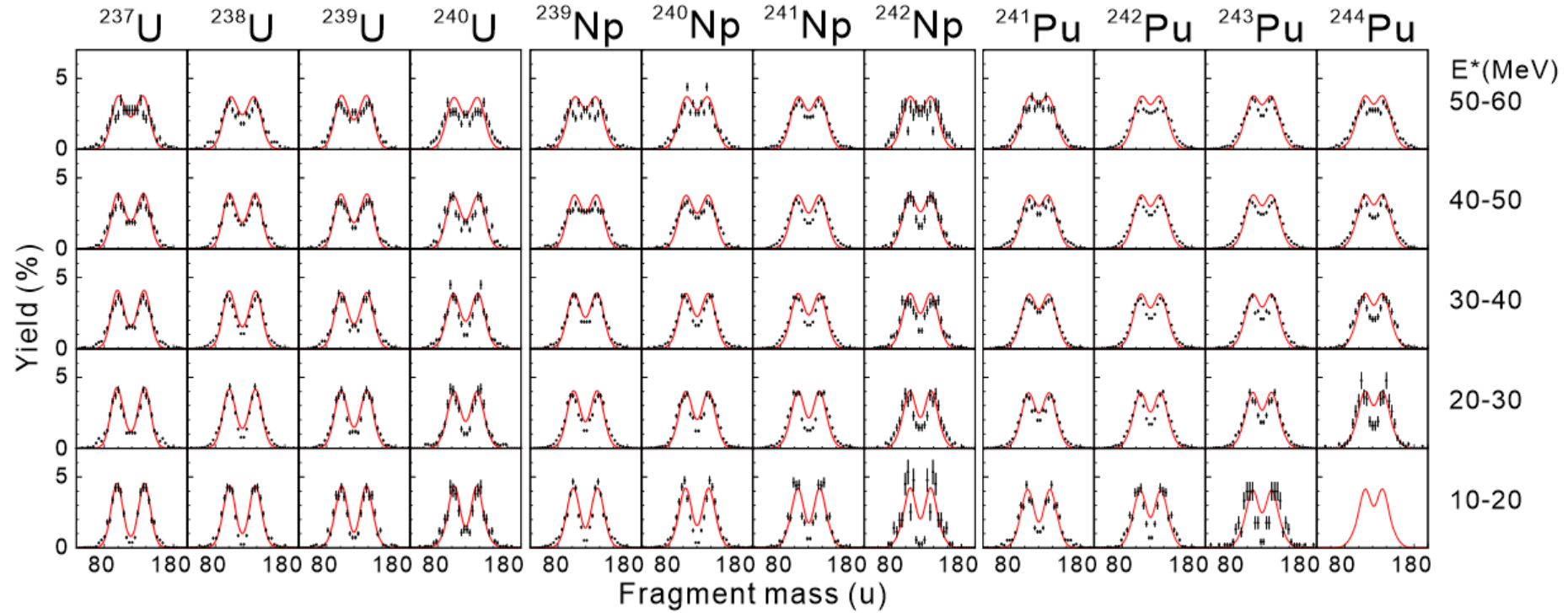
calculation Y. Aritomo and S. Chiba, PRC 88, 044614(2013)

Exp. Phys.Rev. **141**(1966)1146

$$\langle \text{TKE} \rangle_{\text{cal}} = 171.8 \text{ MeV}$$

$$\langle \text{TKE} \rangle_{\text{exp}} = 168.2 \sim 171.3 \text{ MeV} \quad (E^* = 21 \text{ MeV})$$

Langevin Calculation (Multi-chance Fission)



With
Multi-chance Fission

Calculated by S. Tanaka (Riken)

K. Hirose, K. Nishio, S. Tanaka, et al Phys.
Rev. Lett. 119, 222501 (2017)

S. Tanaka, et al.
Phys. Rev. C 100, 064605 (2019)

Research Trigger

*Good agreement with
the experimental data*

*We obtain **calculation results**,
Based on **the trajectory behavior***

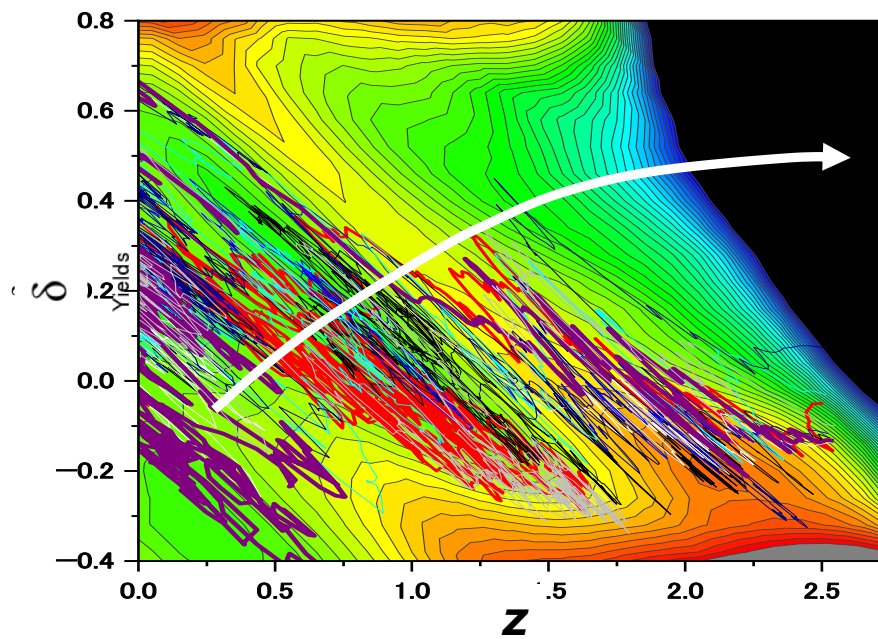
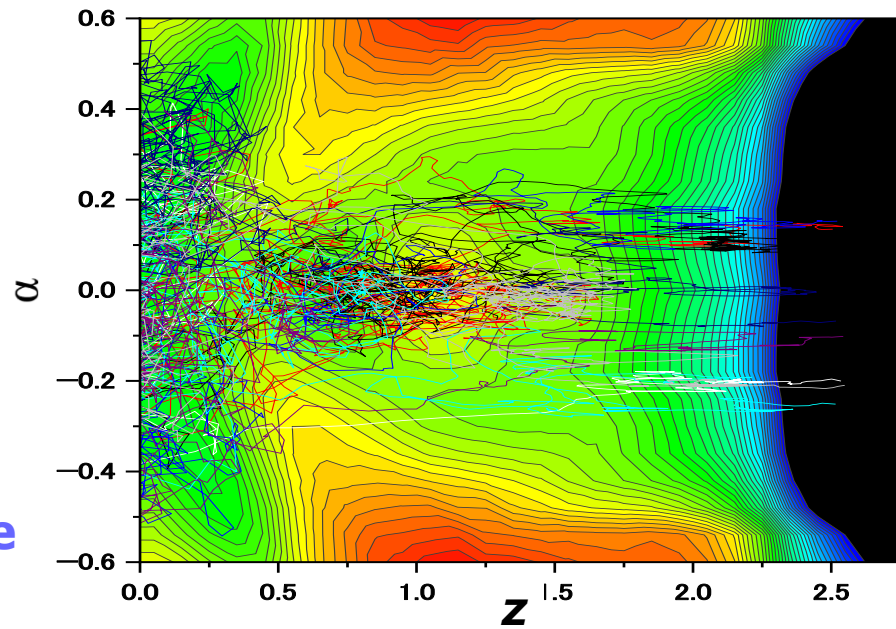


