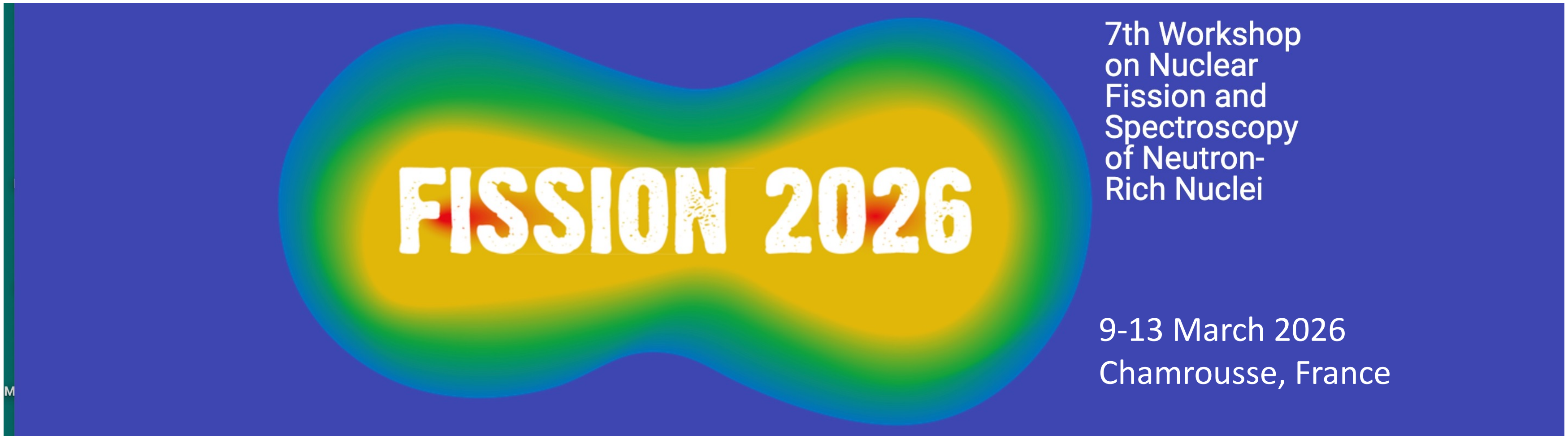


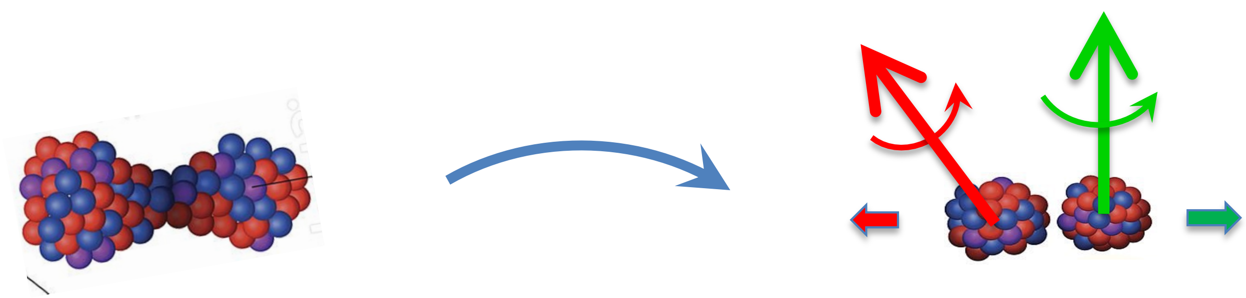
Effect of nucleon exchange on fission fragment angular momenta

Jørgen Randrup

Lawrence Berkeley National Laboratory, University of California

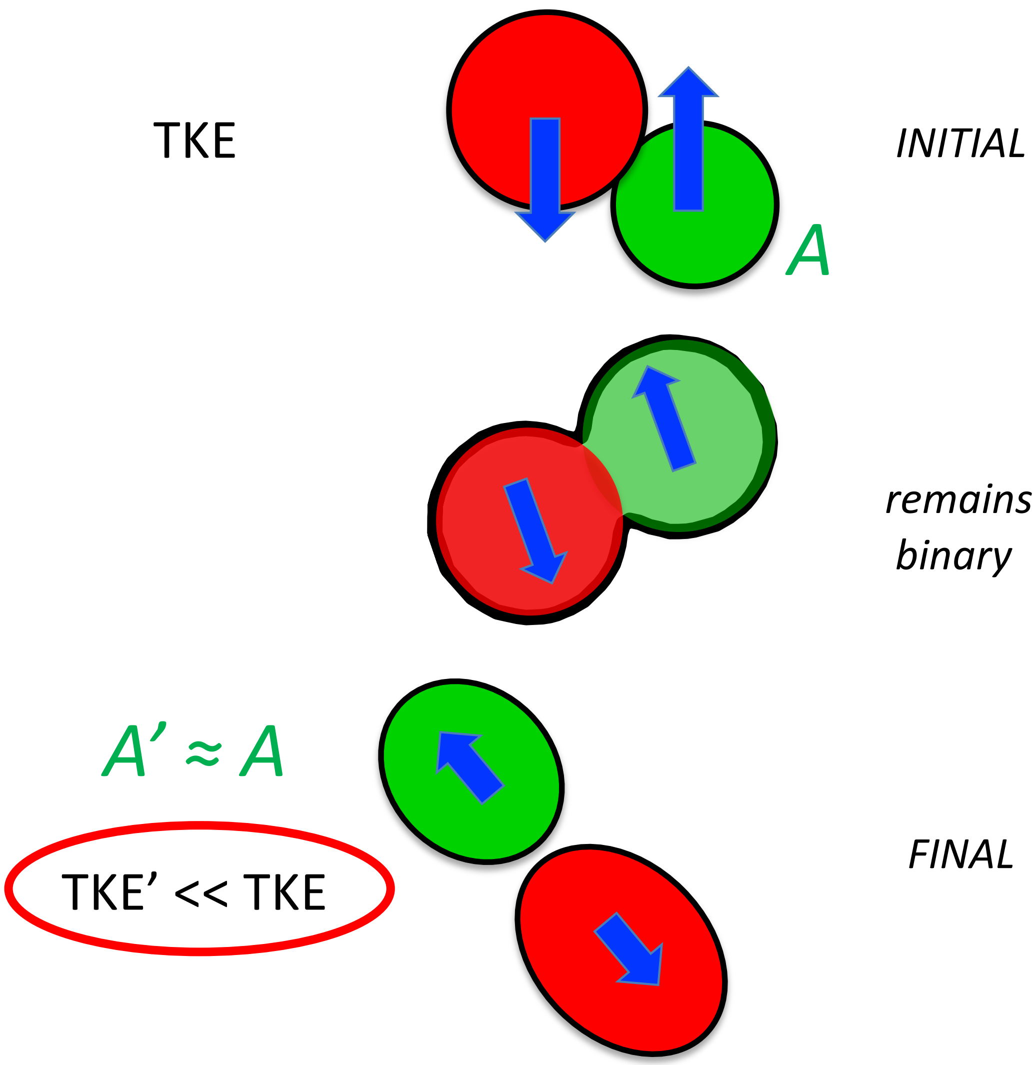
in collaboration with

Pavel Nadtochy, Christelle Schmitt, and Katarzyna Mazurek



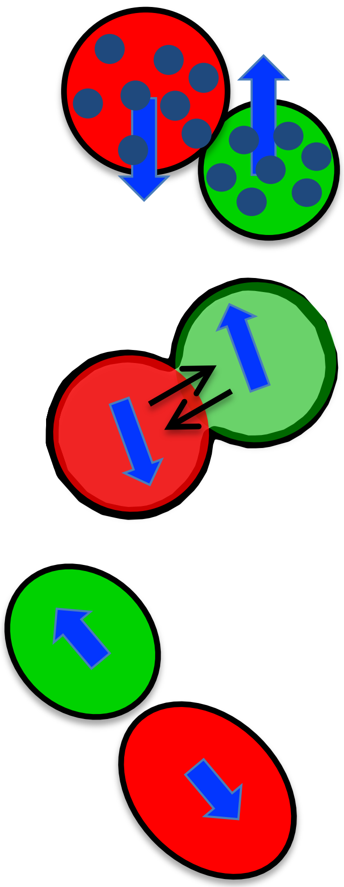
J. Randrup, P. Nadtochy, C. Schmitt, and K. Mazurek, Phys. Rev. C **113,** in press (2026)