Go back to the fission path

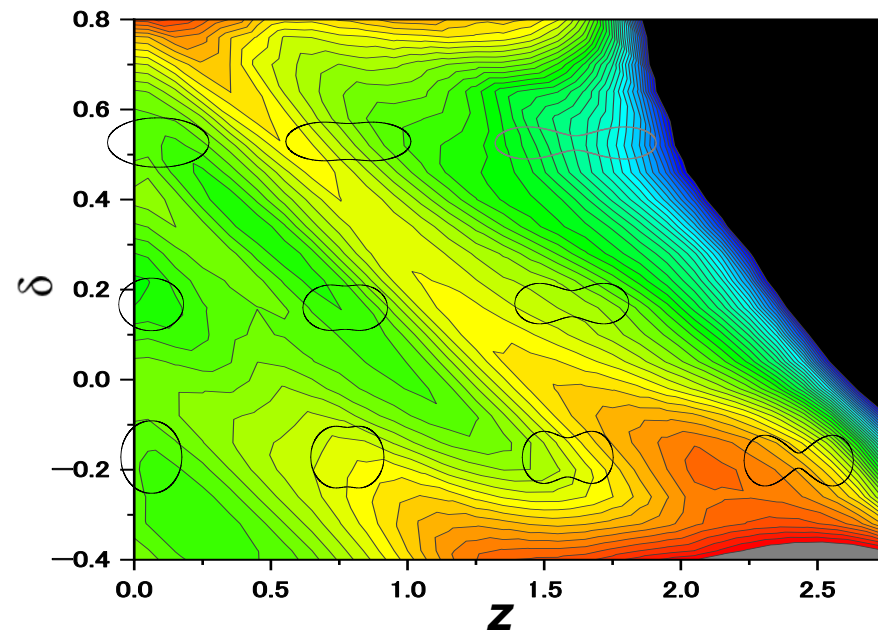
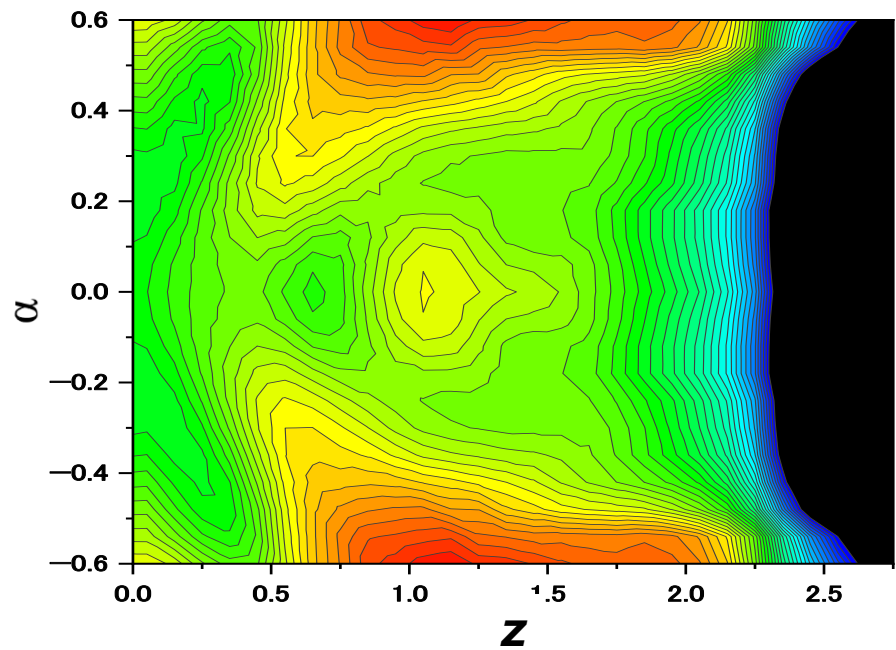
^{236}U

LDM+shell

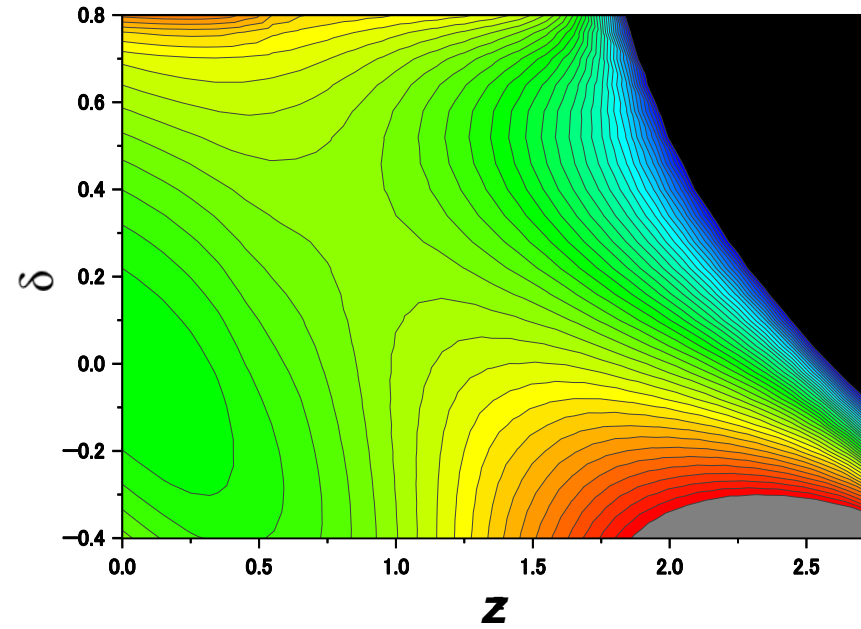
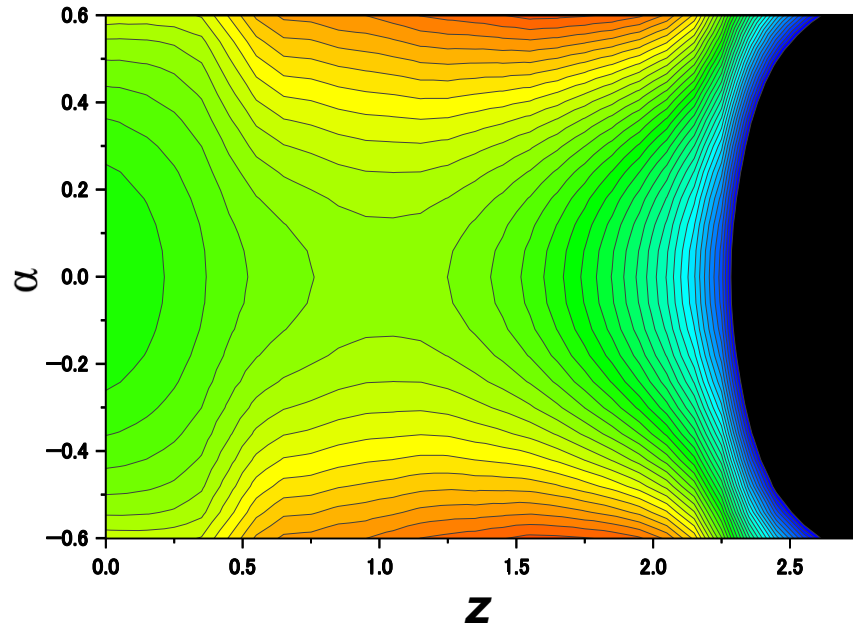
Potential energy surface