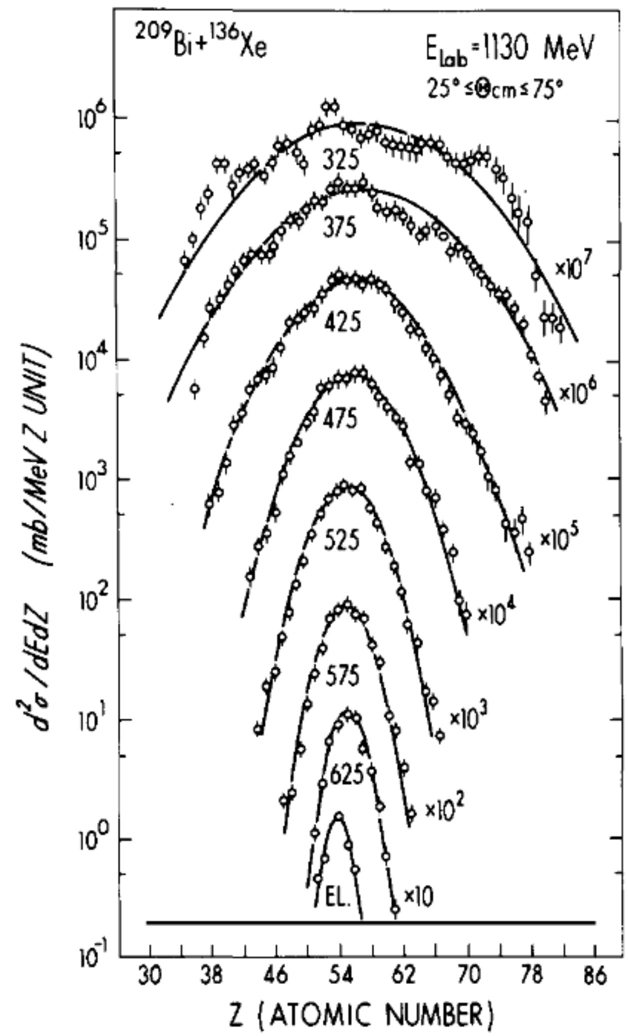
Strongly Damped Nuclear Reactions



Multiple nucleon exchange
was found to be a dominant mechanism
in *damped nuclear reactions*

W.U. Schröder and J.R. Huizenga,
Damped Nuclear Reactions,
Treatise on Heavy-Ion Science II, p. 113,
(1984) Plenum Press, Ed: D.A. Bromley





W.U. Schröder *et al*., PRL **36**, 514 (1975)

Multiple nucleon exchange was found to be a dominant mechanism in *damped nuclear reactions*

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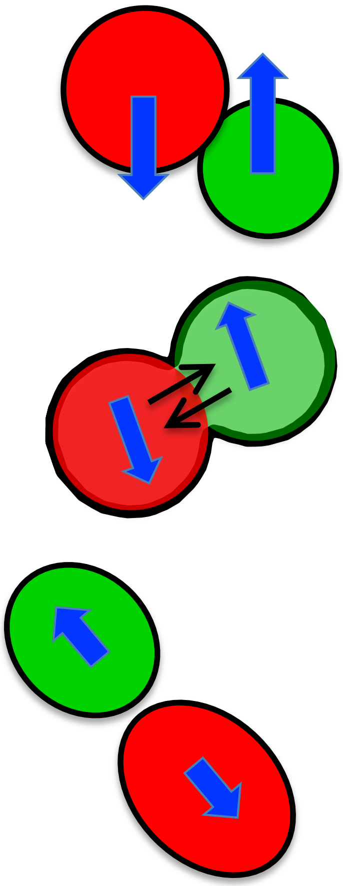
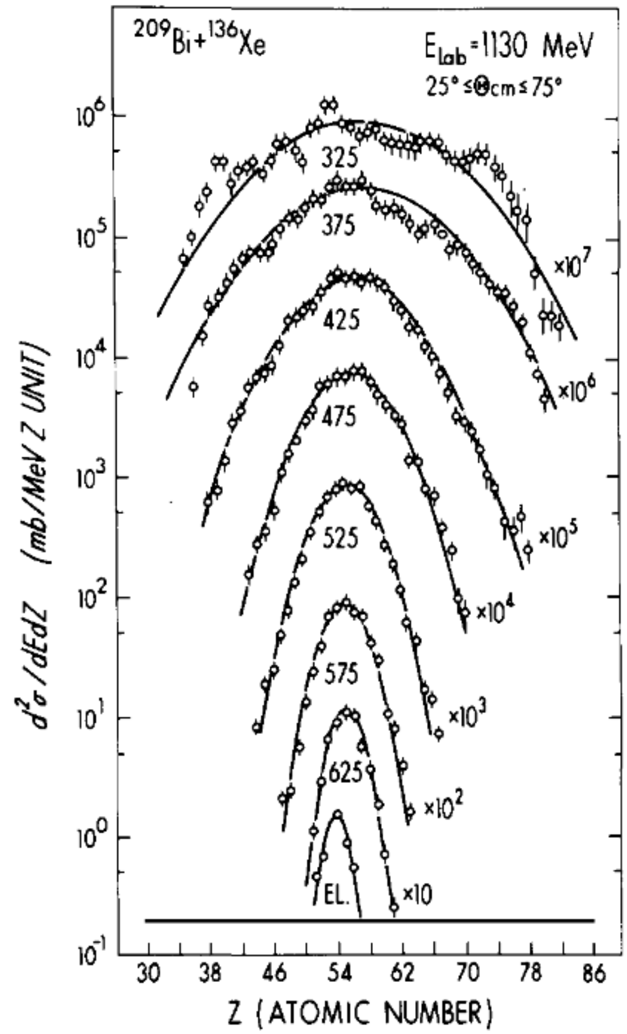


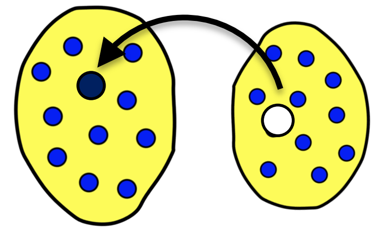


W.U. Schröder *et al.*, PRL **36**, 514 (1975)

The participating nucleons are *near the Fermi surface*

=> The dissipation is *strong!*

W.U. Schröder *et al.*, PRL **44**, 308 (1980)



window formula

Multiple nucleon transfers produce a dissipative force affecting the linear and angular momenta of the binary partners:

Z_H, N_H, $\boldsymbol{P}_H$, $\boldsymbol{S}_H$, T_H, Z_L, N_L, $\boldsymbol{P}_L$, $\boldsymbol{S}_L$, T_L

Nucleon Exchange Transport Theory

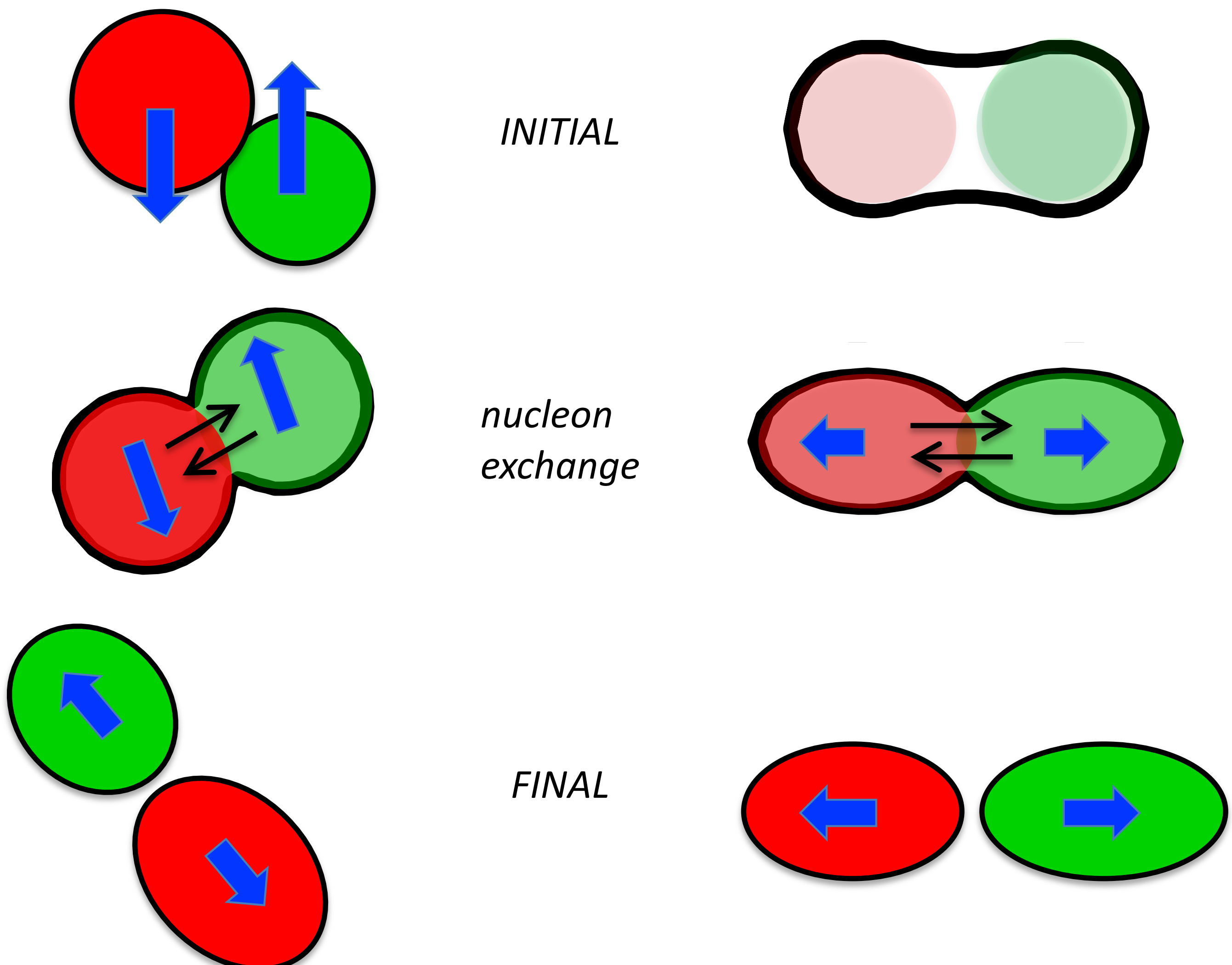
J. Randrup, Nucl. Phys. A **327**, 490 (1979)
J. Randrup, Nucl. Phys. A **383**, 468 (1983)
T. Døssing & J. Randrup, NPA **433**, 215 (1985)