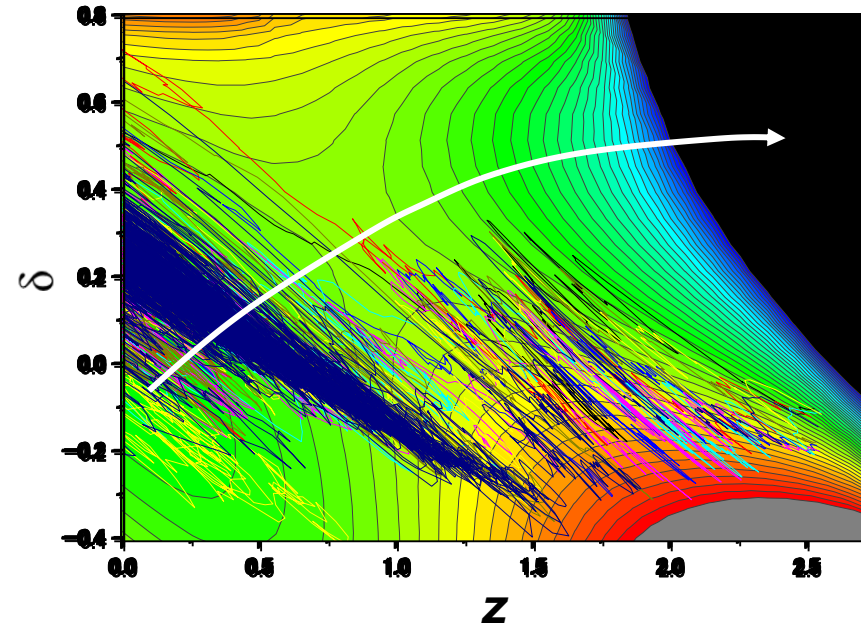
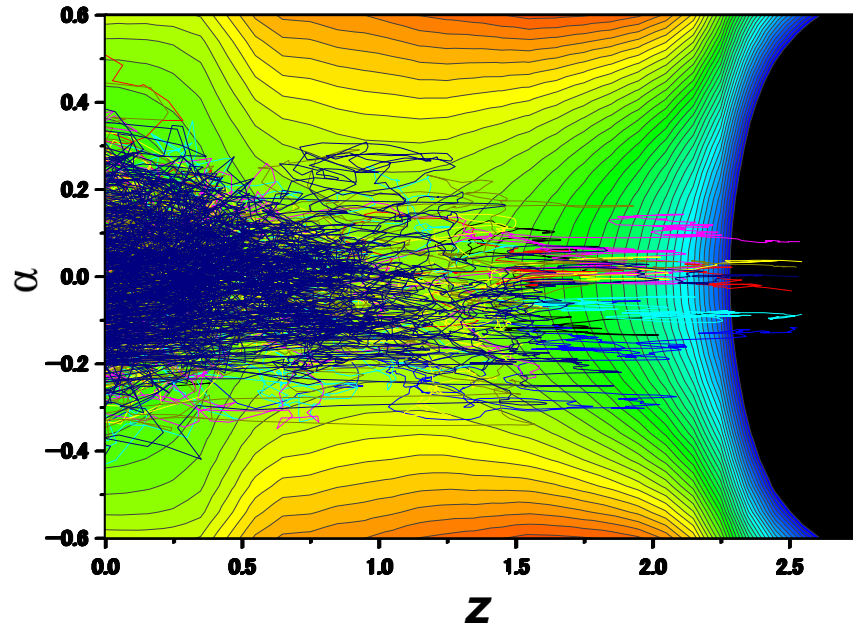
Such trajectory behavior is not observed



What happens with the LDM potential? ^{236}U



What happens with the LDM potential? ^{236}U





verification

Does the trajectory fluctuate
in a **particular direction**?

Test on a flat surface
← **unaffected by potential**

① Equation

$$\frac{dq_i}{dt} = (m^{-1})_{ij} p_j$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2} \frac{\partial}{\partial q_i} (m^{-1})_{jk} p_j p_k - \gamma_{ij} (m^{-1})_{jk} p_k + g_{ij} R_j(t)$$

② Transport coefficients

Includes non-diagonal components
(normal calculation case)

$$\begin{bmatrix} \gamma_{zz} & \gamma_{z\delta} & \gamma_{z\alpha} \\ \gamma_{\delta z} & \gamma_{\delta\delta} & \gamma_{\delta\alpha} \\ \gamma_{\alpha z} & \gamma_{\alpha\delta} & \gamma_{\alpha\alpha} \end{bmatrix} \begin{bmatrix} m_{zz} & m_{z\delta} & m_{z\alpha} \\ m_{\delta z} & m_{\delta\delta} & m_{\delta\alpha} \\ m_{\alpha z} & m_{\alpha\delta} & m_{\alpha\alpha} \end{bmatrix}$$

← Change the appearance
of the motion

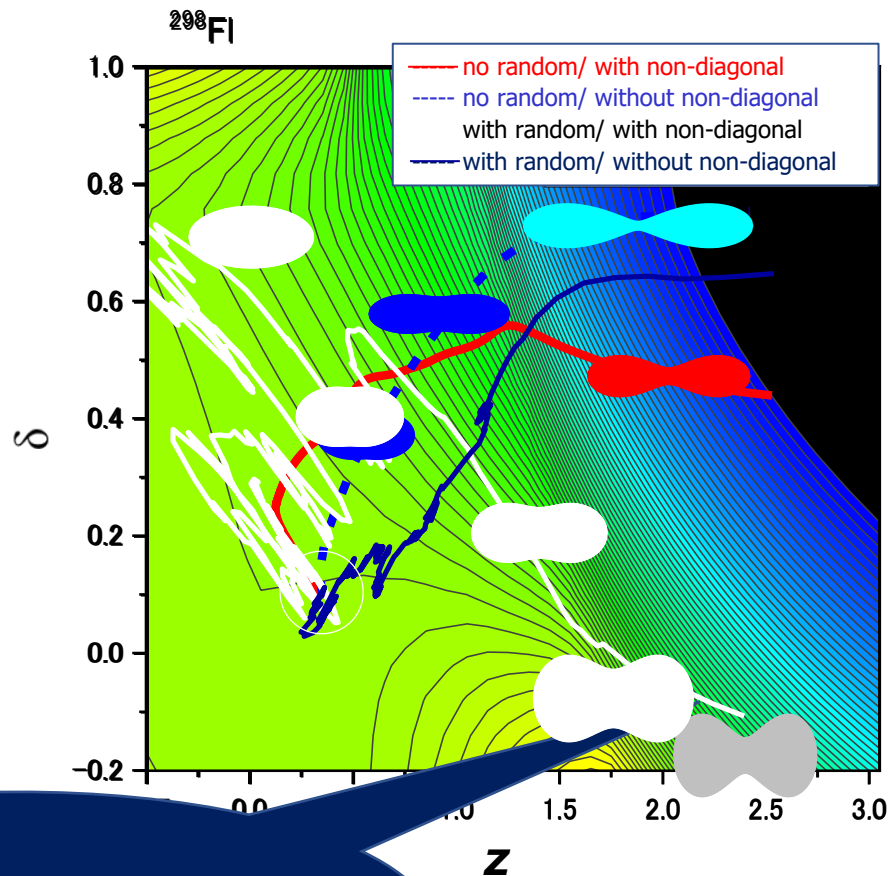
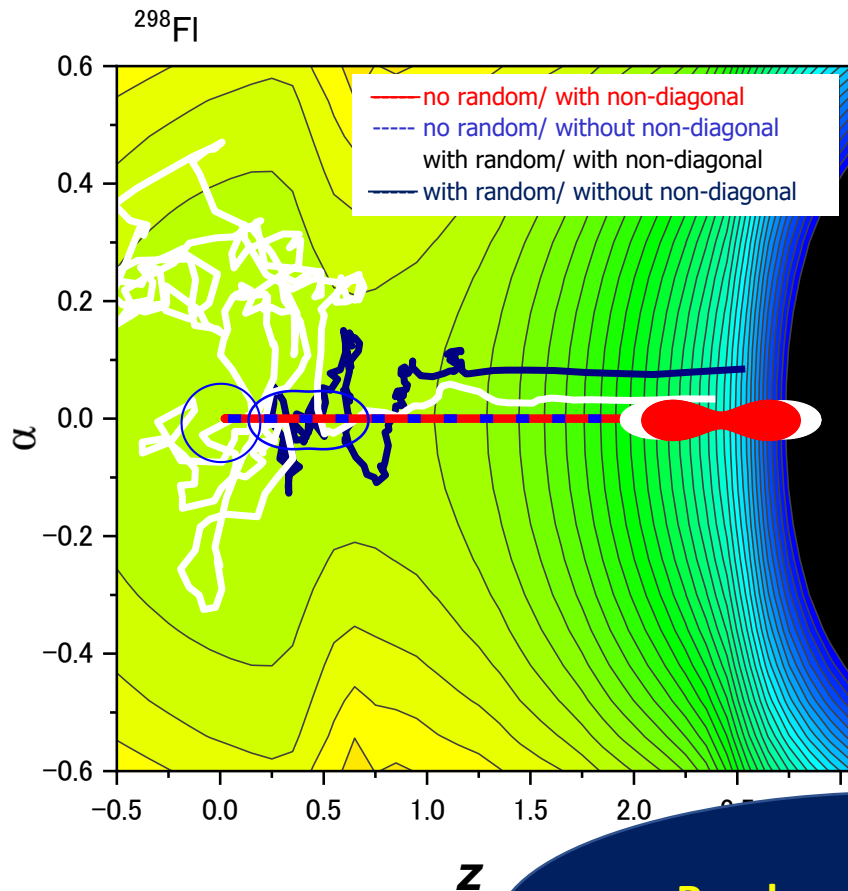
Without non-diagonal component
(test calculation)