Damped reaction

(binary throughout)

Saddle to scission

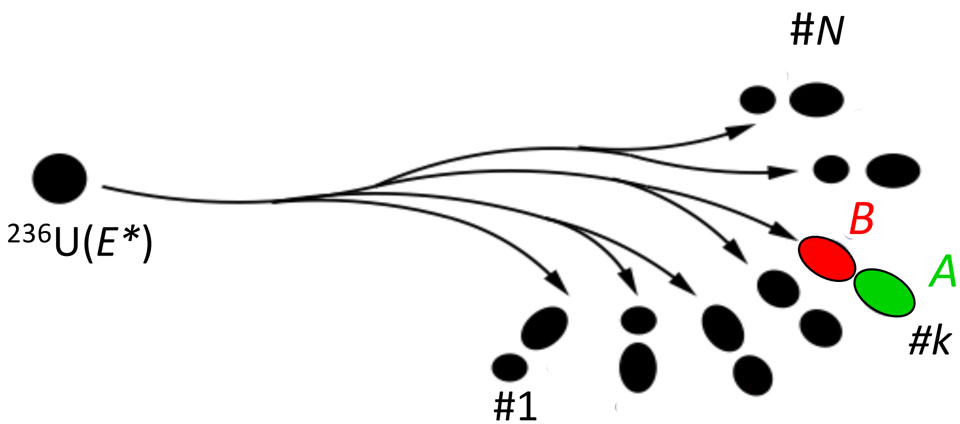
(becomes binary)



Calculational strategy

We consider a variety of cases, such as ^{236}U(E^*=46 MeV)

For each case, N = 10^4 different shape evolutions,
k = 1,…,N, are generated by *Langevin* simulation



For each trajectory, k, the evolution of the correlated fragment spin distribution, $P^{(k)}(\boldsymbol{S}_A, \boldsymbol{S}_B; t)$, is calculated with the *Nucleon Exchange transport theory*

Generate fission events by Langevin simulation

G.D. Adeev, A.V. Karpov, P.N. Nadtochii, and D.V. Vanin,
Physics of Particles and Nuclei **36,** 378 (2005)

8 cases:
$N = 10^4$

- ^{236}U at E^* = 46 & 70 MeV with k_s = 1 & 0.25
- ^{202}Po at E^* = 46 MeV with k_s = 1 & 0.25
- ^{182}Hg at E^* = 46 MeV with k_s = 1 & 0.25

One-body dissipation: $\dot{Q}_{\rm wall} = k_{\rm s} \times \underbrace{m\rho\bar{v} \oint \dot{n}^2 d^2\sigma}_{\text{Standard } wall\ formula}$

k_s = 1: *Standard value*

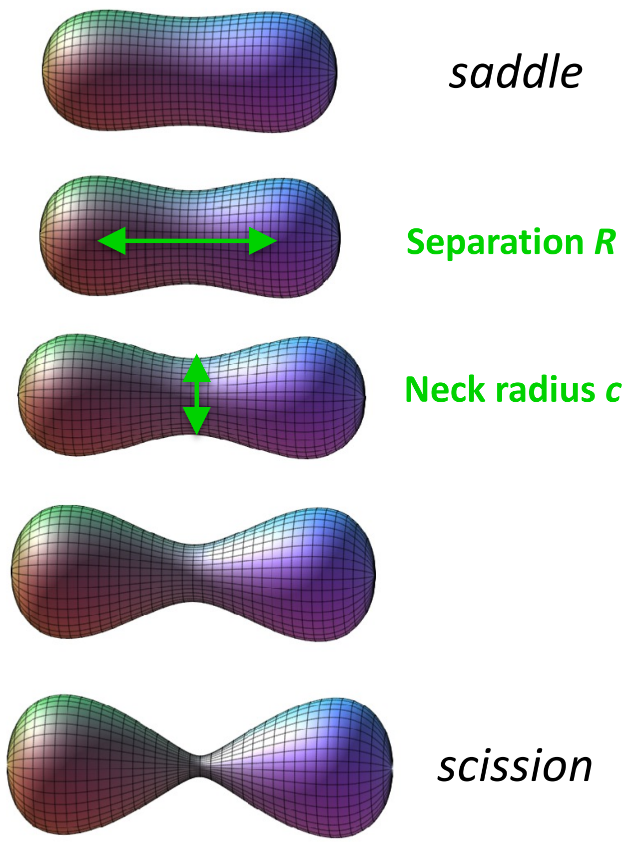
k_s = 0.25: *Fitted value*

Standard *wall formula*

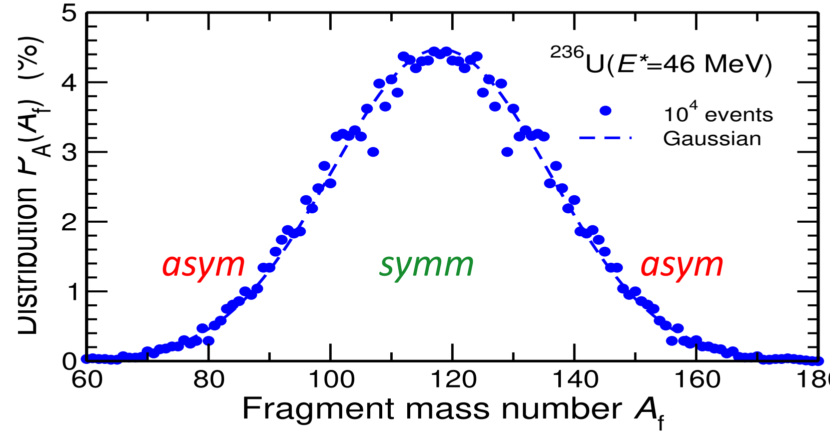
J. Blocki, Y. Boneh, J.R. Nix. J. Randrup, M. Robel,
A.J. Sierk, W.J. Swiatecki, Ann. Phys. **113**, 330 (1978)

Dynamical evolution: *R*(*t*) & *c*(*t*) & α(*t*) - as well as *T*(*t*)