$$\begin{bmatrix} \gamma_{zz} & 0 & 0 \\ 0 & \gamma_{\delta\delta} & 0 \\ 0 & 0 & \gamma_{\alpha\alpha} \end{bmatrix} \begin{bmatrix} m_{zz} & 0 & 0 \\ 0 & m_{\delta\delta} & 0 \\ 0 & 0 & m_{\alpha\alpha} \end{bmatrix}$$

^{298}Fl

V_{DM} : Liquid Drop Model with a random term

— With non-diagonal
- - - Without non-diagonal



Random forces strongly deflect the trajectory

This is not special case

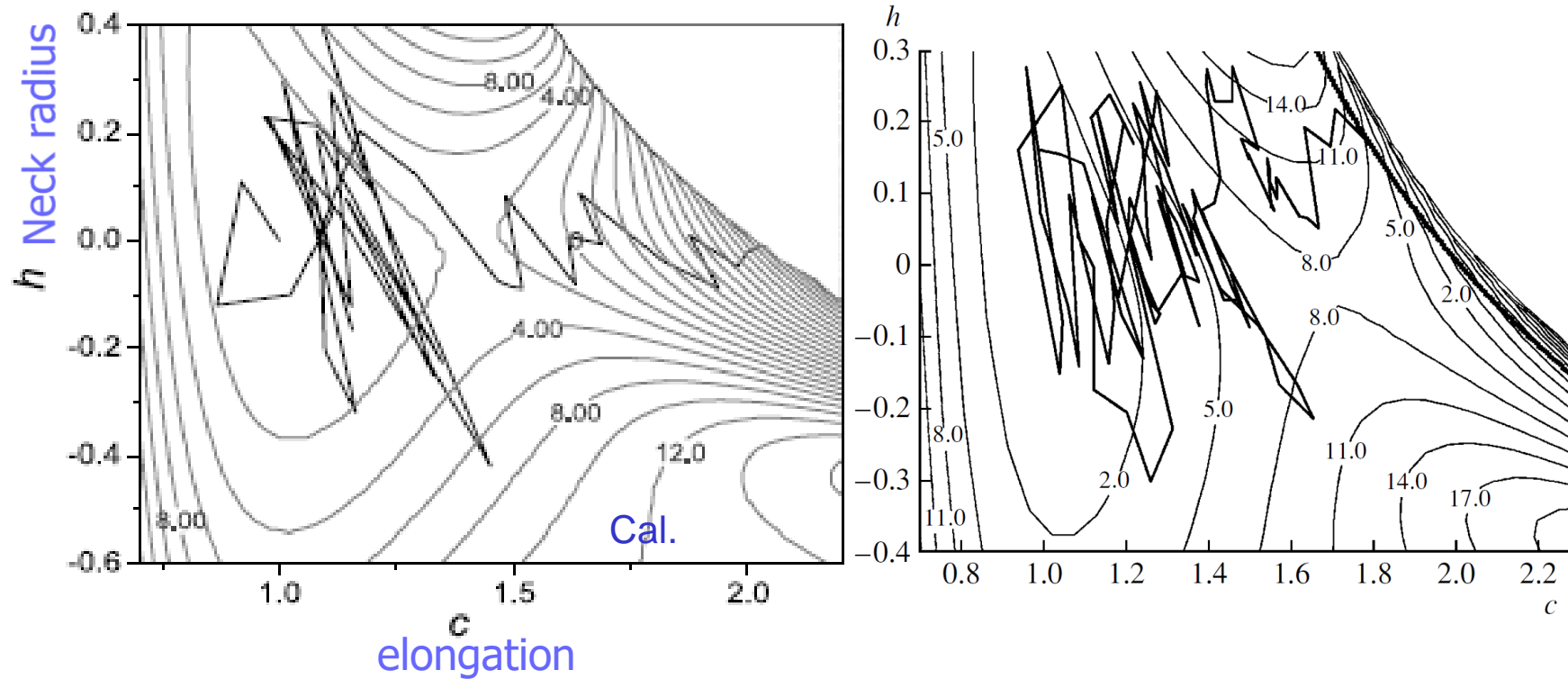
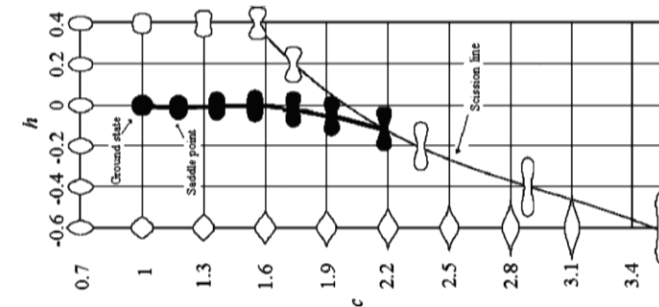


Figure 1. Stochastic Langevin trajectory in the space of the collective coordinates ($c, h, \alpha' = 0$) is shown against the potential-energy background. The numbers at the isolines specify the values of the potential energy in MeV. The solid line in the right upper corner of the figure is the scission line. The trajectory given in this figure represents a fission event.