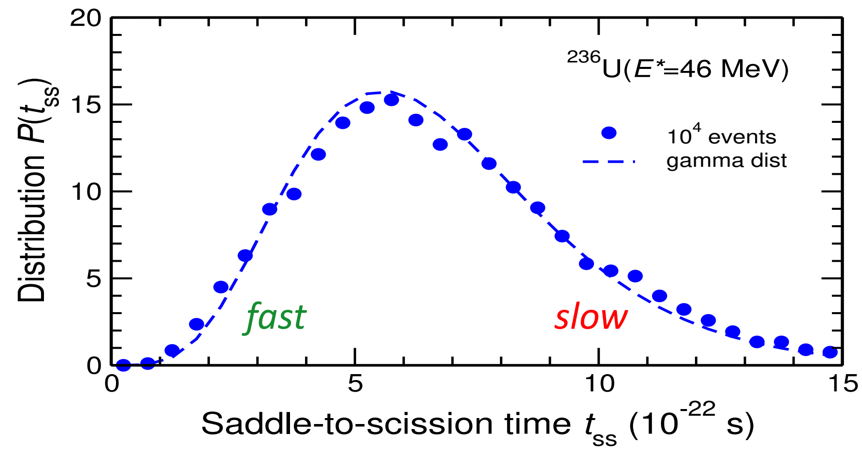
One shape evolution



Fragment mass distribution



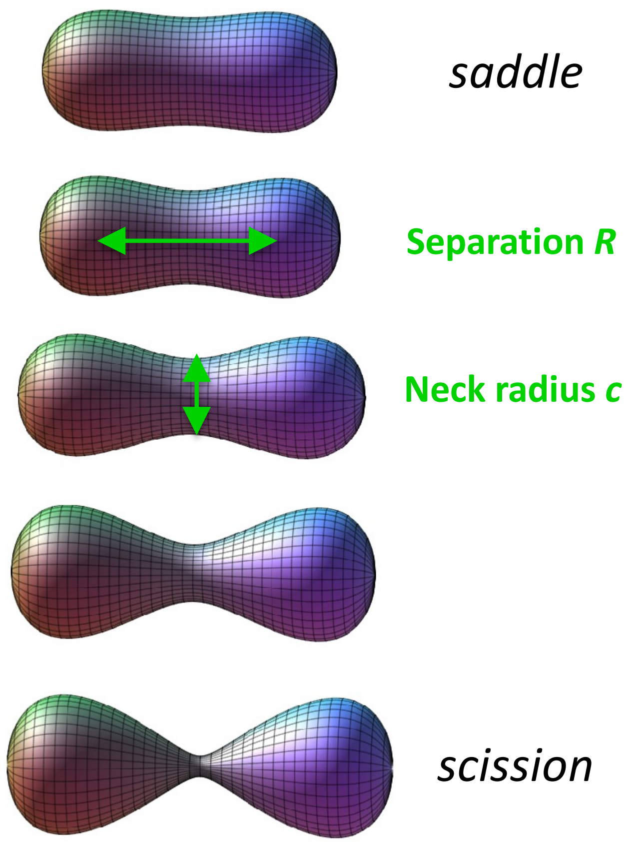
Saddle-to-scission time t_{ss}

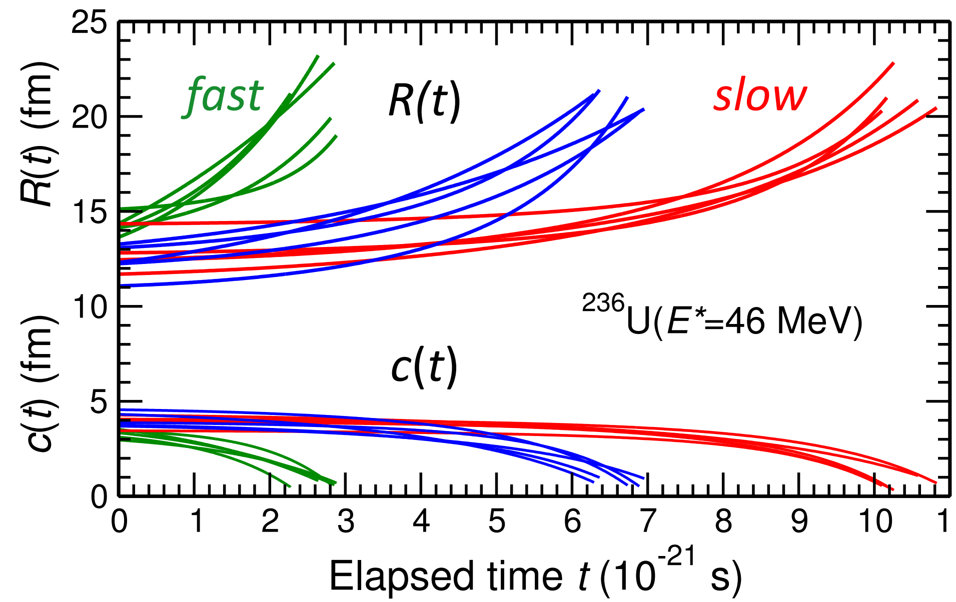


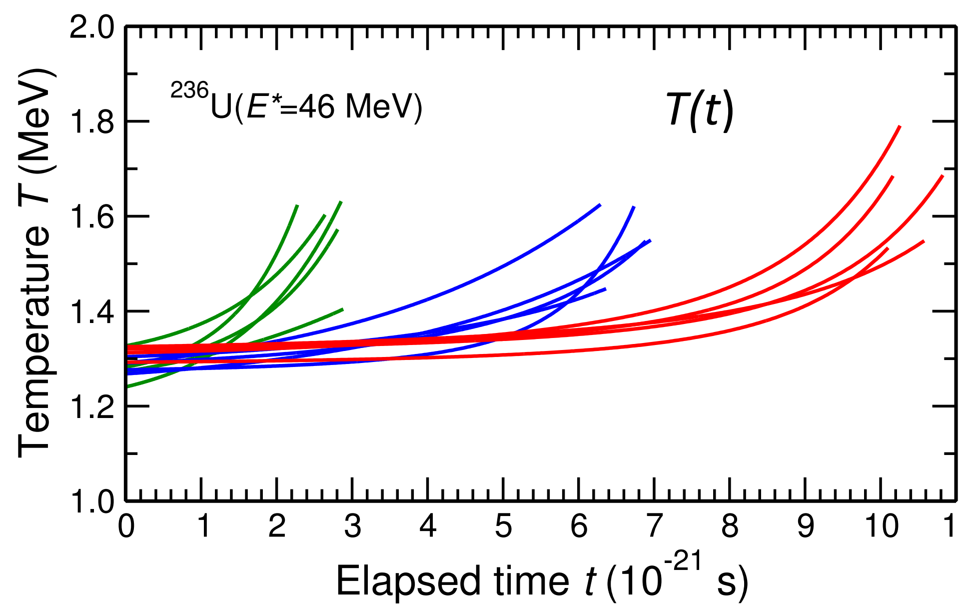
$$\gamma(t;p,t_0) \equiv \frac{1}{t_0}\frac{1}{\Gamma(p)}\left(\frac{t}{t_0}\right)^{p-1} e^{-t/t_0}$$

gamma distribution

Dynamical evolution: *R*(*t*) & *c*(*t*) & α(*t*) - as well as *T*(*t*)







For each shape evolution, we wish to calculate the evolution of the distribution of the correlated fragment spins, $P(\boldsymbol{S}_A, \boldsymbol{S}_B)$

Rotational energy:

overall rotation (J terms) — *internal rotations* (s terms)

$$E_{\rm rot} = \frac{J_\perp^2}{2\mathcal{I}_\perp} + \frac{J_\parallel^2}{2\mathcal{I}_\parallel} + \frac{s_{\rm wrig}^2}{2\mathcal{I}_{\rm wrig}} + \frac{s_{\rm bend}^2}{2\mathcal{I}_{\rm bend}} + \frac{s_{\rm twst}^2}{2\mathcal{I}_{\rm twst}}$$

Normal spins: $s_{\rm wrig}$, $s_{\rm bend}$, $s_{\rm twst}$

For each shape evolution, we wish to calculate the evolution of the distribution of the correlated fragment spins, $P(\boldsymbol{S}_A,\boldsymbol{S}_B)$

Fokker-Planck equation for $P(\boldsymbol{s}_A,\boldsymbol{s}_B;t)$

$$\frac{\partial}{\partial t}P = -\sum_i \frac{\partial}{\partial s_i} V_i P + \sum_{ij} \frac{\partial^2}{\partial s_i \partial s_j} D_{ij} P$$

Drift coefficient $V_i\,(\chi) = \Sigma_j\, M_{ij}\,(\chi)\, f_j$ determines the *mean* evolution of $s_i(t)$:

$$\frac{d\bar{s}_i}{dt} = V_i(\bar{\chi})$$

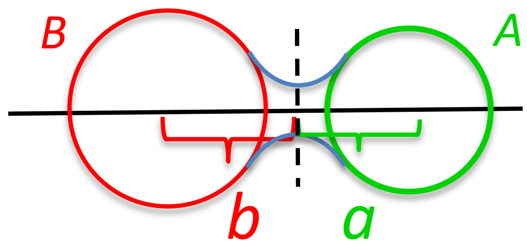
Mobility coefficients

Diffusion coefficient $D_{ij}\,(\chi) = M_{ij}\,(\chi)\; T$ generates *correlated spin fluctuations* $\sigma_{ij}(t)$:

$$\sigma_{ij} \equiv \langle (s_i - \bar{s}_i)(s_j - \bar{s}_j) \rangle :$$

$$\frac{d}{dt}\sigma_{ij} \;=\; 2D_{ij}(\bar{\chi}) - \sum_k \left[\nu_{ik}\sigma_{kj} + \nu_{jk}\sigma_{ki}\right]$$

Mobility coefficients $M_{ij}(\chi)$



$$c_{\rm ave}^2 = \tfrac{1}{2}c^2$$

a + *b* = *R*

Twisting $(s_A^{\parallel}, s_B^{\parallel})$:
$$\boldsymbol{M}_{AB}^{\parallel} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 \begin{pmatrix} c_{\rm ave}^2 & -c_{\rm ave}^2 \\ -c_{\rm ave}^2 & c_{\rm ave}^2 \end{pmatrix}$$

Wriggling / Bending $(s_{\rm wrig}, s_{\rm bend})$:
$$\boldsymbol{M}_{\pm} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 \begin{pmatrix} R^2 & R\delta \\ R\delta & \delta^2 + c_{\rm ave}^2 \end{pmatrix}$$