A.V. Karpov,
 P.N. Nadtochy,
 D.V. Vanin, and G.D. Adeev,
 PRC 63 (2001) 054610



Fission process

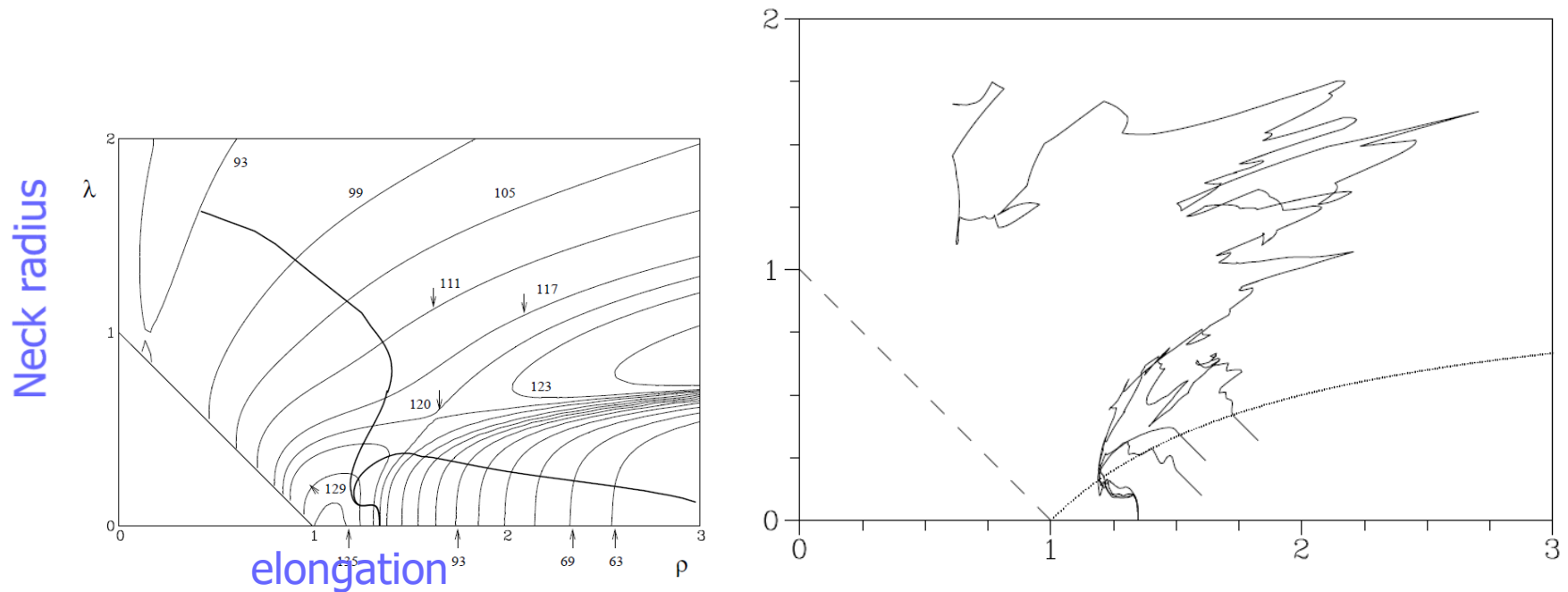
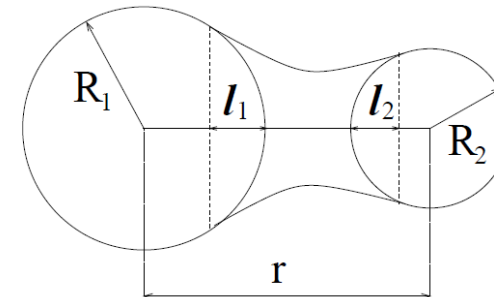


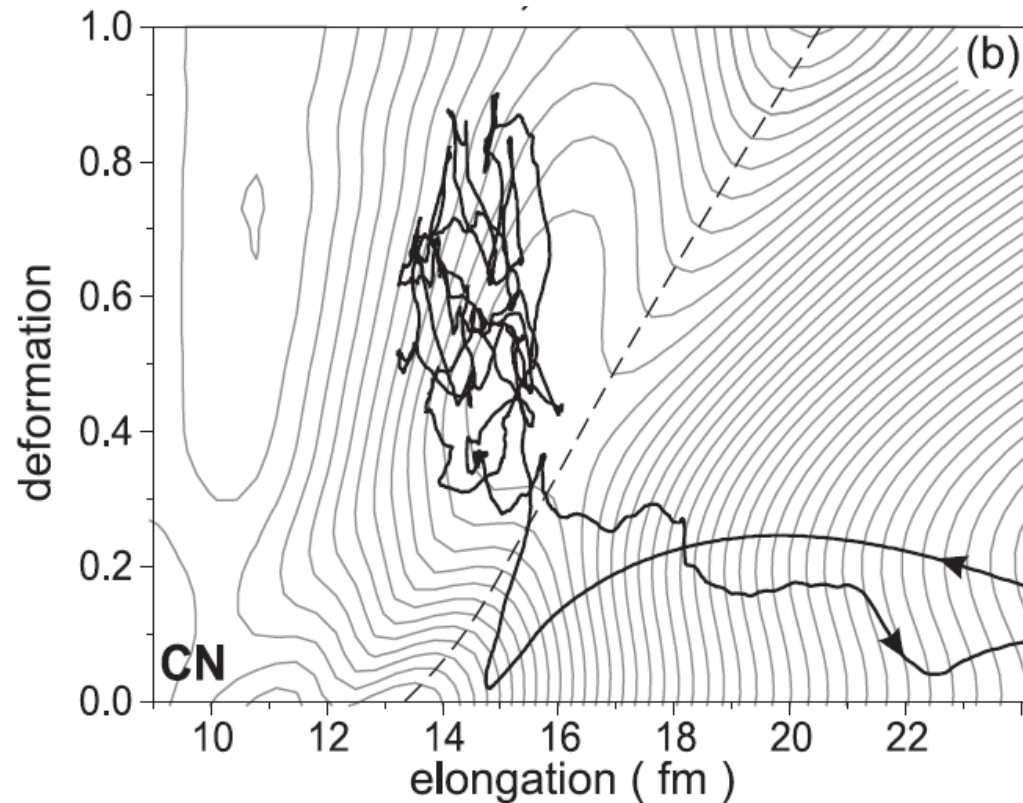
Fig. 6. Two deterministic trajectories for the $^{86}\text{Kr}+^{70}\text{Ge}$ colliding system, starting with initial kinetic energy 132 and 134 MeV and plotted on the potential energy map in (ρ, λ) -space. The energy contours are in MeV.

- 1) the distance variable, $\rho = r/(R_1 + R_2)$,
- 2) the neck variable, $\lambda = (l_1 + l_2)/(R_1 + R_2)$,
- 3) the asymmetry variable, $\Delta = (R_1 - R_2)/(R_1 + R_2)$

Langevin Dynamics in 4-dimensional Model of Nucleus-Nucleus Collisions
 J. Blocki, O. Mazonka, J. Wilczynski, Z. Sosin, and A. Wieloch
 Acta Physica Polonica B 31 1513 (2000)



Fusion-Fission process



V. Zagrebaev and W. Greiner
J. Phys. G. 31, 825-844 (2005)

The fluctuation of the trajectory
does not show particular direction

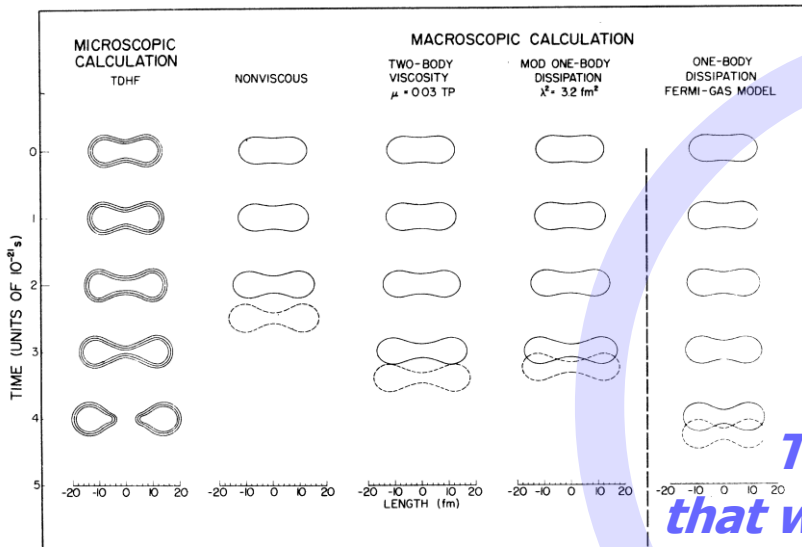
the friction forces which depend generally on the coordinates \vec{x} . For the moment, we ignore the non-diagonal terms of the mass and friction parameters. The so-called ‘sliding friction’ (which

Dynamics of induced fission*

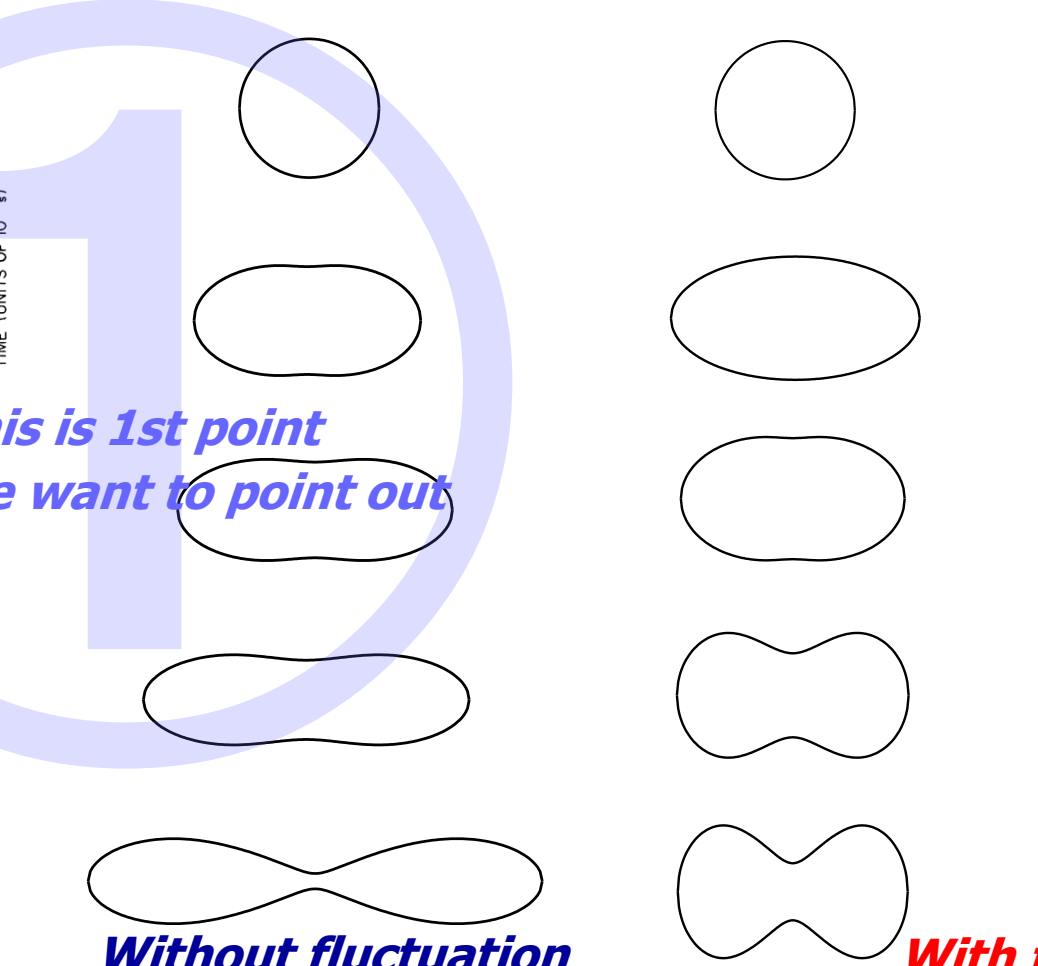
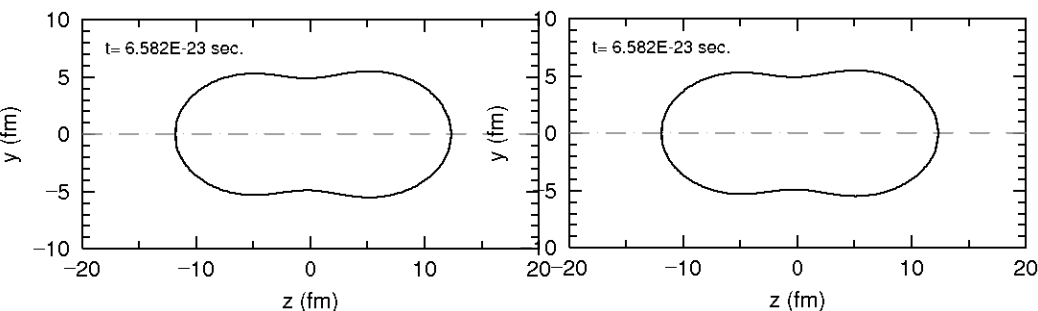
1978

Why calculate from the saddle?

J. W. Negele[†]
 Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
 S. E. Koonin[†]
 California Institute of Technology, Pasadena, California 91125
 Möller[‡] and J. R. Nix[‡]
 Laboratory, University of California, Los Alamos, New Mexico 87545
 A. J. Sierk[†]
 California Institute of Technology, Pasadena, California 91125
 Laboratory, University of California, Los Alamos, New Mexico 87545[§]
 Received 18 October 1977)



This is 1st point that we want to point out



Without fluctuation
 Without non-diagonal com.

With fluctuation
 With non-diagonal com.

The generalized Lagrange equations of motion are^{31,38,59}

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \frac{\partial F}{\partial \dot{q}_i}, \quad i = 1, 2, \dots, N, \quad (25)$$

where

$$L(q, \dot{q}) = T(q, \dot{q}) - V(q) \quad (26)$$

is the Lagrangian for the system. Upon substituting Eqs. (17), (18), and (26) into Eq. (25) and using the symmetry of the inertia and damping tensors, we obtain

$$\sum_j M_{ij}(q) \ddot{q}_j + \sum_{j,k} \left[\frac{\partial M_{ij}(q)}{\partial q_k} - \frac{1}{2} \frac{\partial M_{jk}(q)}{\partial q_i} \right] \dot{q}_j \dot{q}_k + \sum_j \eta_{ij}(q) \dot{q}_j + \frac{\partial V(q)}{\partial q_i} = 0, \quad i = 1, 2, \dots, N, \quad (27)$$

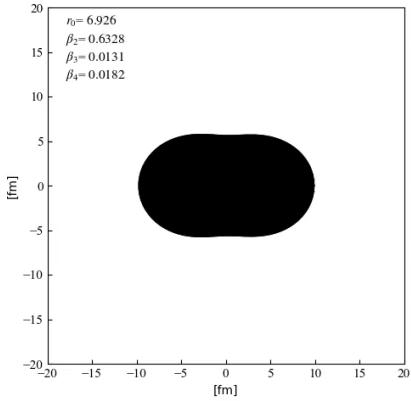


What is induced fission?