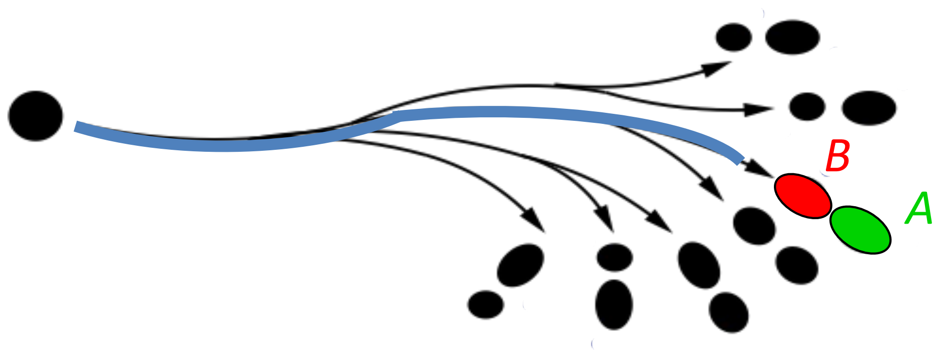
$$\delta = \frac{a\mathcal{I}_B - b\mathcal{I}_A}{\mathcal{I}_A + \mathcal{I}_B}$$

Individual spins $(s_A^{\perp}, s_B^{\perp})$:
$$\boldsymbol{M}_{AB}^{\perp} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 \begin{pmatrix} a^2 + c_{\rm ave}^2 & ab - c_{\rm ave}^2 \\ ab - c_{\rm ave}^2 & b^2 + c_{\rm ave}^2 \end{pmatrix}$$

$$\boldsymbol{s}_A = \frac{\mathcal{I}_A^{\perp}}{\mathcal{I}_A^{\perp} + \mathcal{I}_B^{\perp}}\boldsymbol{s}_{\rm wrig} + \boldsymbol{s}_{\rm bend}$$
$$\boldsymbol{s}_B = \frac{\mathcal{I}_B^{\perp}}{\mathcal{I}_A^{\perp} + \mathcal{I}_B^{\perp}}\boldsymbol{s}_{\rm wrig} - \boldsymbol{s}_{\rm bend}$$

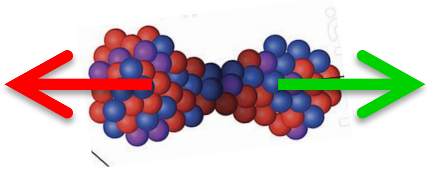
J. Randrup, Nucl. Phys. A **327** (1979) 490; **383** (1983) 468
T. Døssing and J. Randrup, Nucl. Phys. A **433** (1985) 215

Results for a single shape evolution



Results for a single shape evolution:

Spin components *along* the fission axis



Twisting

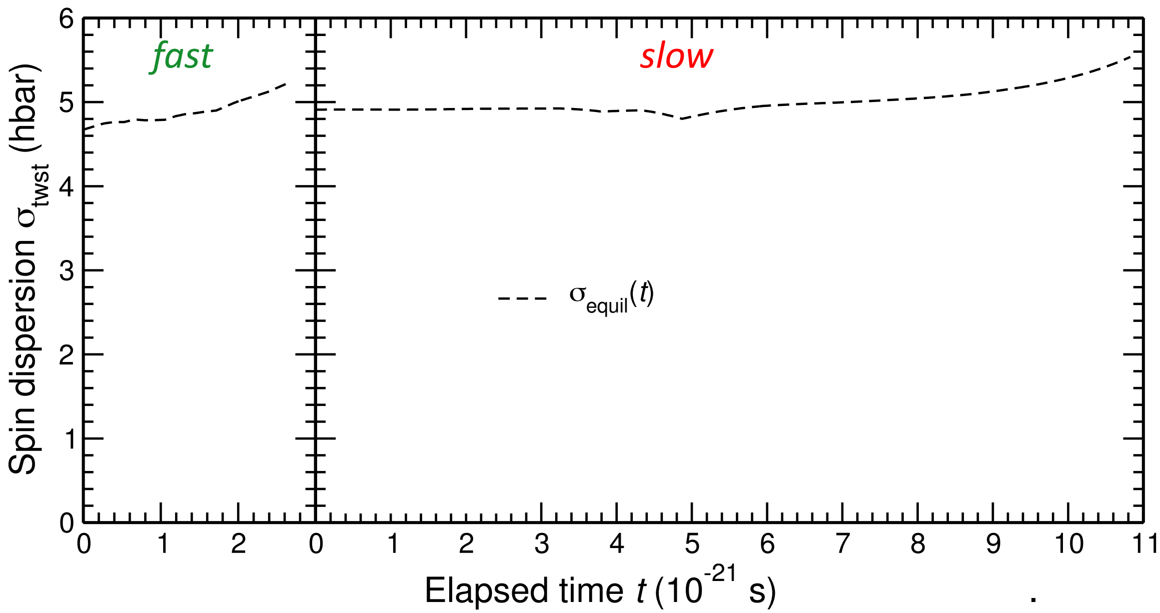
Moment of inertia: $\mathcal{I}_{\rm twst} = \frac{\mathcal{I}_A^{\|}\mathcal{I}_B^{\|}}{\mathcal{I}_A^{\|}+\mathcal{I}_B^{\|}}$

Equilibrium variance: $\tilde{\sigma}^2_{\rm twst} = \mathcal{I}_{\rm twst}T$

Individual fragments:

$s_A = s_{\rm twst}$

$s_B = -s_{\rm twst}$



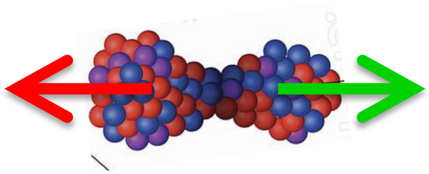
$\sigma_A = \sigma_B = \sigma_{\rm twst}$

$\sigma_{AB} = -\sigma_{\rm twst}^2$

$$M_{\rm twst} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 c_{\rm ave}^2 \qquad \boldsymbol{M}_{AB}^{\|} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 \begin{pmatrix} c_{\rm ave}^2 & -c_{\rm ave}^2 \\ -c_{\rm ave}^2 & c_{\rm ave}^2 \end{pmatrix}$$

Results for a single shape evolution:

Spin components <u>*along*</u> the fission axis



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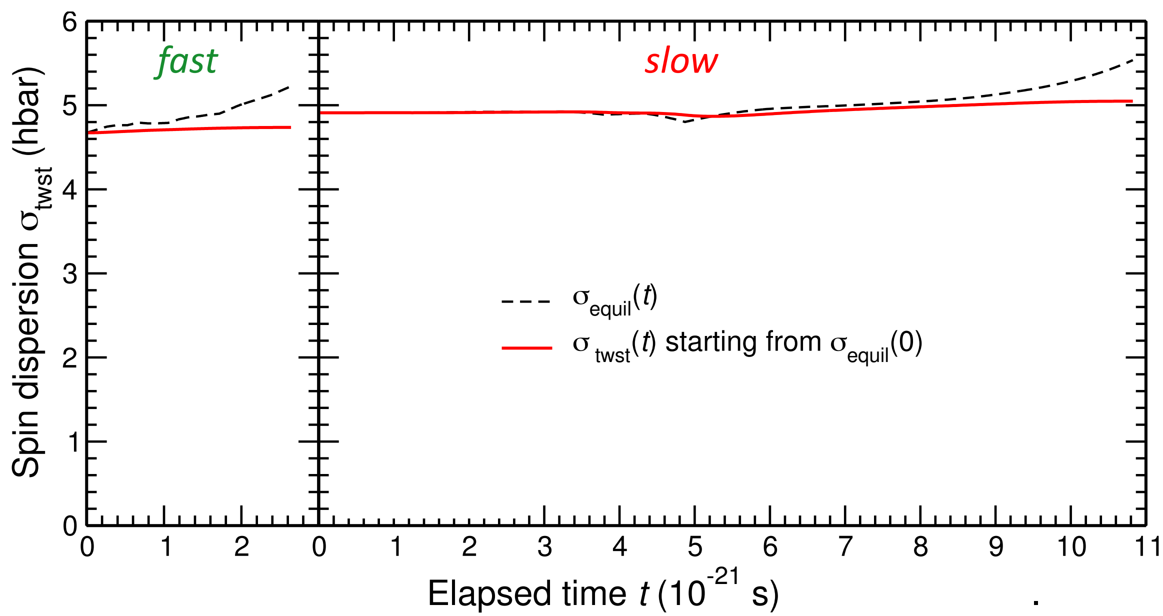


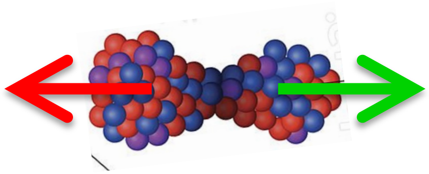
$\sigma_A = \sigma_B = \sigma_{\rm twst}$

$\sigma_{AB} = -\sigma_{\rm twst}^2$

$$M_{\rm twst} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 c_{\rm ave}^2 \qquad \boldsymbol{M}^{\parallel}_{AB} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 \begin{pmatrix} c_{\rm ave}^2 & -c_{\rm ave}^2 \\ -c_{\rm ave}^2 & c_{\rm ave}^2 \end{pmatrix}$$

Results for a single shape evolution:

Spin components *along* the fission axis



Twisting

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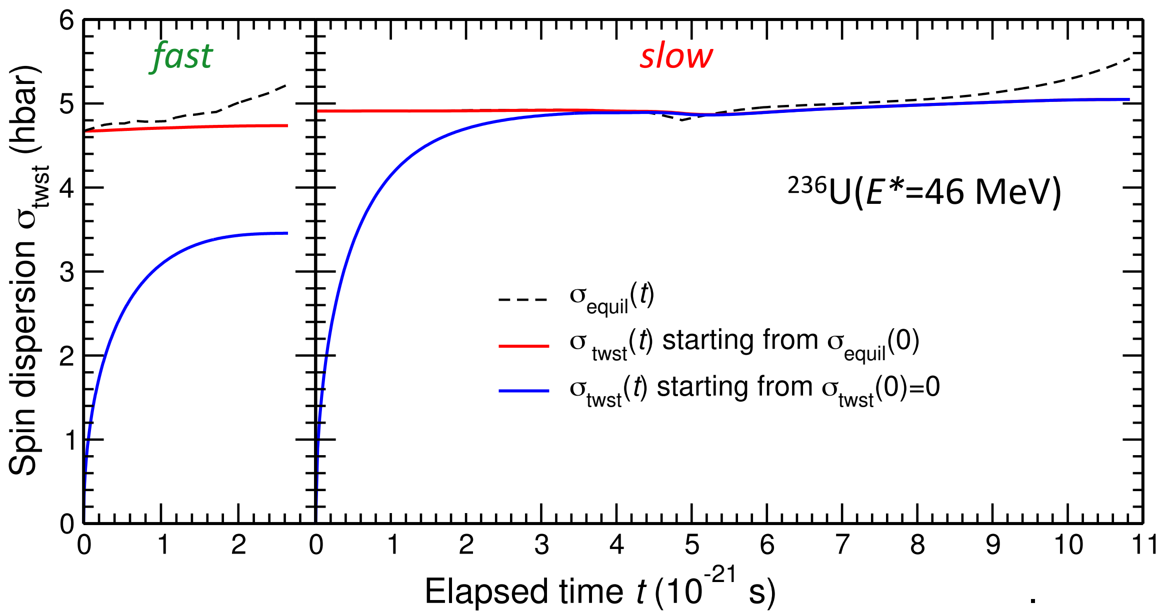
Equilibrium variance: $\tilde{\sigma}^2_{\rm twst} = \mathcal{I}_{\rm twst}T$

Individual fragments:

$s_A = s_{\rm twst}$

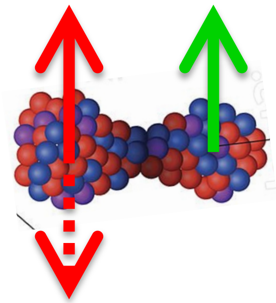
$s_B = -s_{\rm twst}$



$\sigma_A = \sigma_B = \sigma_{\rm twst}$

$\sigma_{AB} = -\sigma_{\rm twst}^2$

$$M_{\rm twst} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 c^2_{\rm ave} \qquad \boldsymbol{M}^{\|}_{AB} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 \begin{pmatrix} c^2_{\rm ave} & -c^2_{\rm ave} \\ -c^2_{\rm ave} & c^2_{\rm ave} \end{pmatrix}$$



Results for a single shape evolution:

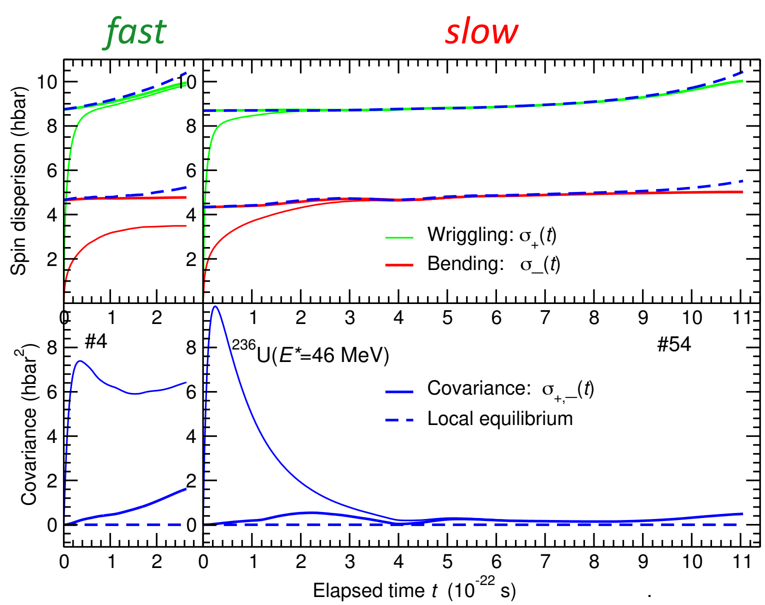
Spin components *perpendicular* to the fission axis

Wriggling & bending

$$\mathcal{I}_{\rm wrig} = \frac{(\mathcal{I}_A^\perp + \mathcal{I}_B^\perp)\mathcal{I}_R}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp + \mathcal{I}_R} \qquad \tilde{\sigma}^2_{\rm wrig} = \mathcal{I}_{\rm wrig} T$$

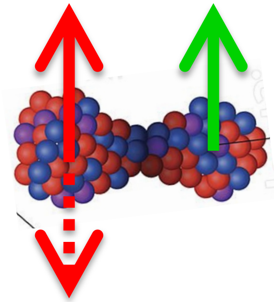
$$\tilde{\sigma}_{\rm wrig,bend} = 0$$

$$\mathcal{I}_{\rm bend} = \frac{\mathcal{I}_A^\perp \mathcal{I}_B^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp} \qquad \tilde{\sigma}^2_{\rm bend} = \mathcal{I}_{\rm bend} T$$



$$\boldsymbol{M}_\pm = \tfrac{1}{4} m\rho\bar{v}\,\pi c^2 \begin{pmatrix} R^2 & R\delta \\ R\delta & \delta^2 + c^2_{\rm ave} \end{pmatrix}$$

$$\delta = (a\mathcal{I}_B - b\mathcal{I}_A)/(\mathcal{I}_A + \mathcal{I}_B)$$ small



Results for a single shape evolution:

Spin components <u>*perpendicular*</u> to the fission axis

Wriggling & bending

$$\mathcal{I}_{\rm wrig} = \frac{(\mathcal{I}_A^\perp + \mathcal{I}_B^\perp)\mathcal{I}_R}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp + \mathcal{I}_R} \qquad \tilde{\sigma}^2_{\rm wrig} = \mathcal{I}_{\rm wrig} T$$