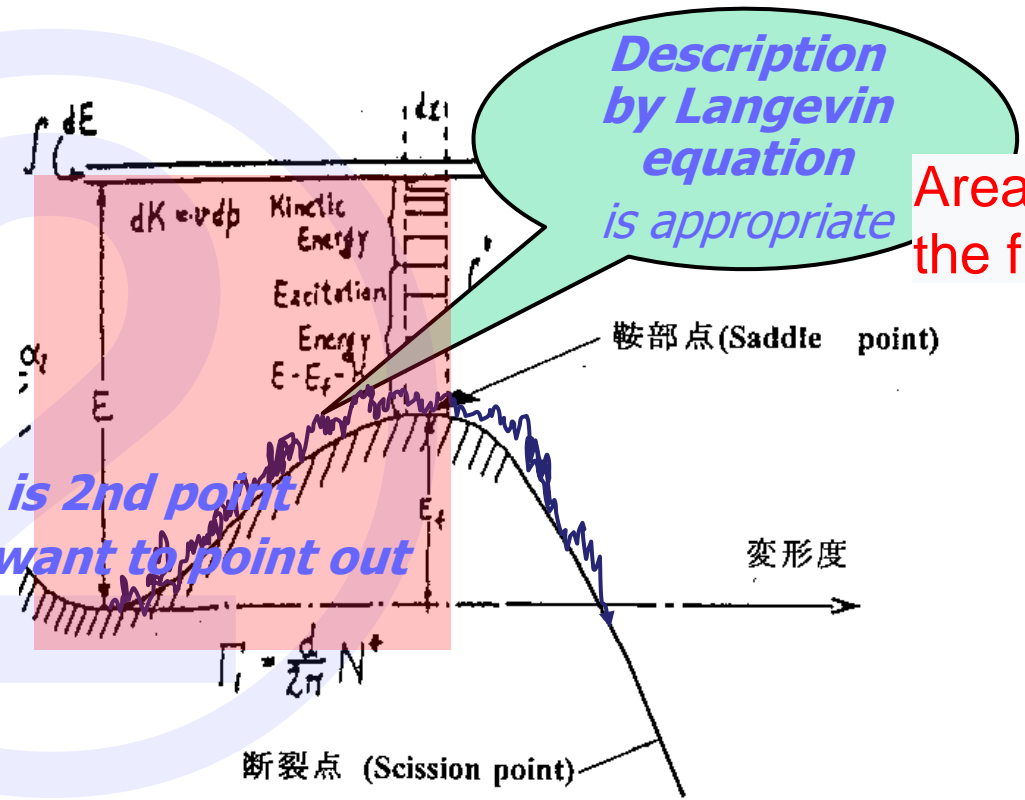
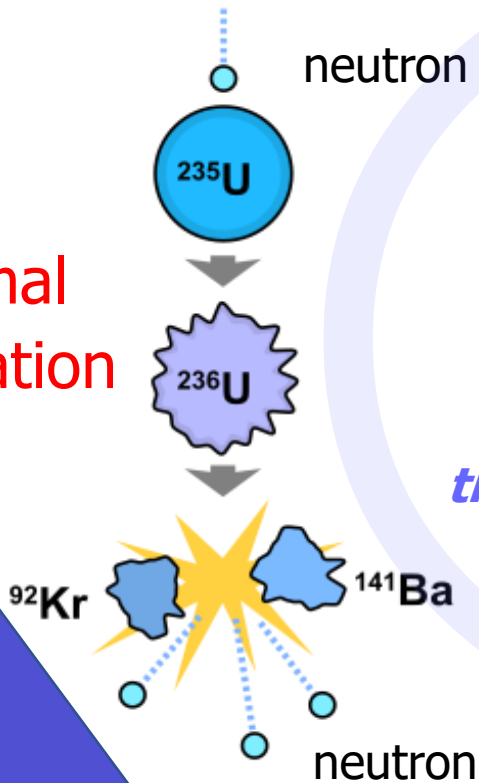
Induced fission occurs by thermal fluctuation of nuclei

To overcome the fission barrier, Thermal fluctuation of shape is required

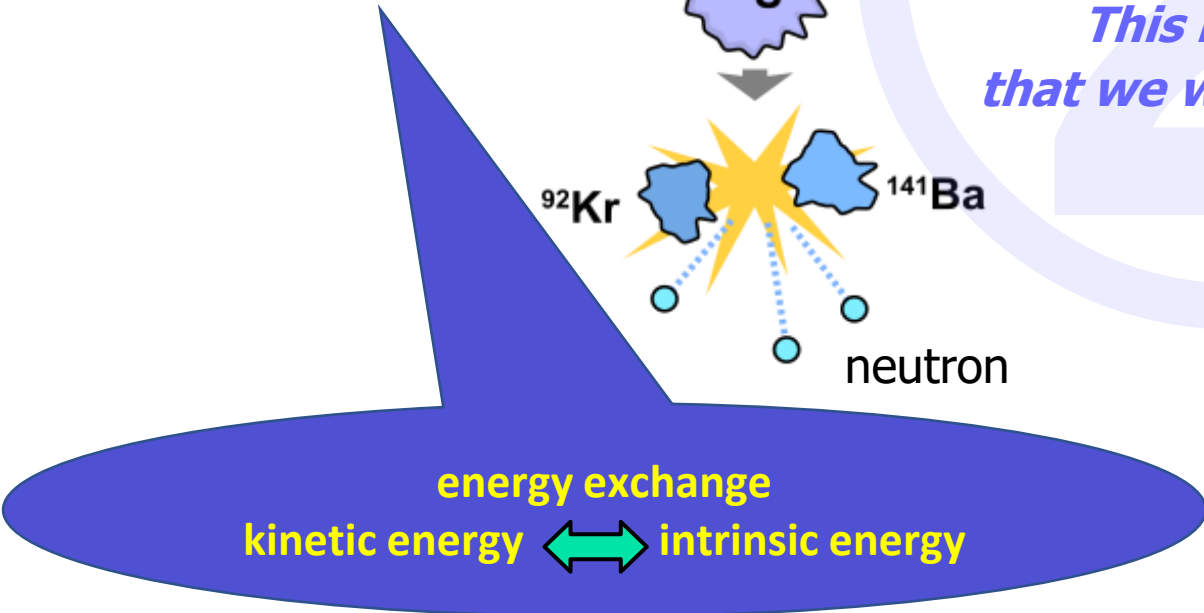
Under the strong friction



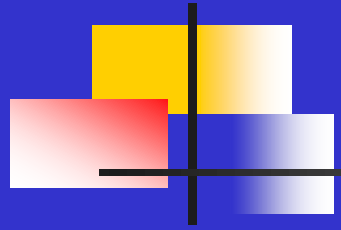
Thermal Fluctuation



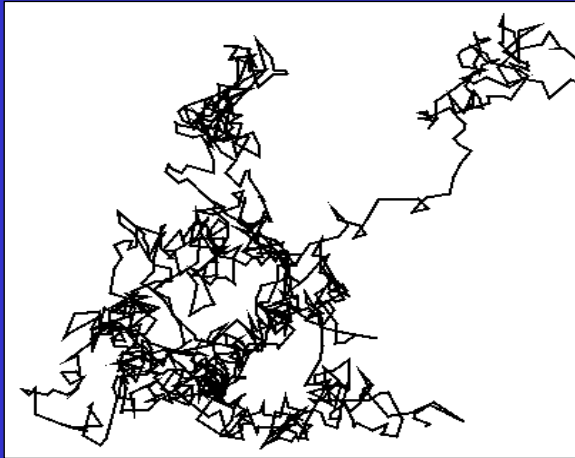
Area inside the fission barrier



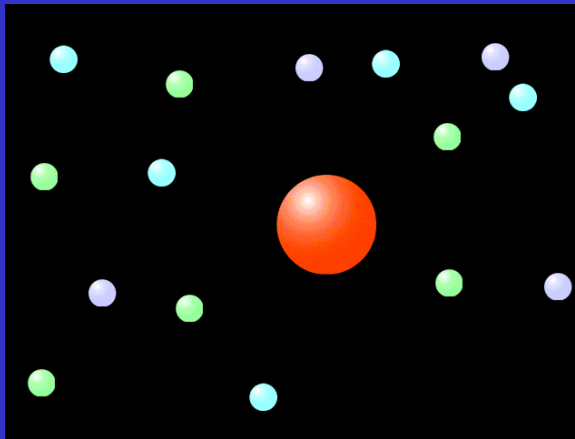
The Mechanism of Nuclear Fission
 NIELS BOHR
 University of Copenhagen, Copenhagen, Denmark, and The Institute for Advanced Study, Princeton, New Jersey
 AND
 JOHN ARCHIBALD WHEELER
 Princeton University, Princeton, New Jersey
 (Received June 28, 1939)



Brownian motion



Trajectory of a particle from pollen



..., the general existence of the **active molecules** in inorganic as well as organic bodies, their apparent indestructibility by heat, ...