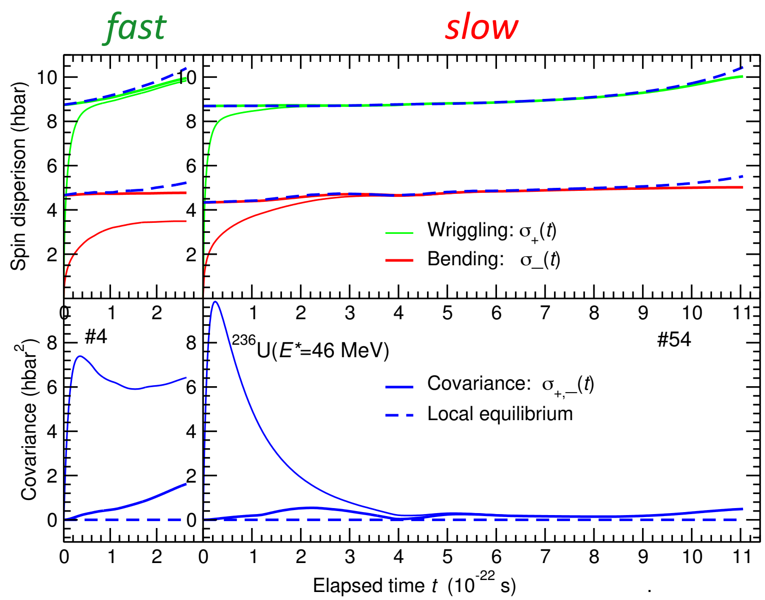
$$\tilde{\sigma}_{\rm wrig,bend} = 0$$

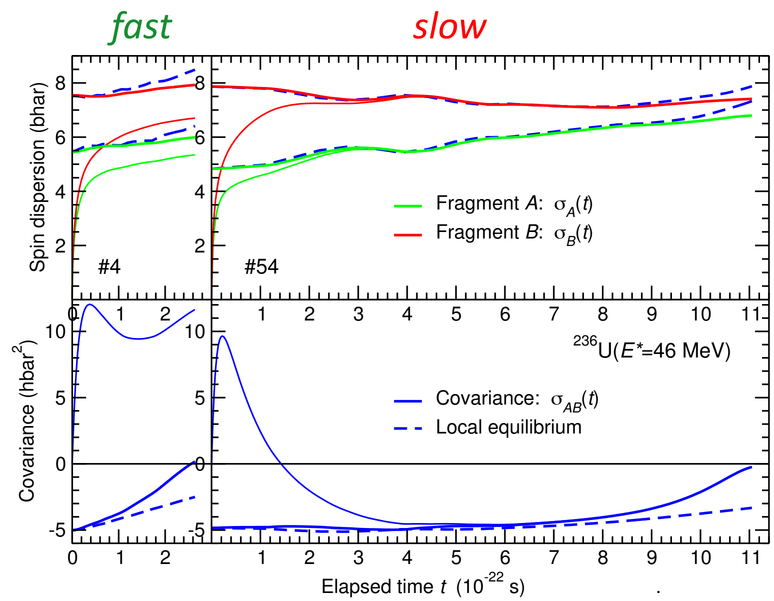
$$\mathcal{I}_{\rm bend} = \frac{\mathcal{I}_A^\perp \mathcal{I}_B^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp} \qquad \tilde{\sigma}^2_{\rm bend} = \mathcal{I}_{\rm bend} T$$

Individual fragments:

$$\boldsymbol{s}_A = \frac{\mathcal{I}_A^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp} \boldsymbol{s}_{\rm wrig} + \boldsymbol{s}_{\rm bend}$$

$$\boldsymbol{s}_B = \frac{\mathcal{I}_B^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp} \boldsymbol{s}_{\rm wrig} - \boldsymbol{s}_{\rm bend}$$

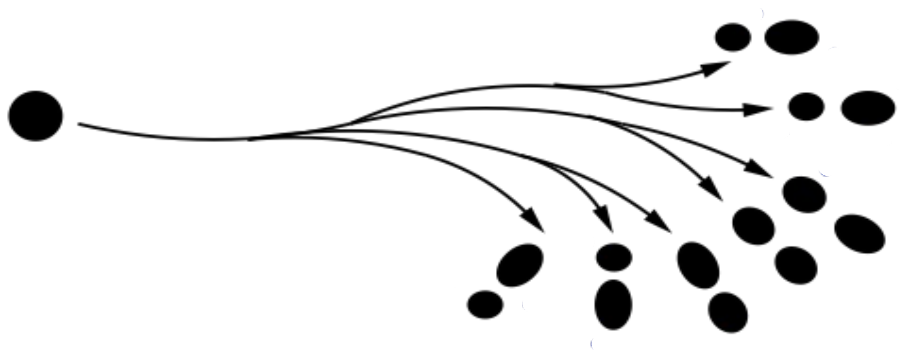




$$\boldsymbol{M}_\pm = \tfrac{1}{4} m\rho\bar{v}\,\pi c^2 \begin{pmatrix} R^2 & R\delta \\ R\delta & \delta^2 + c^2_{\rm ave} \end{pmatrix} \qquad \boldsymbol{M}^\perp_{AB} = \tfrac{1}{4} m\rho\bar{v}\,\pi c^2 \begin{pmatrix} a^2 + c^2_{\rm ave} & ab - c^2_{\rm ave} \\ ab - c^2_{\rm ave} & b^2 + c^2_{\rm ave} \end{pmatrix}$$

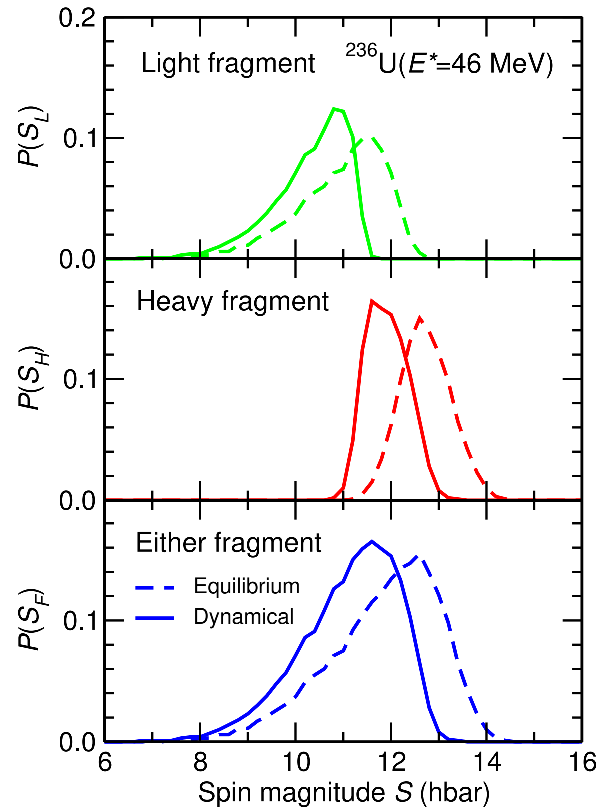
$$\delta = (a\mathcal{I}_B - b\mathcal{I}_A)/(\mathcal{I}_A + \mathcal{I}_B)$$ small

Results for an ensemble of evolutions



Results for an ensemble of evolutions:

Fragment spin magnitude

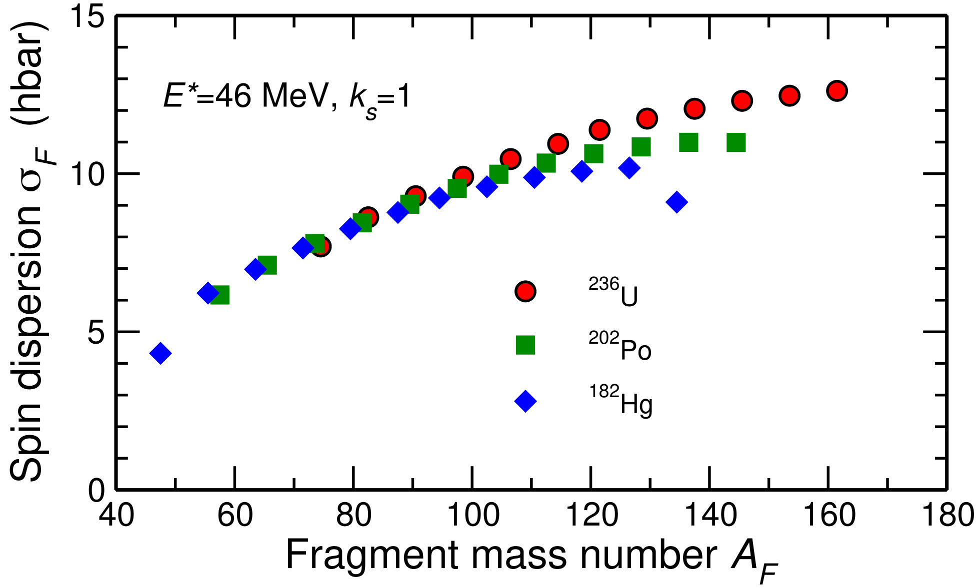


The fragment spins relax quickly to the local equilibrium values throughout most of the evolution from saddle towards scission

But near scission the spin equilibrium magnitudes grow rapidly (due to the increasing temperature), while the mobility coefficients decease (because the neck closes), so the dynamical spin evolution effectively freezes out before scission, leading to smaller magnitudes

Results for an ensemble of evolutions:

Mass dependence of spin magnitude



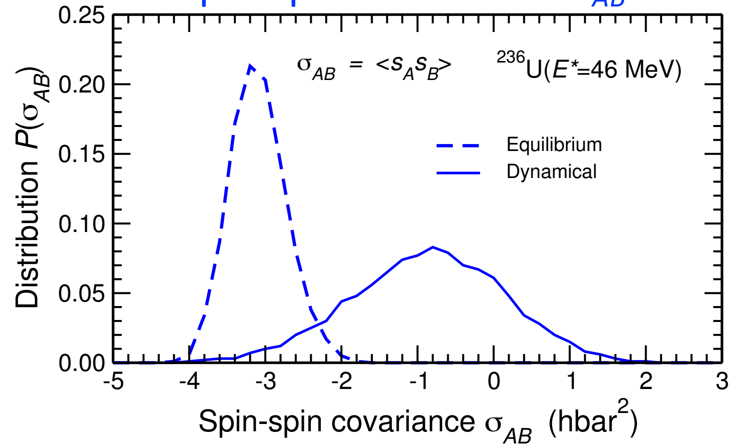
Gating on fragment mass may be informative

Can be measured!

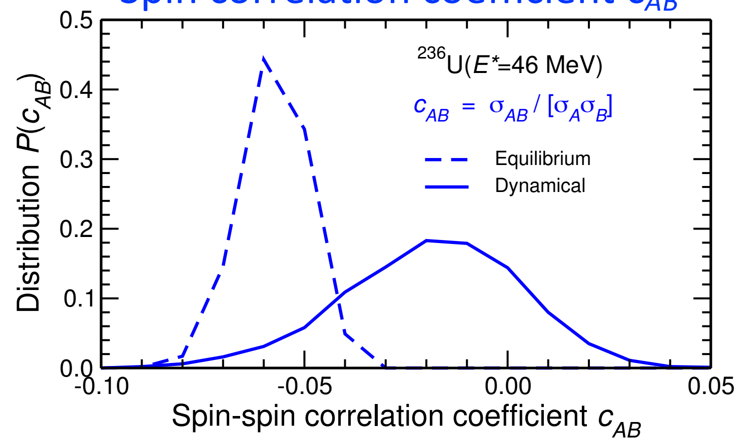
Results for an ensemble of evolutions:

Fragment spin correlation

Spin-spin covariance σ_{AB}



Spin correlation coefficient c_{AB}



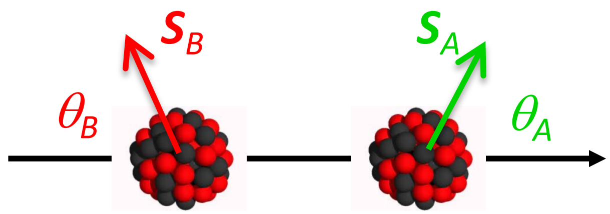
The spin-spin correlation is quite small in equilibrium
and it is even smaller when generated dynamically
=> The fragment spins are practically uncorrelated ← J. Wilson *et al.*, Nature **590** (2021)

Results for an ensemble of evolutions:

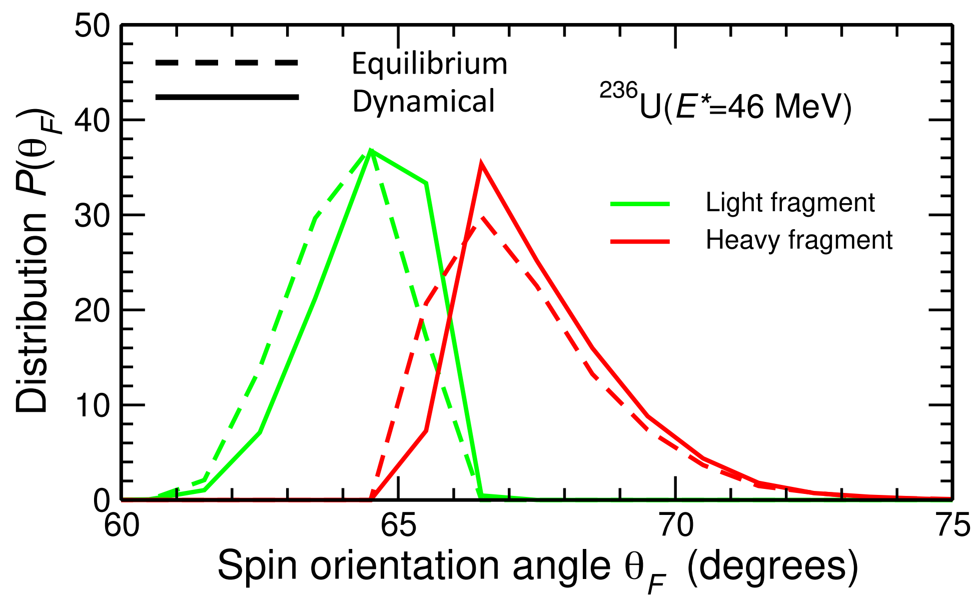
Fragment spin orientation

J.B. Wlhelmy *et al.*,
Phys. Rev. **C 5**, 2041 (1972):

The fragment spins are *approximately perpendicular* to the fission direction



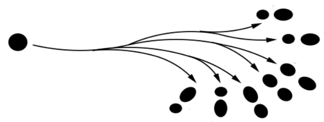
$$\cot^2\theta_F = \frac{(S_F^{\parallel})^2}{(S_F^{\perp})^2} \Rightarrow \cot^2\theta_F^{(k)} \approx \frac{(\sigma_{\parallel}^{(k)})^2}{(\sigma_{\perp}^{(k)})^2}$$



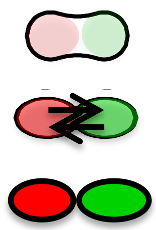
Can be measured ->
Should be measured ->
[Henrik Haug]

While the spin orientation angle θ_F is large,
it is significantly smaller than 90°,
both in equilibrium and dynamically

Effect of nucleon exchange on fission fragment angular momenta

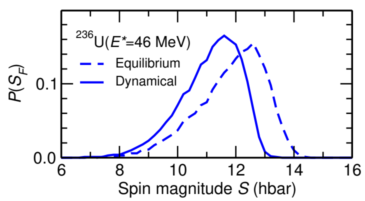


Generate ensembles of shape evolutions by Langevin simulation 10^4

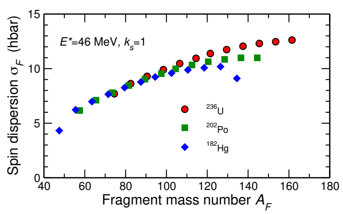


Obtain the correlated fragment spin evolution $P^{(k)}(\boldsymbol{S}_A, \boldsymbol{S}_B; t)$
In each from the *Nucleon Exchange* transport model: $\boldsymbol{M}_{AB}(t)$

Wrigling	fast
Bending	slow
Twisting	Slower

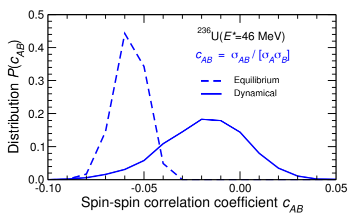


The fragment spins relax to the local equilibrium values throughout most of the evolution from saddle towards scission



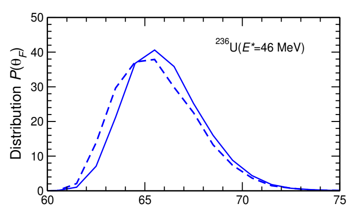
But near scission the spin equilibrium magnitudes grow rapidly (due to the increasing temperature), while the mobility coefficients decrease (because the neck closes), so the dynamical spin evolution effectively freezes out before scission, resulting in smaller magnitudes

Gating on fragment mass may be informative



The spin-spin correlation is quite small in equilibrium and it is even smaller when generated dynamically
=> The fragment spins are practically uncorrelated

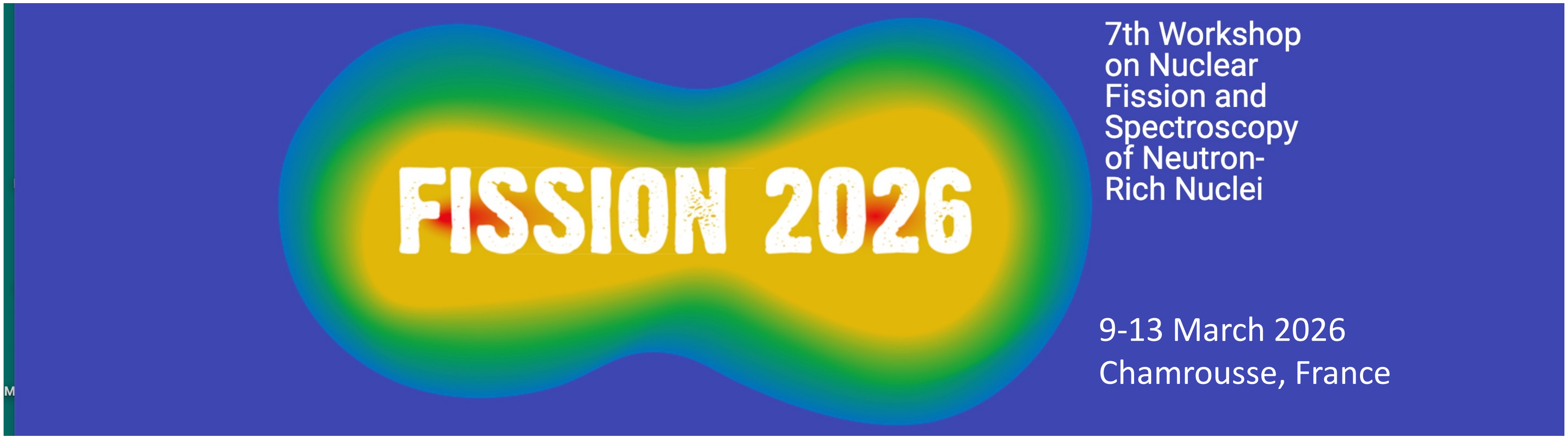
J. Wilson *et al.*, Nature **590** (2021)



The spin orientation angle θ_F is about 70°, both in equilibrium and dynamically

Can be measured!

[Henrik Haug]



Effect of nucleon exchange on fission fragment angular momenta

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Thank You!