R. Brown, Philosophical Magazine, vol.4, 1828

Motion of the tiny particles (0.3μm) from the pollen grains of flowers (30-50μm) on water (0.3nm)

Microscopic degree of freedom (water molecules)

← Thermal motion

Heat bath

→ Interaction

Macroscopic degree of freedom (particles of pollen)

Brownian motion

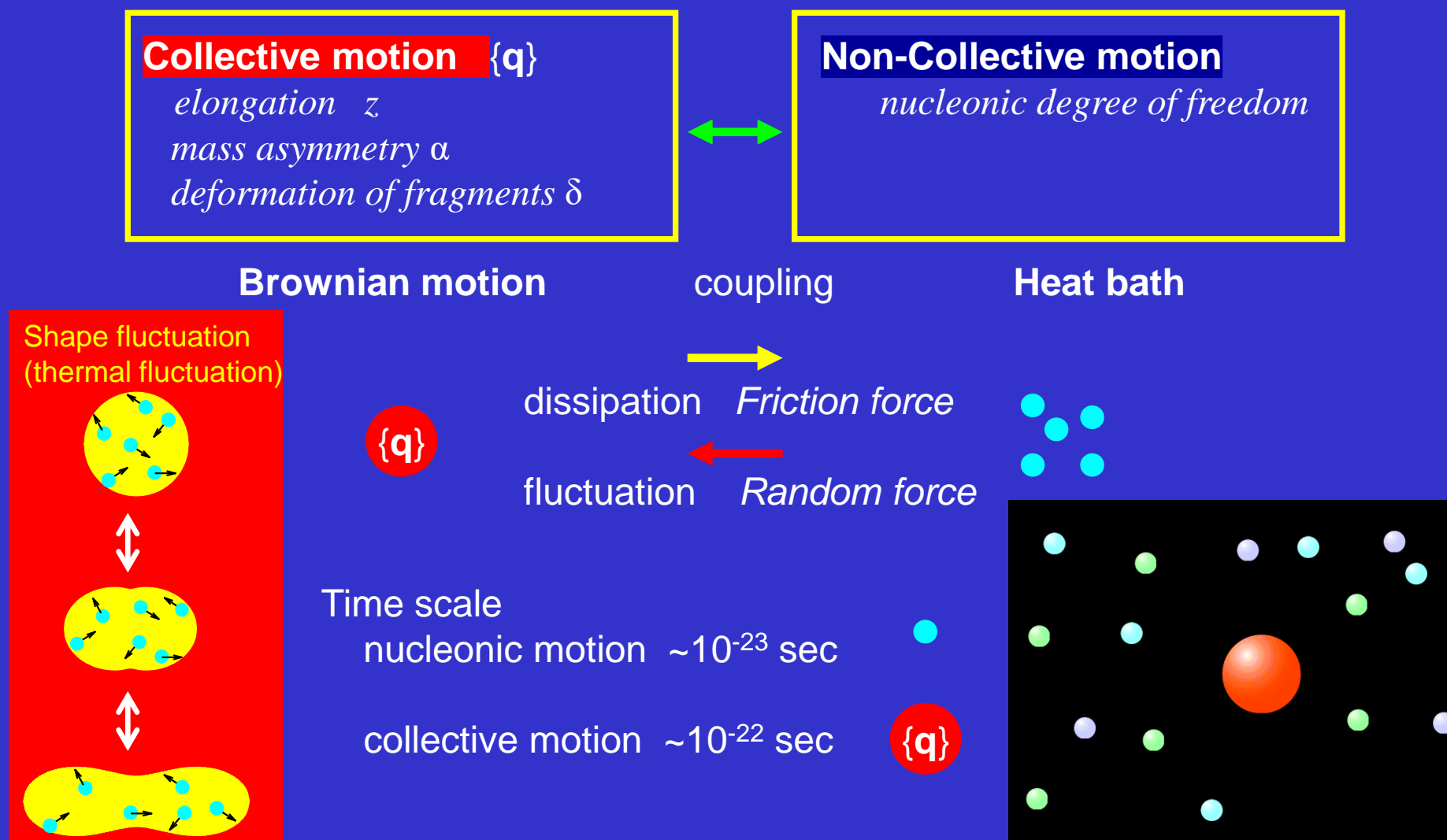
$$m \frac{dv}{dt} = -\gamma v + gR(t)$$

$$g = \sqrt{\gamma T}$$

Nuclear Fission : Employed the concept of Brownian motion

原子核の運動を2つのカテゴリーに分ける

Kramers (1940)



Summary

What governs nuclear fission

Nuclear structure (microscopic properties)

fission valley

Liquid Droplet properties (macroscopic properties)

Characteristics of shape (collective motion) fluctuation

inertia mass
friction (viscosity)

tensor

← nuclear density ρ

← *Requires
detailed analysis*

Affects direction of deformation is important

, which is controlled by strength and direction of random forces

What we want to convey at this workshop

No Induced Fission occurs without fluctuation
(thermal fluctuation)

Fluctuation has two directions, one is easy to
deform and the other is hard to deform

**It is not a property of potentials
It is the nature of the fluid**

*It's up to
the saddle*

Important physical quantities in trajectory analysis

Langevin Equation

$$\frac{dq_i}{dt} = (m^{-1})_{ij} p_j$$

$$\sum_k g_{ik} g_{jk} = T \gamma_{ij}$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2} \frac{\partial}{\partial q_i} (m^{-1})_{jk} p_j p_k - \gamma_{ij} (m^{-1})_{jk} p_k + g_{ij} R_j(t)$$

macroscopic property
macroscopic property

Macroscopic + Microscopic model

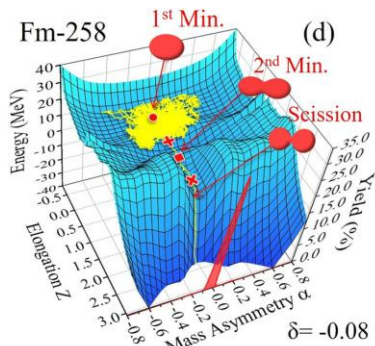
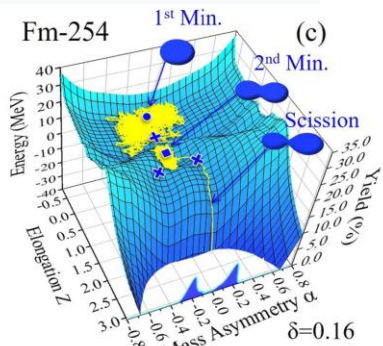
term where Shell structure are taken into account

Macroscopic model

Term where motion is governed by fluid properties

- Difficulty in motion
- Difficulty in changing a certain shape

Area inside the fission barrier



Collaborators

K. Nishio, K. Hirose, A. Iwamoto

Advanced Science Research Center, Japan Atomic Energy Agency



M. Ohta

Konan University



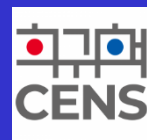
S. Amano, S. Takagi,
K. Nakajima, K. Kawai
Kindai University



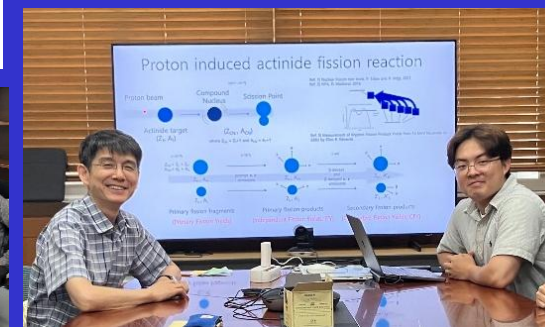
N. Nishimura
S. Tanaka
Riken



Y. Kim, C. Song,
C. Lee, I. Shin
*IBS, CENS, PNU
Korea*



F. Minato
*Kyushu
University*





Snapshot of water droplets

No charge

